GENERATION AND ANALYSIS OF HANDWRITING SCRIPT WITH THE BETA-ELLIPTIC MODEL

[1]HALA BEZINE, [1]ADEL M. ALIMI AND [2]NASSER SHERKAT

[1]Research Group on Intelligent Machines, National School of Engineers of Sfax, BP. W, 3038, Sfax, Tunisia. Phone: +216-74-274.088, Fax: +216-74-275.595
[2]Research Group of Intelligent Recognition and Interactive Systems School of Computing and Mathematics of Nottingham Trent University, U-K
Phone: +115-848.6032, Fax: +115-848.6518
Emails: Hala.Bezine@ieee.org; Adel.Alimi@ieee.org; Nasser.Sherkat@ntu.ac.uk

Abstract: recent developments in the field of human movements modelling supply new ways in which to view complete models for analysing and understanding complex movements.
Based on a kinematic theory and an algebraic beta-elliptic model, a new way for understanding the inherent mechanisms that govern handwriting movement generation is presented here. This paper describes an approach for analysing simple as well as complex movements such as cursive handwriting. Cursive handwriting is described as the superimposition of basic strokes with elliptic form that results from the algebraic summation of beta velocity profiles. The overall approach is based upon the hypothesis that complex human movements can be segmented into basic and simple strokes. Each stroke is totally described by a set of ten parameters that characterizes the movement both in the kinematics and the static domains.
The paper then treats the extraction of the model parameters using the beta profiles, and its application to the case of both simple rapid human movements and complex handwriting movements.
The experiments showed that the beta-elliptic model could be applied for the case of latin handwriting scripts as well as arabic handwriting scripts. New ways are proposed for the application of the beta-elliptic model such as: signature verification, style classification, shape recognition, etc…

Keywords: handwriting movement modelling, beta-elliptic model, parameters extraction, on-line handwriting.

1. INTRODUCTION

Handwriting stands among the most complex tasks presented by literate human beings.
Handwriting is affected by sensory motor control mechanisms, which are in turn influenced by emotions and communications. As such, the study of the generation of handwriting movements constitutes a very broad field that permits researches with various backgrounds and interests to collaborate and interact at multiple levels with different but complementary objectives [8, 23, 45, 74].
In the literature, the study of hand movements was based on proposed models. Several modelling techniques were used to derive motor driving signals and to simulate the handwriting process [1]. Two general methodologies of handwriting modelling become apparent from the review of the literature.
The first methodology taken into account computational models which are aimed to replicate some features of Human handwriting movements such as velocity profiles and some relations between different aspects of the movements dynamics; such as curvature and curvilinear velocity. Such methodology includes oscillators models [55, 90], which combine various velocity sinusoids to yield different movement shapes. The oscillation model of Hollerback uses kinematic parameters such as velocities and amplitude of motion to represent the handwriting movement.
Denier Van der Gon proposed a second order linear system to represent the handwriting process [26, 27].
Optimization models refer also to such methodology [31, 36, 58, 102]. Flash and Hogan [35, 36, 54] proposed that a global kinematic optimization approach- the minimum jerk model- defines a solution for the trajectory determination problem, in other words, which trajectory (hand path and velocity) should be used for handwriting movement. The solution predicts that the horizontal and vertical components of the trajectory x and y are quintic functions of time. In fact, the minimum-jerk model, takes into account only the desired hand movement, and completely ignores the dynamic properties of the arm [31, 36]. A second and more complicated approach, the minimum-torque change model [98], takes into account the dynamics of the arm, but emphasizes only the torques at the joints, and completely ignores the properties of the muscles [98]. Wada and Kawato use the torque minimization (minimum-torque-change model) as a trajectory

criterion as well as a system for choosing and optimising the number of via-points needed to regenerate a given shape [103].

The second methodology of handwriting modelling focuses on psychologically descriptive models [38]. Such methodology includes models that usually summarize many of the requirements of a handwriting system by addressing as much data as possible. Consequently they do address such issues as learning, movement memory, which are often omitted from the first methodology [20, 21, 25, 33, 34, 39, 40, 50].

The Feedback Error Learning (FEL) Neural Network [59] of Kawato et al., which when trained, learns the inverse dynamics of a controlled plant. The total control effort U applied to the plant (hand-muscle system) is the sum of the feedback control output and the network control output. The ideal configuration of the neural network would correspond to the universal mathematical model of the plant. The network is given the information of the desired position and its derivatives and it will calculate the control effort necessary to make the output of the system to follow the desired trajectory [59].

The VITE model of Bullock and Grossbergh, is a neural network model, which has been used to explain many kinematic properties of synchronous multi-joint movement, such as bell-shaped velocity profiles, peak acceleration as a function of movement amplitude [22, 41, 43].

Uno, Kawato and Suzuki (1989) extended the purely kinematic minimum-jerk model and proposed a dynamic optimization model (minimum-torque-change model) to account for wider range of human behaviours [98]. Recently, Wada and Kawato (1995) developed the FIRM (the Forward-Inverse Relaxation Model) for optimal trajectory generation and control. FIRM can generate an arm trajectory within a few iterations. It contains both the forward dynamics model and the inverse dynamics model of the controlled object [58, 103].

All these models, although succeeded in synthesizing the control pulses that drive the muscles, are rather simplified representations of the real physical model of the hand-muscle system. They are used under some assumptions and certain conditions, verifying some principles such as the 2/3 power law. In fact when human draw planar curves the instantaneous tangential velocity of the hand decreases as the curvature increases [100]. This relationship is more described as the power law, where tangential velocity is proportional to the 1/3 power of the radius of curvature. This power law was originally reported to hold mainly for elliptical movements [61]. Viviani and Schneider (1991) have reported that drawing movements are more variable for portions of an ellipse with a large radius of curvature than for portions with a small radius of curvature [99].

In this paper, we are interested in understanding handwriting generation movement at the global neuromuscular level, focusing mainly on the development of a stroke generation model which is sufficient to explain the origin of some basic psychophysical laws of simple human movements and to show how human subjects can take advantage of this representation to control the sensory motor interaction involved in the generation of more complex trajectories. The overall approach is based upon the hypothesis that complex human movements can be segmented into basic and simple components. In fact, due to the intrinsic properties of the neuromuscular systems involved in a rapid writing task, there is a class of simple movements, called strokes. The general case of complex movements is thus considered as an algebraic overlapping of simple strokes. Each stroke is totally described by a set of parameters that characterises the movement both in the kinematics and the static domains.

The notion of fundamental component of human movements generation is not new. It found its root in the Lashley experiments [62], and has been adapted to the analysis of handwriting movements in the sixties [68, 69]. Analogously, the idea of overlapping strokes to study handwriting movements has been put forward by Morasso and Mussa Ivaldi (1982), in the early eighties [71]. These latter concepts have been considered as the base upon which the handwriting generation models have been built.

A few years ago, Plamondon and Guerfali (1998) presented a handwriting model, which refers to the first methodology as mentioned previously [85]. Their model uses the “delta-lognormal synergies”. This name refers to the authors’definition of the velocity of a muscle synergy as a gaussian function of the movement parameters that varies logarithmically with time. They have been interested to the kinematic properties of handwriting generation process and omitted the relationship between kinematics and the involved handwriting trajectory. Plamondon and Guerfali (1993) suggest that stroke timing is considered as the solely crucial factor in determining the trajectory shape [83].

Using real handwriting data, it is obvious that some models perform better than others. So the decision of manipulating a proper model depends on the goal of the research. Our objective in this paper is to use the kinematics approach of rapid human movements to explain and understand the handwriting generation process. In fact, kinematic properties involved in this paper perform to joint angle trajectory, which obeys an elliptic form. The parameters characterizing an elliptic trajectory are performed according to the Beta curvilinear velocity profile [2, 5].

This article deals with two themes. Firstly, in section 2, we describe the handwriting-generated scripts in the context of kinematic and static points of view that obey both the Beta law and the elliptic form. Simple stroke movements, as well as complex handwriting movements are considered [16].

In Section 3, we are interested in the trajectory generation, where a trajectory is planned according to the arm position *(x, y)* in an intrinsic kinematic space, which depends on the musculoskeletal dynamics. In addition an elementary movement called stroke is represented by an elliptic arc having a null curvilinear velocity at the start and the end of the movement. The curvilinear velocity corresponding to this elementary movement is in turn approximated by a Beta profile in the kinematic space.

Finally, in Sections 4 and 5, we discuss the adequacy of the Beta elliptic model for the case of complex handwriting movements and examine some computer simulations. With different handwriting scripts, we show their fitting with the Beta elliptic theory, both in dynamic and statistic domains. Some applications of the Beta elliptic model are proposed in addition, such as: signature verification and shape recognition.

2. HANDWRITING GENERATION THEORY: THE BETA-ELLIPTIC MODEL

The main goal of the experiments reported in this work is to further the investigation of the constraints between trajectory and kinematics, which provide a clue to both the degrees of freedom problem and the computational complexity problem.

The handwriting generation theory is aimed essentially at understanding the generation of simple and complex handwriting movements. Several approaches have been shown in the past few years such as the delta-lognormal theory proposed by Plamondon [85].

2. 1 Simple Rapid Human Movements

The proposed model is based upon some assumptions: Firstly, it considered that handwriting movement, like any other highly skilled motor process, is partially programmed in advance. Secondly, it supposes that movements are represented and planned in the velocity domain, since the most widely accepted invariant in movement generation is the Beta shape of the velocity profiles.

As in the Delta Lognormal theory of Plamondon and Guerfali [81, 82, 84, 85], we have suggested that a neuromuscular synergy is composed of two parallel and global systems that represent, respectively, the set of neural and muscular networks involved in the generation of the agonist and antagonist activities resulting in a specific movement.

Supposing that each of these two systems is composed internally of n neuromuscular subsystems characterized by an impulse response that is real normalized and non-negative. If n is sufficiently large, applying the central limit theorem, the global impulse response can be described by a Beta function $\beta(t, t_0, t_1, t_c, p, q)$ where t_0 is the starting time, t_1 is the ending time, p and q are intermediate parameters, as shown in equation (1), [2, 3, 6, 7].

with:

$$\beta(t, t_0, t_1, p, q) = \left(\frac{t_1 - t}{t_1 - t_c}\right)^p \left(\frac{t - t_c}{t_c - t_0}\right)^q \quad (1)$$

and

$$t_c = \frac{p * t_0 + q * t_1}{p + q} \quad (2)$$

t_c is the instant where the curvilinear velocity reaches its maximum.

Exhaustive comparisons with other models using large database have been done previously, demonstrating the advantages of the Beta model for an accurate description of a velocity profile [2, 3]. Comparing to the Delta-lognormal model, it appears that the more general hypothesis of the Beta model leads to a velocity profile description that is more flexible, and therefore that can fit better the diversity of the experimental data. The Delta-lognormal function puts some constraints on the shape of the profile because it's an unbounded function [6, 46, 48]. However, the Beta function allows some variation of its symmetry, which depends on the values of the two parameters p and q.

As shown in figure 1, different Beta shapes are depicted for different values of the two parameters p and q.

A Beta shape is intrinsically asymmetric as compared, for example, with the symmetrical velocity profiles of the minimum jerk models or minimum snap [31, 36], sinusoidal functions [32, 55], or the models that use cubic splines [72].

Cubic splines functions have also been used by Sherkat N. et al. [8, 51, 52], for the approximation of the zone lines separating the writing zones. In fact, zone information has been used to classify singular points, which can be applied to a variety of handwriting recognition problems, i.e. classification of strokes and words and the verification of letter candidates. In this context, the spline-based method has been demonstrated to perform well: the superior performance is mainly caused by extending the zoning scope to lines of text instead of individual words. On the other hand, the use of cubic splines functions is constrained by normalization of word size and position and consequently recognition can be simplified by providing an image of constant size to the recognizer [13, 53, 93].

Using normalization of word size and position to the fitting of curvilinear velocity signals may decrease the dynamic information, which is contained in the curvilinear velocity signal while the potential of dynamic handwriting compared to the static handwriting is concentrated on the fruitful information that can be exploited. Dynamic data contains the information on how the shapes were written, static data conveys the result of the writing process i.e. what has been written.

Motor control research focuses on the process that produces handwriting and tries to establish the hidden parameters, e.g. [7, 9, 16, 23, 29, 42, 44, 55, 56, 59, 60, 71, 73, 75, 80, 101, 104]. Many researchers use 'significant' or 'critical' points to split the pen-path into smaller entities. Commonly used significant points are local extrema in horizontal or vertical direction [56], local extrema in velocity [16], local extrema in curvature [9, 13, 19, 24, 35, 68, 71], and points of inflexion [56]. For our case we have used both local extrema and points of inflexion in velocity profiles.

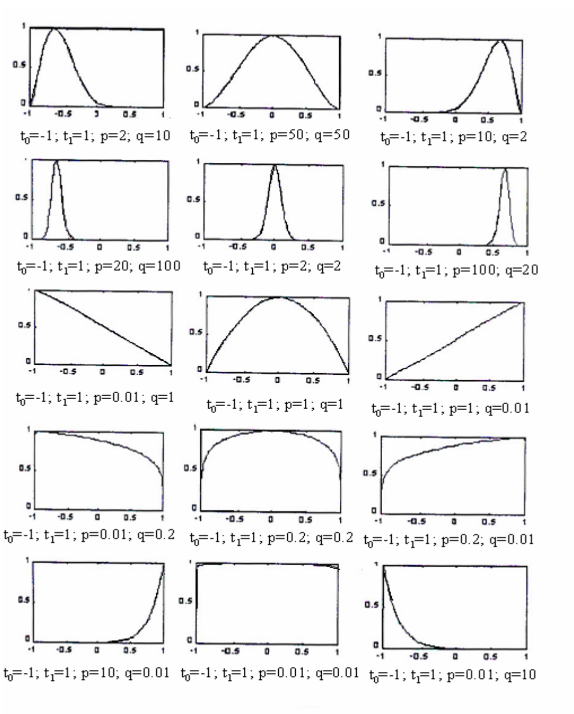


Figure 1**:** Different Beta profiles for distinct values of *p* and *q* parameters.

In this context, a stroke executed from an arbitrary starting position is characterized by five parameters. A part from these five parameters of the Beta function, each elementary component called "stroke" is also characterized in the space domain by five statistic parameters which globally reflect the geometric properties of the set of muscles and joints used in a particular handwriting movement: a and b are respectively the dimensions of the large and the small axes of the elliptic shape, x_0 and y_0 are the Cartesian coordinates of the elliptic center relative to the orthogonal reference *(o, x, y)*. The angle θ defines the deviation of the elliptic portion according to the orthogonal reference *(o, x, y)*.

Consequently, a single movement, also called stroke is represented in the space and velocity domains by a curvilinear velocity starting at time t_0 at an initial point in the domain space, and moving along an elliptic path. This elliptic path obeys a variable curvature C not a constant one as it was proposed by few models in this direction by the literature [46, 76, 77, 78, 84]. Since characters are drawn by using conic arcs and straight segments, it has been shown that the arc of circle has enough descriptive power to substitute curves of different shapes but with the same semantic value.

Freeman direction codes approximate the pen-path by small linear segments. As the segments directions are quantized, e.g. into 8, 16, or 32 distinct directions, the precision of direction codes decreases with increasing length of the polygon [37, 96].

Plamondon and Guerfali (1998) presented a handwriting model using Delta-Lognormal synergies [85, 86]. The model uses superposition of strokes toward "virtual" via-points to generate continuous curves, where a stroke is represented by a velocity vector moving along a circular path characterized by an initial orientation θ_0 and a constant curvature C_0. Circular arc representation is widely used, grouping segments of common curvature [10, 49, 75, 79, 88]. Piecewise linear modulation models are used in [24, 26], wavelets in [64].

Analysing the curvature function, we remark that: firstly, the curvilinear velocity varies with handwriting cycle: curvilinear velocity decreases for the parts of small curvature of handwriting and increases for the parts of large curvature, secondly, if the trajectory of the hand is a circle or a combination of circles, then the horizontal and vertical components of the movement are necessarily harmonic functions of equal frequency and amplitude, which is not usually verified experimentally.

The case of elliptic trajectories is particularly interesting because many portions of hand trajectories can be approximated fairly accurately by elliptic segments; a complex movement may be seen as a splined sequence of such segments [70].

As reported in the early eighties, the correlation that exists between the kinematics handwriting and the movement trajectory was formalized in terms of equation, known as the 1/3 power law. This law links the radius of curvature *R(t)* which is the inverse of curvature C and the tangential velocity *V(t)* of the handwriting movement by a 1/3 power [61].

Since then, the law has been shown to be valid for a certain class of movements and it has been slightly modified and adapted over the years to cover an even large set of movements. Numerous studies have shown that the power of 1/3 holds mainly for elliptical movements [61, 99, 100].

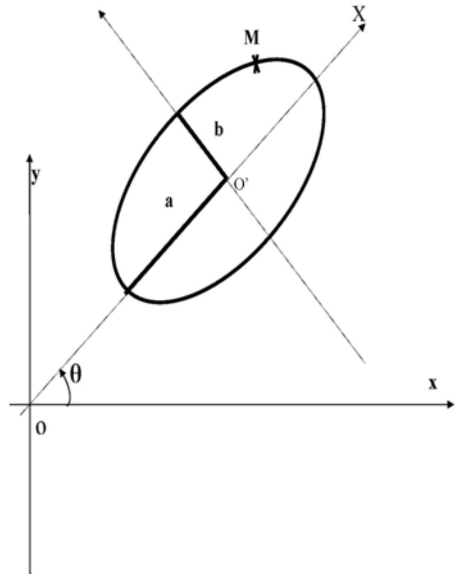


Figure 2: Descriptive scheme of an ellipse.

So we have proposed this more general structure of the contour curvature: not the circular path but an elliptic path verifying equation (3) [16, 17, 18], where *X(t)* and *Y(t)* are the cartesian coordinates of a point *M* along the elliptic stroke. *a* and *b* are the small and large axis dimensions, as shown in figure 2.

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \tag{3}$$

A point *M* having the coordinates *X(t)* and *Y(t)* verified the set of equations according to the reference *(O, X, Y)* related to the elliptic stroke (see equation (4)).
where *(x(t)*, *y(t))* are the coordinates of the point *M* according to the orthogonal spatial axis in Cartesian space *(o, x, y)* [11, 14, 15].

$$\begin{cases} X = (x - x_0)\cos\theta + (y - y_0)\sin\theta \\ Y = (x_0 - x)\sin\theta + (y - y_0)\cos\theta \end{cases}$$
(4)

Theoretically, to design an elliptic stroke, we must have at least 5 points: we are based on the dynamic aspect of this stroke, in other words the Beta velocity profile fitting the original curvilinear velocity signal [63, 65, 66].

For the case of simple handwriting movement, the five points are deduced from the Beta profile approximating the original curvilinear velocity signal; the starting point M_1 where the curvilinear velocity is near 0. The ending point M_3 is defined such as the curvilinear velocity reaches 0. The point that corresponds in the timing space to the maximum velocity characterizes the intermediate point M_2. The lasting two points M_4 and M_5 are constrained to belong to the considered Beta profile.
Obtaining the 5 points *(M_1, M_3, M_2, M_4,* and *M_5)*, the problem now is to check the different parameters characterising the elliptic segment *(x_0, y_0, a, b, θ)*, which are determined numerically by resolving the set of equations (see equation 10) [65, 66, 95, 97].
Rewrite equation (3) using the *(x, y)* coordinates, then we have:

$$[(x - x_0)\cos\theta + (y - y_0)\sin\theta]^2 / a^2$$
$$+ [(x_0 - x)\sin\theta + (y - y_0)\cos\theta]^2 / b^2 = 1$$
(5)

$$((x - x_0)^2 \cos^2\theta)/a^2 + ((y - y_0)^2 \sin^2\theta)/a^2$$
$$+ (2\,\mathrm{sos}\theta \sin\theta (x - x_0)(y - y_0))/a^2 + ((x_0 - x)^2 \sin^2\theta)/b^2$$
$$+ ((y - y_0)^2 \cos^2\theta)/b^2 - (2\sin\theta\cos\theta (x - x_0)(y - y_0))/b^2$$
$$= 1$$

(6)

$$x^2\cos^2\theta/a^2 + x_0^2\cos^2\theta/a^2 - 2xx_0\cos^2\theta/a^2$$
$$+ y^2\sin^2\theta/a^2 + y_0^2\sin^2\theta/a^2 - 2yy_0\sin^2\theta/a^2$$
$$+ \sin 2\theta/a^2 (xy - xy_0 - x_0y + x_0y_0)$$
(7)
$$+ y^2\cos^2\theta/b^2 + y_0^2\cos^2\theta/b^2 - 2yy_0\cos^2\theta/b^2$$
$$+ x^2\sin^2\theta/b^2 + x_0^2\sin^2\theta/b^2$$
$$- 2xx_0\sin^2\theta/b^2 - \sin 2\theta/b^2 (xy - xy_0 - x_0y + x_0y_0) = 1$$

$$(\cos^2\theta/a^2 + \sin^2\theta/b^2)x^2 + (\cos^2\theta/b^2 + \sin^2\theta/a^2)y^2$$
$$+ xy(b^2 - a^2)/(a^2b^2)\sin 2\theta$$
$$- x[2x_0(\cos^2\theta/a^2 + \sin^2\theta/b^2) + (1/a^2 - 1/b^2)\sin 2\theta\, y_0]$$
$$- y[2y_0(\cos^2\theta/b^2 + \sin^2\theta/a^2) + (b^2 - a^2)/(a^2b^2)\sin 2\theta\, x_0] + x_0^2(\cos^2\theta/a^2 + \sin^2\theta/b^2)$$
$$+ y_0^2(\cos^2\theta/b^2 + \sin^2\theta/a^2)$$
(8)
$$+ x_0y_0(1/a^2 - 1/b^2)\sin 2\theta - 1 = 0$$

Let us represent a general conic by an implicit second order polynomial:

$$F(x, y) = x^2 + \alpha y^2 + \beta xy + \gamma x + \delta y + \varepsilon = 0;$$
(9)

If we replace the different coordinates of the five points M_1, M_2, M_3, M_4 and M_5 in equation (9), we construct 5 linear equations, which look like:

$$\underbrace{\begin{bmatrix} y_1^2 & x_1y_1 & x_1 & y_1 & 1 \\ y_2^2 & x_2y_2 & x_2 & y_2 & 1 \\ y_3^2 & x_3y_3 & x_3 & y_3 & 1 \\ y_4^2 & x_4y_4 & x_4 & y_4 & 1 \\ y_5^2 & x_5y_5 & x_5 & y_5 & 1 \end{bmatrix}}_{(A)} \underbrace{\begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \\ \varepsilon \end{bmatrix}}_{(K)} = \underbrace{\begin{bmatrix} -x_1^2 \\ -x_2^2 \\ -x_3^2 \\ -x_4^2 \\ -x_5^2 \end{bmatrix}}_{(B)} \quad (10)$$

where $\alpha = K(1)$, $\beta = K(2)$, $\gamma = K(3)$, $\delta = K(4)$ and

$\varepsilon = K(5)$.

For the elliptic contour we may simply incorporate the constraint and impose the inequality constraint $\beta^2 - 4\alpha < 0$, then we have the 5 parameters:

$\alpha = (a^2os^2\theta+b^2\sin^2\theta)/(b^2 \cos^2\theta+ a^2 \sin^2\theta)$ (11)
$\beta= (b^2-a^2)\sin 2\theta/(b^2\cos^2\theta+a^2\sin^2\theta)$ (12)
$\gamma = -[2x_0 + (b^2 - a^2) \sin 2\theta / (b^2 \cos^2\theta + a^2 \sin^2\theta)]$ (13)
$\delta = -[2y_0 (a^2 \cos^2 \theta + b^2 \sin^2\theta)/(b^2 \cos^2\theta + a^2 \sin^2\theta) +x_0 (b^2-a^2) \sin 2\theta/ (b^2 \cos^2\theta + a^2\sin^2\theta)]$ (14)
$\varepsilon = x_0^2 + y_0^2 (a^2 \cos^2\theta + b^2 \sin^2\theta)/(b^2 \cos^2\theta+ a^2 \sin^2\theta) + x_0y_0 (b^2 - a^2) \sin 2\theta / (b^2\cos^2\theta +a^2 \sin^2\theta)$ (15) $- a^2b^2 / (b^2 \cos^2 \theta + a^2 \sin^2\theta)$

If we consider $m= b/a$, we get from equations (11) and (12):

$\alpha = (\cos^2\theta + m^2 \sin^2\theta) / (m \cos^2\theta + \sin^2\theta)$. (16)

$m^2 = (\cos^2\theta - \alpha \sin^2\theta) / (\alpha \cos^2\theta - \sin^2\theta)$. (17)

$\beta = (m^2 - 1) \sin 2\theta / (m^2 \cos^2\theta +\sin^2\theta)$. (18)
$= (1 - \alpha) \sin 2\theta / (\cos 2\theta)$.

and the deviation angle θ will be:

$\theta = ½ \text{Atn} (\beta / (1 - \alpha))$. (19)

The center coordinates (x_0, y_0) of an elliptic contour are deduced from equation (13) and (14):

$x_0 = -\delta\beta + 2 \alpha\gamma / (\beta^2 - 4\alpha)$ (20) $y_0 = -\gamma\beta + 2 \delta / (\beta^2 - 4\alpha)$ (21)

The axis lengths a and b of an elliptic arc are deduced by using equation (15).

$a = ((x_0^2 + \alpha y_0^2 +\beta x_0y_0 - \varepsilon)(m^2\cos^2 \theta+\sin^2 \theta))^{1/2} /m$ (22)
$b = ((x_0^2 + \alpha y_0^2 +\beta x_0y_0 - \varepsilon)(m^2\cos^2\theta +\sin^2 \theta))^{1/2}$ (23)

The solution of equations system (10) subject to the constraint $\beta^2 - 4\alpha < 0$ admits exactly one elliptical solution which provides the set of parameters (a, b, x_0, y_0, θ).

In our case, a simple stroke is approximated by a beta profile in the dynamic domain which corresponds in turn to an elliptic arc in the static domain such that M_1M_3 is the large axis dimension a. As reported by Viviani and al. for human drawing curves, that the instantaneous tangential velocity of the hand decreases as the curvature increases [98], and then M_1 and M_3 which are characterized by minimum tangential velocity correspond to the maximum of curvature in the static domain.

Consequently, a stroke is characterized by ten parameters; the first five parameters reflect the global timing properties of the neuromuscular networks involved in generating the movement, whereas the last five parameters describe the global geometric properties of the set of muscles and joints recruited to execute the movement.

Figure 3 shows a typical one-stroke (solid line) as reproduced by the Beta-elliptic model. The curvilinear velocity relative to this stroke depicted in figure 4, provides a typical example of a single movement, within single-peak velocity, which might be reproduced with a two set of parameters. On each graph, the predictions of the kinematic theory are reported using dashed lines imposed on the continuous lines that extrapolate the original data, sampled at 200 Hz, by using a digitizer tablet type Wacom 4.

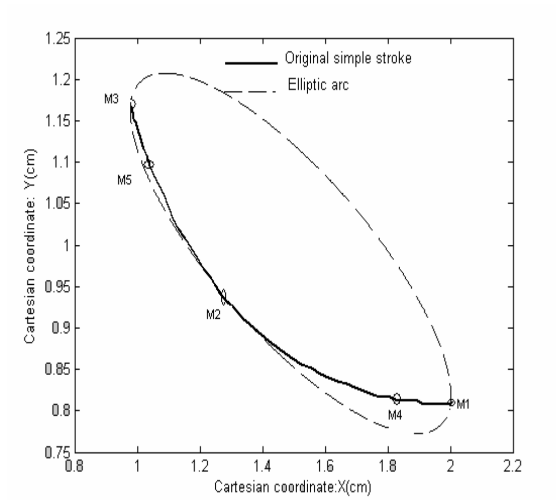


Figure 3: Approximation of simple handwriting trajectory with elliptic contour.

As can be seen, the Beta-elliptic theory is able to reproduce a stroke, both in the image and in the kinematic domain. Similar results have been reported over different databases for more than 5000 strokes in all.

2. 2 Complex Handwriting Movements

As shown previously, the Beta-elliptic model considers a simple movement as the response of the

neuromuscular system, which is described by an elliptic trajectory and a Beta velocity profile.
The serial nature characterizing the central representation of a movement is also detectable from a kinematic point of view during the generation of the movement in the external environment, where the velocity profile of the end effector is decomposable into a series of basic, approximately Beta, asymmetric profiles overlapping each other as a function of the speed of the whole movement [57, 59, 68, 74].
Then the generation of handwriting scripts can be viewed as an algebraic summation in time of different strokes, such expressed in equation (24), where *n* stands for the total number of different Beta shape.

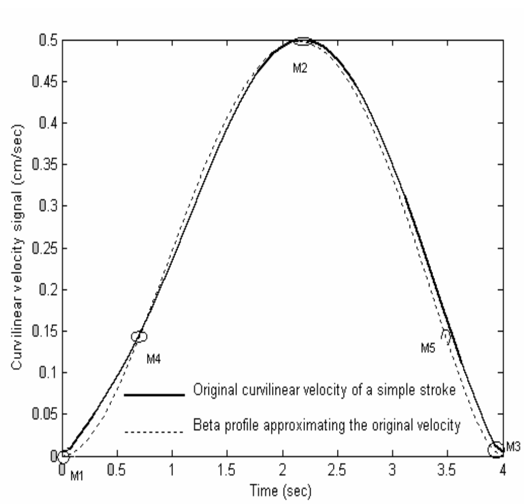


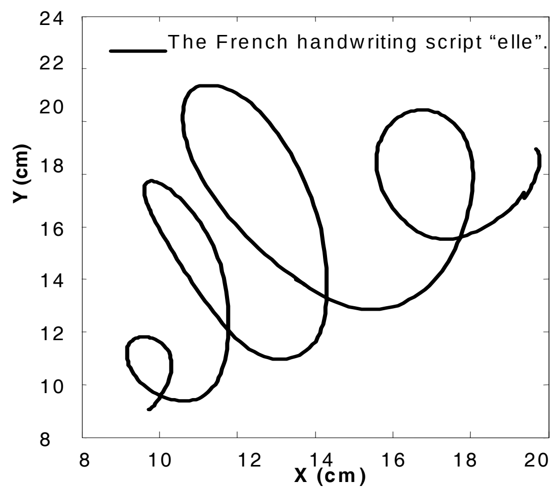
Figure 4: Approximation of curvilinear velocity signal with Beta model.

$$V(t) = \sum_{i=1}^{n} \beta_i(t) \qquad (24)$$

Superimposition of elementary strokes is a common assumption among modelling of handwriting (e.g. Morasso and Mussa Ivaldi 1982, Edelman and Flash 1987, Plamondon 1989, Schomaker 1989) [30, 74, 75, 91]. Models differ in the constraints they place on stroke superimposition. Schomaker et al. 1989 [92], as well as Plamondon assumes essentially arbitrary timing relation between onsets of overlapping movement strokes [80]. Whereas, Morasso et al (1982), as well as for our case, constrain stroke superimposition by limiting the number of strokes that are concurrently executed to two [16, 17, 18, 71].
A simulated case of a complex handwriting script is depicted in figures 5A and 5B.

For the case of the figure 5A, we can see the original handwriting script, which is the French word "elle" ("she" in English), as generated by a human subject. The curvilinear velocity signal associated to this real data is depicted in figure 5B. As previously, the original data was acquired by using a digitizer tablet type Wacom4, sampled at 200 Hz. In figure 5B, we remark, the presence of an inflexion point which will participate in the modelling process by adding a supplementary Beta profile.
In Plamondon 98, the resulting curvilinear velocity of a handwriting trace, for a sequence of strokes is obtained by summing the vectors representing each individual stroke curvilinear velocity [85].

(A)



(B)

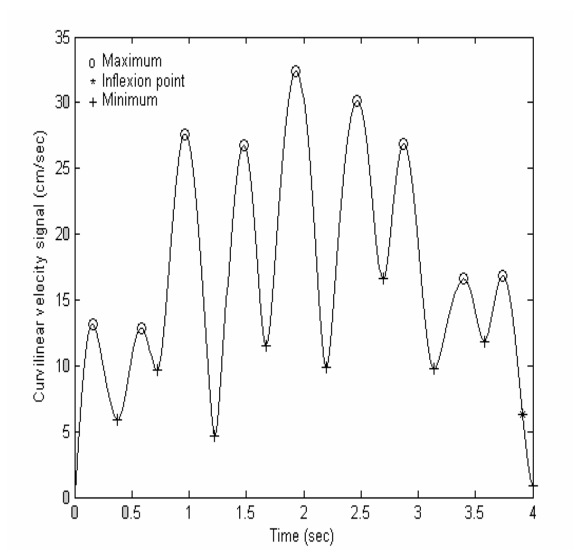


Figure 5: (A) The French handwriting script "elle".
(B) The curvilinear velocity signal related to the French handwriting script "elle".

The same idea is applied for the case of the angular velocity: the resulting angular velocity is obtained by summing the vectors representing each individual angular velocity stroke.
In order to eliminate the phase shift between angular and curvilinear velocity, which is implicit in the delta-lognormal model, the vectorial summation was investigated.
For our case, we assume that handwriting movements are synthesized simply by using curvilinear velocity movements generator.
A handwriting trace is segmented into curvilinear strokes, which are partially hidden in the signal. We are based on the Beta-law [2, 3] which describes the curvilinear velocity profile of a single stroke, and on strategies for the superimposition of strokes, without the investigation of angular velocity and the notion of vectorial velocity summation.
A stroke is defined as a trajectory between two extrema curvilinear velocity (minimum and maximum) and the inflexion points are considered as maximum of curvilinear velocity. Every two consecutives strokes are superimposed and the resulting curvilinear velocity signal is obtained by summing the Beta profiles representing each individual stroke curvilinear velocity.

3. SIMPLE MOVEMENT ANALYSIS

To extract the set of Beta model parameters that best fits the handwriting movements under study, an analysis-by-synthesis procedure was used. Obviously, the extracted parameters reflect the temporal properties of the neuromuscular networks involved in the handwriting movement and the geometric characteristics of the movement are analysed [7, 9, 47, 75].
The extraction of the set of parameters that best fits a simple movement *i* called also one stroke might be done in two steps: firstly, we have to estimate the parameters that describe the kinematics of the considered movement: $t_{o(i)}$, $t_{1(i)}$, $t_{c(i)}$, $p_{(i)}$ and $q_{(i)}$; and secondly, is to estimate the static parameters which are: $a_{(i)}$, $b_{(i)}$, $x_{0(i)}$, $y_{0(i)}$ and $\theta_{(i)}$. These earlier describe the geometric properties of the movement in 2 dimensional space. Having obtained this set of ten parameters, a proper simple movement called stroke can be identified and completely characterized in both the kinematic and spatial domains.
In order to use the optimal set of parameters that best fit the considered movement, we have to use some optimization methods that guarantee algorithm convergence. Because of the non linearity of the Beta function, non linear regression techniques are required to extract the optimal parameters from the original curvilinear velocity signals. Several techniques exist to solve non linear regression problems, for our case, we are limited to a brief over view of one of the most popular techniques [46, 67].

3. 1 Non Linear Regression Method

Considering a regression problem, first of all we have a dependent variable *y* with *n* measures $(y_1, y_2, \ldots\ldots, y_n)$. For our case *y* is identified by the curvilinear velocity signal, that earlier depends on *k* independent variables $(x_1, x_2, \ldots\ldots, x_k)$, (the curvilinear velocity depends only on the time *t* in our case) [12].
Then the relationship between the variable *y* and the independent variables x_i is determined by a class of functions (a Beta function), which depends on *m* parameters, which are (t_0, t_1, t_c, p, q) [86, 89, 94].
The regression techniques look for the set of functions parameters that best fits the measures according to some criteria: like the least squares fit, for example [46]. When the class of functions is nonlinear according to some of its parameters and no easy transformation is known to linearize the function, which is the case for the Beta function, non linear regression techniques might be used. Based on the derivability of the considered function, optimization methods can be used, such as the Levenberg-Marquardt [67, 94].
Around a set of initial conditions of the parameters, an iterative non linear regression algorithm was used to find the least square estimate of the parameters for the tuning functions given by equation (25):

$$F = V(t) - \sum_{i=1}^{n} \beta_i(t) \tag{25}$$

where the resulting velocity *V(t)* of the pentip, for a sequence of *n* strokes is obtained by summing the Beta profiles representing each individual stroke velocity.
An iterative process is then necessary until the error becomes smaller than a certain fixed value, or a certain amount of computational time has exceeded [17, 46, 49].

3. 2 Estimation of the Spatial Parameters

To check the spatial parameters related to the elliptic trajectory that best fit the simple movement called "stroke" and referring to figure 4, we have used this algorithm:

1. According to the Beta profile checked in the kinematic domain, we have to determine the two parameters $t_{0(i)}$ and $t_{1(i)}$ which correspond both to the curvilinear velocity near to zero.
2. Having obtained the two instants: $t_{0(i)}$ and $t_{1(i)}$, we determine their equivalent in the space domain respectively $M_{1(i)}$ and $M_{3(i)}$, in other words the starting time and the ending time of the stroke.
3. $M_{2(i)}$ corresponds to the instant $t_{c(i)}$ where the curvilinear velocity is maximum.
4. $M_{4(i)}$ and $M_{5(i)}$ are constrained to belong to the same Beta profile β_i, in the timing domain,

respectively the ascendant part and the descendant part of the considered Beta profile. In practical situations, $M_{4(i)}$ and $M_{5(i)}$ correspond respectively to the quarter of the maximum curvilinear velocity. The curvature of a generated elliptic arc is enormously affected by the choice of both $M_{4(i)}$ and $M_{5(i)}$.

Comparing to ($M_{1(i)}$, $M_{2(i)}$ and $M_{3(i)}$), $M_{4(i)}$ and $M_{5(i)}$ have the most effect on the variation of the generated elliptic stroke curvature. If $M_{4(i)}$ and $M_{5(i)}$ are respectively far from $M_{1(i)}$ and $M_{3(i)}$, the curvature of an elliptic generated arc increases.

5. Having located these 5 points ($M_{1(i)}$, $M_{2(i)}$, $M_{3(i)}$, $M_{4(i)}$, and $M_{5(i)}$), described above, the five parameters defining the elliptic trajectory: a, b, x_0, y_0, and θ can be evaluated by resolving the set of equations defined by equation (10). Then we will have the different characteristics of an elliptic arc, as searched previously in section 2.

$$a = \sqrt{(x_0^2 + \alpha y_0^2 + \beta x_0 y_0 - \varepsilon)(m^2 \cos^2\theta + \sin^2\theta))/m}$$

$$b = \sqrt{\left(x_0^2 + \alpha y_0^2 + \beta x_0 y_0 - \varepsilon\right)\left(m^2 \cos^2\theta + \sin^2\theta\right)}$$

$$x_0 = -\,\delta\beta + 2\alpha\gamma \,/\, (\beta^2 - 4\alpha)$$

$$y_0 = -\,\gamma\beta + 2\delta \,/\, (\beta^2 - 4\alpha) \qquad (26)$$

$$\theta = ½\, \mathrm{Atn}\,(\beta / (1 - \alpha))$$

In practical situations, depending on the location of the points $M_{4(i)}$ and $M_{5(i)}$, multiple solutions of elliptic trajectories can be obtained.

4. HANDWRITING MOVEMENT ANALYSIS

The analysis of complex movements such as handwriting poses more difficulties than that of simple movements. These difficulties are related to the fact that complex movements are composed of time-overlapped strokes. So we have firstly to estimate the number of strokes that might be involved to generate the observed complex movement. In other words we have to check the Beta profiles that compose the curvilinear velocity signal [17, 18]. For this purpose we have to search the different extrema: minima and maxima related to the curvilinear velocity signal. The inflexion points are considered as maxima and the number of Beta profiles is equal to the total of maxima points.

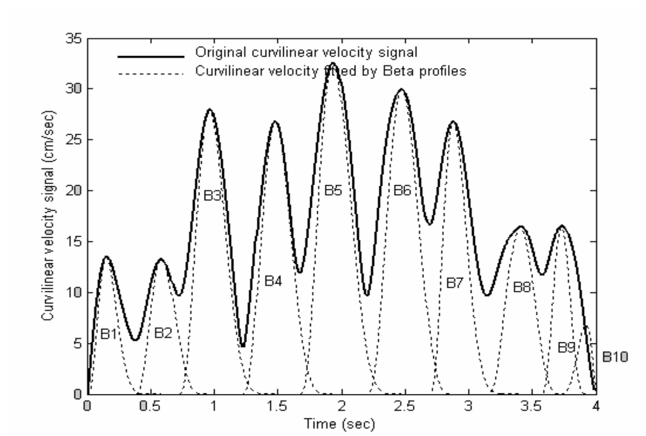
The second problem is related to the optimization of the parameters according to the different strokes. The complexity of this problem grows rapidly when the number of strokes increases, so the convergence problems become more serious [46, 67].

The development of robust parameter extraction techniques for the general problem of multiple strokes extraction presents several difficulties. One of the major difficulties is to ensure the convergence of numerical non-linear regression techniques when there are a large number of parameters to estimate. Furthermore, these numerical methods require initial conditions for the iterative procedure, so a judicious choice of initial conditions is required [67, 87].

Usually, handwriting analysis has been limited to direct measurements x and y sampled at a fixed frequency from digitizing tablets, such for our case.

The overlapping effect was taken into consideration; it is observed each time a particular simple movement *(i)* starts before another simple movement *(i-1)* achieves its final target. Very different shapes can be generated with the same basic movements by changing only the starting time of the second stroke. For the most cases, the starting time of a particular movement *(i)* correspond to a dip in the original curvilinear velocity, when the particular movement *(i-1)* curvilinear velocity is near *0*. If there exist two consecutives maxima -because of the presence of an inflexion point- the starting time of a particular movement *(i)* is equal to the time when the particular movement *(i-1)* reaches its maximum curvilinear velocity.

(A)



(B)

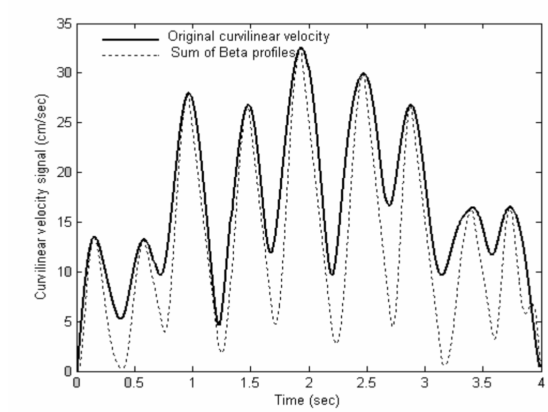


Figure 6: (A) The curvilinear velocity signal related to the French handwriting script “elle” approximated

by Beta profiles: a Beta profile for each single stroke.
(B) The curvilinear velocity signal related to the French handwriting script “elle” approximated by Beta profiles: the summation of the different Beta profiles.

According to our model, each handwritten script is generated by using the curvilinear velocity signal fitted by Beta profiles. Each handwritten script is thus made up of a set of strokes that are superimposed in time to produce the rapid movement.

Figure 6 and 7 show the effect of the time overlapping of multiple stroke movements on the resulting curvilinear velocity and path corresponding to the French word “elle”.

In figure 6A and 6B, the dashed lines show the hidden trajectory of each single stroke as taken separately and the continuous line shows the original curvilinear velocity signal related to the French word “elle”.

For the case of the figure 6A, we can see, the Beta velocity profiles for each single stroke (squared lines). Every stroke is characterised by its own specific activation and target time respectively: $t_{0(i)}$ and $t_{1(i)}$.

For two consecutive strokes, each one having its own specific activation and target time, the increase in the velocity of the second stroke compensates for the decrease in the velocity of the first stroke during the time shift of $t_{1(i)}$ - $t_{0(i+1)}$ between the two strokes.

The number of Beta profiles is equal to the maxima points observed in figure 5B. Then, the summation of the different Beta profiles of the entire movement is depicted in figure 6B (squared lines).

Figure 7A shows the different ellipses associated to the different strokes of the handwriting script “elle”. As we can see, different sizes of ellipses are depicted on this figure. This fact proved the variation of curvature during the handwriting process, then the variation of handwriting velocity.

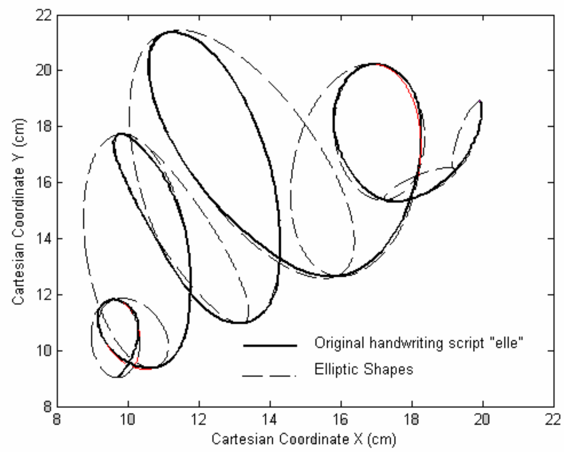
In figure 7B, we have elaborated the original handwriting script “elle” with the different elliptic strokes; we can remark that the different elliptic strokes (dashed lines) are superimposed on the original script.

In this context, we have used the following procedure to generate a handwriting trace with elliptic arcs. Consider a handwriting trace, which have a curvilinear velocity signal $V(t)$ that can be approximated by an algebraic sum of Beta profiles as shown in figure 6. To generate the elliptic arcs related to the continuous trace, which is the French word “elle” -as shown in figure 7B- we have used the different Beta profiles fitting the original curvilinear velocity. We have *10* Beta profiles from $n=1$ to $n=10$.

The ascendant part of the first Beta profile $Beta_{1A}$ will generate solely the first elliptic arc started at the first letter “*e*”. $M_{2\ (1)}$ corresponds to the instant where $Beta_1$ reaches the maximum curvilinear velocity in the timing domain.

In addition to the ascendant part of the second Beta profile $Beta_{2A}$, the descendant part of the first Beta profile $Beta_{1D}$ will partially contribute to the generation of the first overlapped arc for the period of time that corresponds to the time shift of $t_{1(i)}$ - $t_{0(i+1)}$. Thus for the time shift between two consecutive Beta profiles, two elliptic strokes are executed and the mean of both will approximate the original handwriting script for that period of time.

(A)



(B)

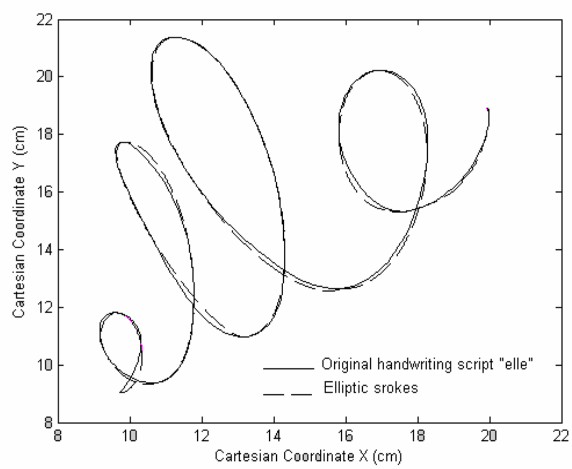


Figure 7: (A) The French handwriting script “elle” approximated by elliptic portions-The different ellipses are designed.
(B) The French handwriting script “elle” approximated by elliptic portions-The different elliptic arcs are designed.

In other words two portions of different Beta profiles, respectively $Beta_{(i)D}$ and $Beta_{(i+1)A}$ are partially integrated for the generation of the original handwriting script: each portion will generate an elliptic arc and the mean of both fit the original handwriting script during the time shift between them. Each handwritten component is thus made up of a set of strokes that are overlapped in time to produce a fluent trace. According to the model, these strokes constitute the basic units of human writing movements and serve as the coding elements in the motor planning of complex trajectories.

(A)

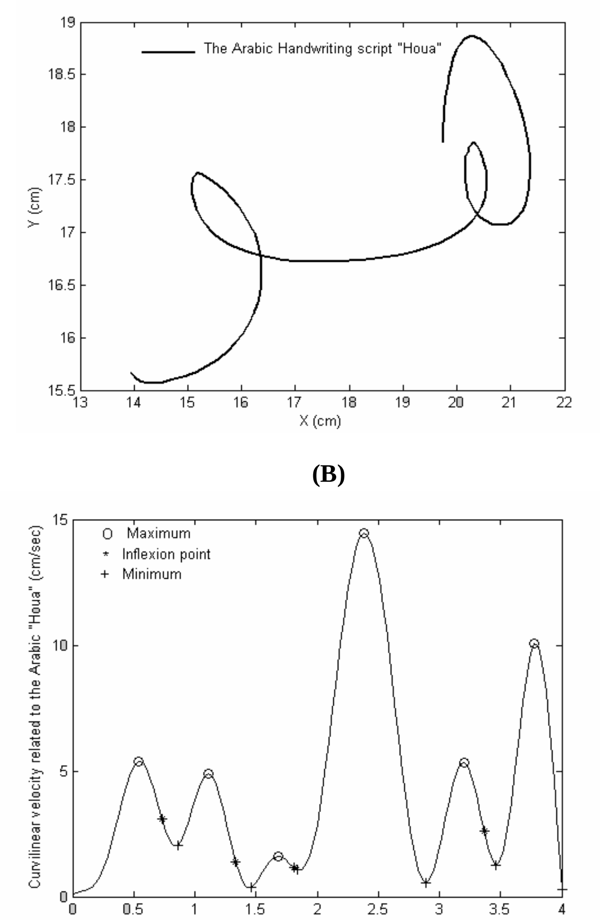


Figure 8: (A) The Arabic handwriting script "Houa".
(B) The curvilinear velocity signal related to the Arabic handwriting script "Houa".

- Firstly, stroke superimposition is constrained by limiting the number of strokes that are concurrently executed to two,

- Secondly, the time shift is fixed to $t_{1(i)}$ - $t_{0(i+1)}$ because of the existence of hidden strokes and mainly the existence of inflexion points.

The same procedure of modelling was applied with a second example of handwritten script, which is the word "Houa" in Arabic, as shown in figure 8.
Figure 8A, shows the original handwriting script, as it was acquired from a digitizer tablet type Wacom4 with a sampling frequency of 200Hz.
The curvilinear velocity signal relative to this earlier handwriting script "Houa" is depicted in figure 8B. We remark the presence of inflexion points which will participate to add further Beta profiles, as shown in figure 9A.

(A)

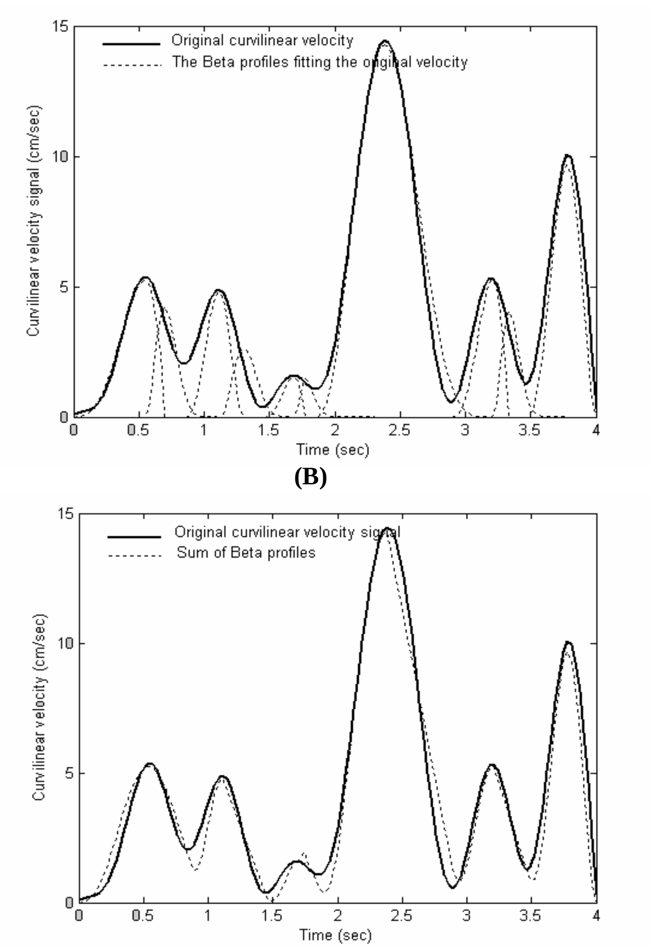
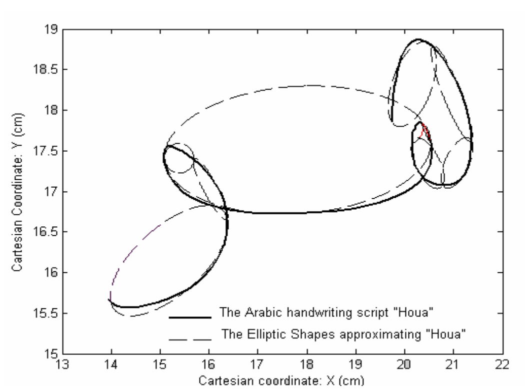


Figure 9: (A) The curvilinear velocity signal related to the Arabic handwriting script "Houa" approximated by Beta profiles: a Beta profile for each single stroke.
(B) The curvilinear velocity signal related to the Arabic handwriting script "Houa" approximated by Beta profiles: the summation of the different Beta profiles.

In figure 9A, we have the different Beta profiles approximating the original curvilinear velocity, which are depicted (dashed lines). Analysing these earlier, we remark that the original curvilinear velocity is dynamically fruitful: we have *4* inflexion points. The overlapping effect was taken into consideration; it's observed that the starting time of a particular Beta profile *(i)* depends on the presence of the inflexion points: the activation time of a Beta profile may corresponds to a dip in the original curvilinear velocity and it may corresponds to the time when a previous Beta profile *(i-1)* reaches its maximum.

The summation of the different Beta profiles is depicted in figure 9B (dashed lines). We remark that the approximated sum is related to the optimization of the parameters according to the different strokes. The complexity of this problem grows rapidly when the number of strokes increases.

(A)



(B)

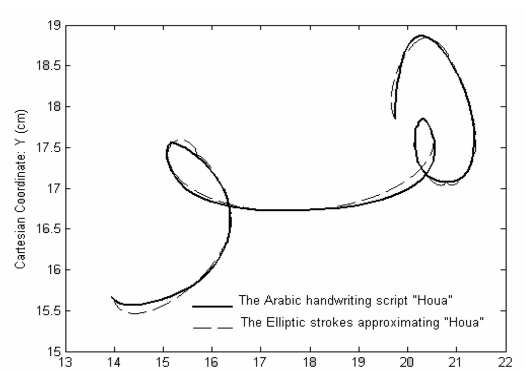


Figure 10: (A) The Arabic handwriting script "Houa" approximated by elliptic portions -The different ellipses are designed.

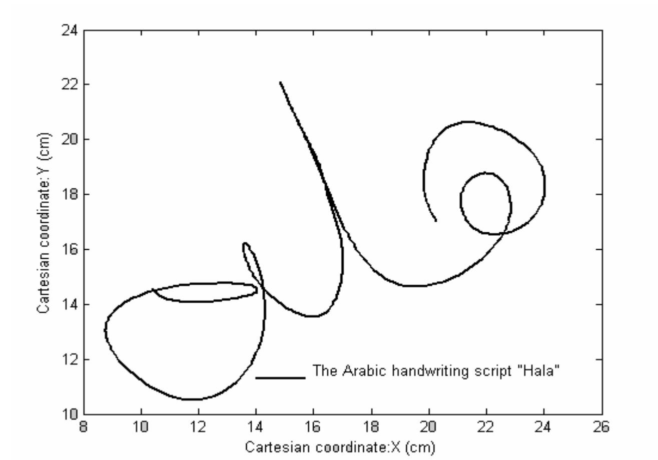
(B) The Arabic handwriting script "Houa" approximated by elliptic portions-The different elliptic arcs are designed.

The same procedure of modelling was applied with a third example of handwritten script, which is the word "Hala" in Arabic, as shown in figure 11.

Figure 11A, shows the original handwriting script, as it was acquired from a digitizer tablet type Wacom4 with a sampling frequency of 200Hz.

The curvilinear velocity signal relative to this earlier handwriting script "Hala" is depicted in figure 11B. Comparing to the last example of the Arabic word "Houa", we remark that the curvilinear velocity signal related to the handwritten script "Hala" contains more discontinuities.

(A)



(B)

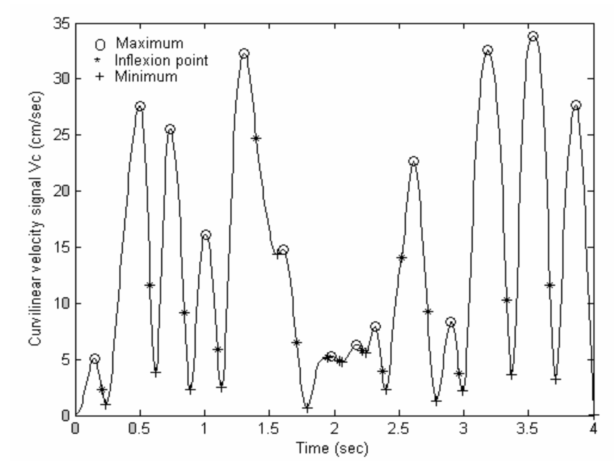


Figure 11: (A) The Arabic handwriting script "Hala".

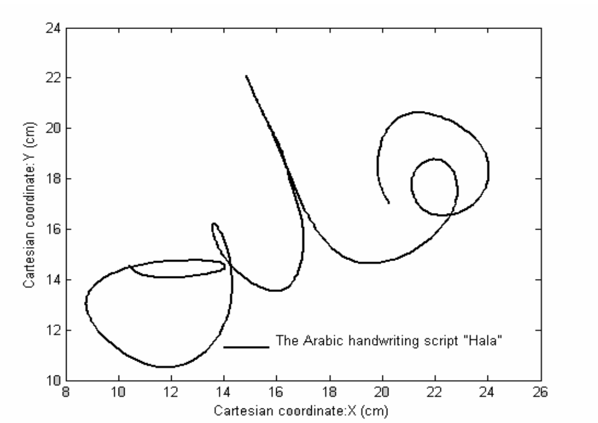
(B) The curvilinear velocity signal related to the Arabic handwriting script "Hala".

The velocity amplitude varies with time, which reflects the form of the generated letter during the handwriting process. The minimum of velocity corresponds in the space domain to the letter "Alif".

In figure 12A, we have the different Beta profiles approximating the original curvilinear velocity relative to the word "Hala", which are depicted (dashed lines). Analysing these earlier, we remark that the original curvilinear velocity is dynamically fruitful: every maximum is succeeded by an inflexion point and the number of curvilinear strokes will be multiplied.

In figure 12B, we have elaborated the sum of the different Beta profiles. We can remark that the Sum obtained for the case of the word "Houa" is more satisfactory, which could be explained by the increase of the strokes number and the fruitful dynamics.

(A)



(B)

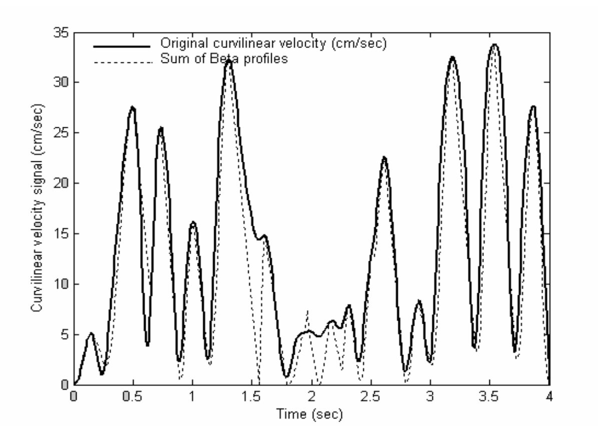


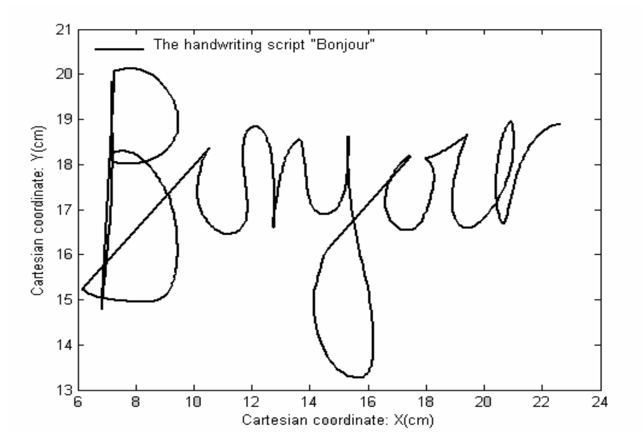
Figure 12: (A) The curvilinear velocity signal related to the Arabic handwriting script "Hala" approximated by Beta profiles: a Beta profile for each single stroke.

(B) The curvilinear velocity signal related to the Arabic handwriting script "Hala" approximated by Beta profiles: the summation of the different Beta profiles

In figure 13A, we remark the different ellipses approximating the original handwriting script "Hala". Using the decomposition scheme, different elliptic strokes are depicted in figure 13B. These strokes are not apparent directly in the image of the handwriting script "Hala". They are partially hidden in the trajectory as a consequence of superimposition process.

Despite the presence of the inflexion points, an important error between the original handwritten trace and the 7^{th} stroke was observed in figure 13B, compared to the other strokes.

(A)



(B)

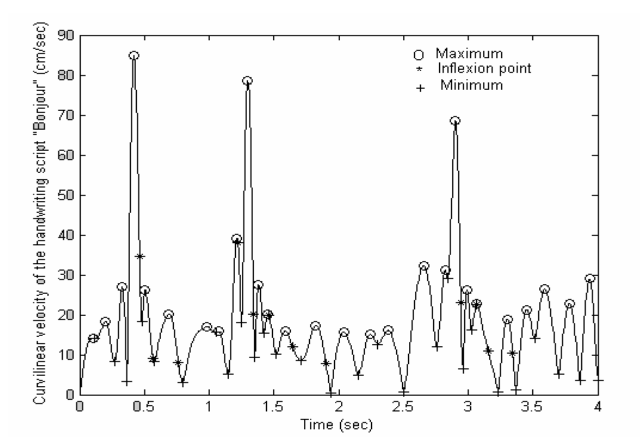


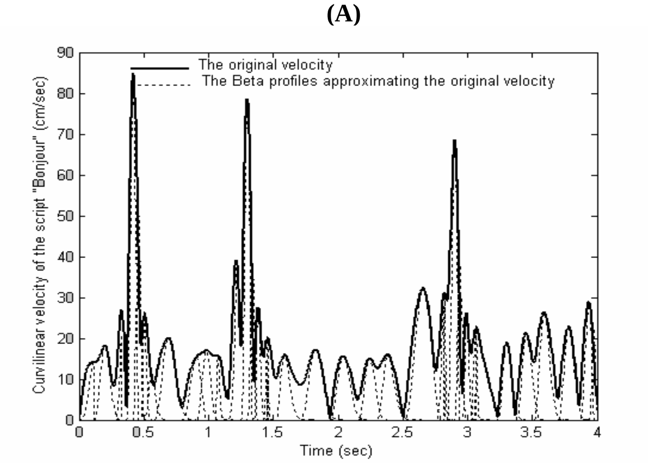
Figure 14: (A) The French handwriting script "Bonjour".

(B) The curvilinear velocity signal related to the French handwriting script "Bonjour".

This is due to mainly to the fruitful dynamics in this part of the fluent trace.

For the case of the figure 14A, we can see the original handwriting script, which is the French word "Bonjour", as generated by a human subject. The curvilinear velocity signal associated to this real data is depicted in figure 14B. As previously, the original data was acquired by using a digitizer tablet type Wacom4, sampled at 200 Hz.
The curvilinear velocity signal presents three interesting parts where the amplitude is greater comparing to the other parts of the signal. As we have seen previously that the curvilinear velocity varies from a letter to another, depending on the form of this earlier.
In figure 15A, we have the different Beta profiles approximating the original curvilinear velocity related to the script "Bonjour", which are depicted (dashed lines). Analysing these earlier, we remark that the original curvilinear velocity becomes more complicated comparing to the previous cases, from the point of view that it contains more inflexion points and hidden strokes.

(A)



(B)

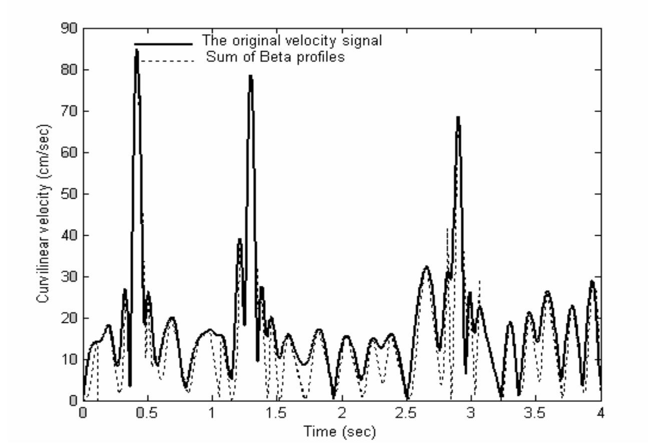
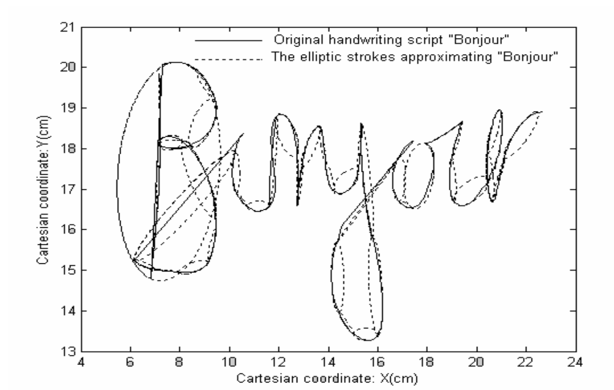


Figure 15: (A) The curvilinear velocity signal related to the French handwriting script "Bonjour" approximated by Beta profiles for each single stroke. (B) The summation of the different Beta profiles.

In figure 16A, we remark the different ellipses approximating the original handwriting script "Bonjour". We observe that the dynamics observed in the original curvilinear velocity signal had significant effect on the regeneration of the original script. Using the decomposition scheme, different elliptic strokes are depicted in figure 16B. These strokes are not

(A)



(B)

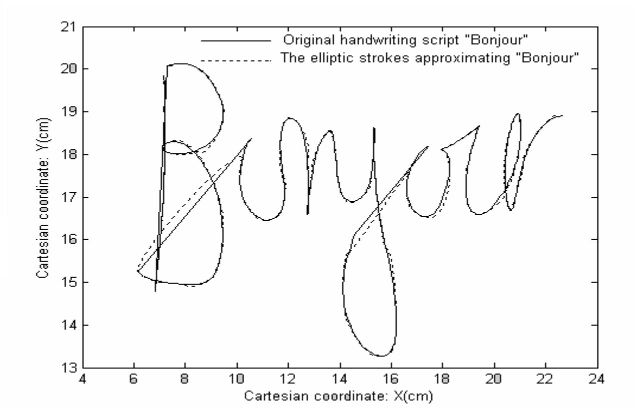


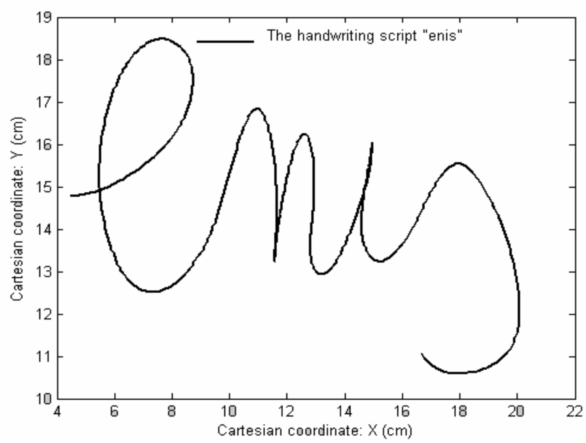
Figure 16: (A) The French handwriting script "Bonjour" approximated by elliptic portions-The different ellipses are designed.
(B) The French handwriting script "Bonjour" approximated by elliptic portions-The different elliptic arcs are designed.

For the last example of handwritten script, we have used the fluent trace "enis" as depicted in figure 17A. The curvilinear velocity related to this script is represented on figure 17B. We remark that the curvilinear signal have less dynamics comparing to the precedent cases of handwriting scripts “Bonjour and Hala”.

As shown in figure 18A, the different Beta profiles approximating the curvilinear velocity related to the script “enis”. One can see that the absence of an inflexion point for the second Beta profile have considerable effect on the obtained sum of the different Beta profiles in figure 18B.

In figure 19A, we remark the different ellipses approximating the original handwriting script “enis”. For each Beta profile of figure 18A corresponds an elliptic stroke in figure 19B. We remark that the approximation with elliptic strokes is more satisfactory for the case of smooth handwritten scripts, where the dynamics are rather absent.

(A)



(B)

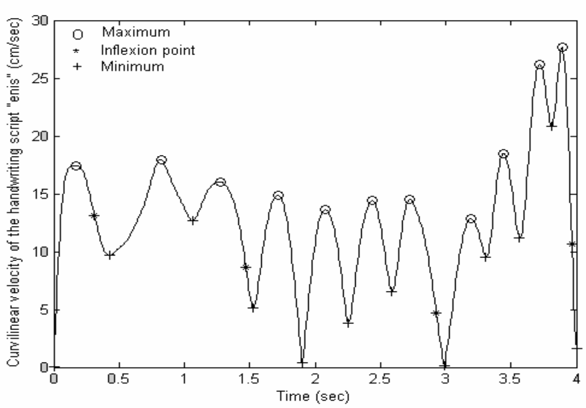
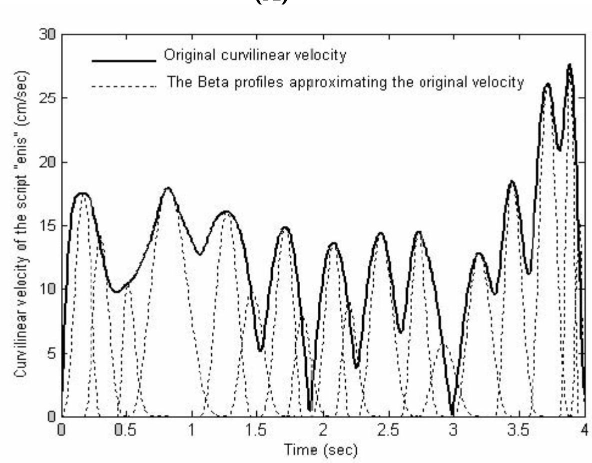


Figure 17: (A) The French handwriting script “enis”. (B) The curvilinear velocity signal related to the French handwriting script “enis”.

In figure 19A, we remark the different ellipses approximating the original handwriting script “enis”. For each Beta profile of figure 18A corresponds an elliptic stroke in figure 19B. We remark that the approximation with elliptic strokes is more satisfactory for the case of smooth handwritten scripts, where the dynamics are rather absent.

One note worthy feature of the Beta elliptic model is that the via-points are necessary reached. It’s not the same case for the delta-Lognormal model of Plamondon and Guerfali [49, 86], where the via-points are not necessarily ever reached: a new stroke may be launched toward a via-point in a different direction and super-imposed on the prior stroke so that the first “virtual” via point is not reached. Instead of choosing a via-point, which is far away and does not to be reached, we preferred to choose a closer via-point and reach it

(A)



(B)

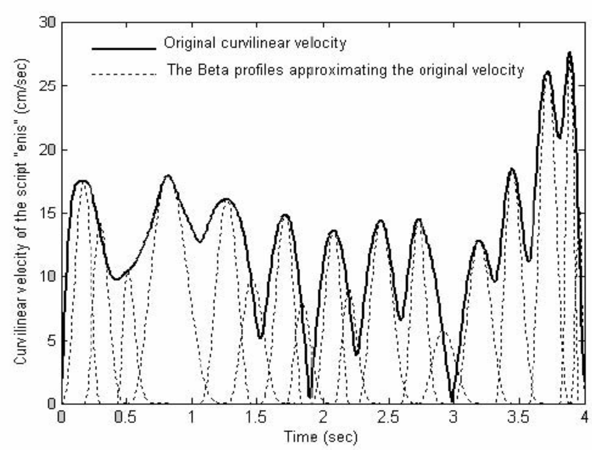


Figure 18: (A) The curvilinear velocity signal related to the French script “enis” approximated by Beta profiles.
(B) The summation of the different Beta profiles.

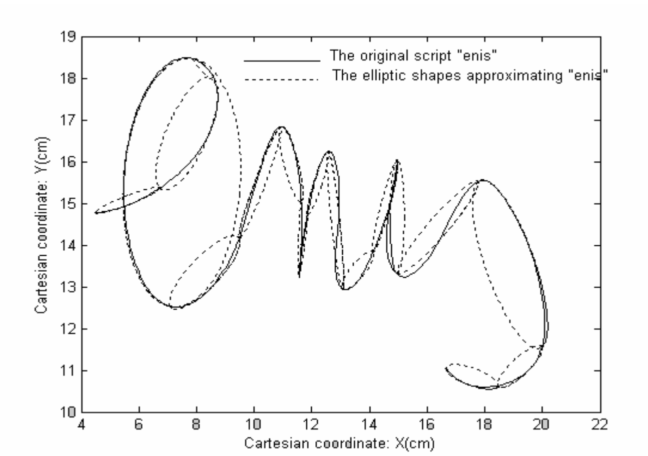
5. CONCLUSION

In this paper, we present a novel generation modelling approach for the analysis and understanding of online cursive handwriting. Our scheme is based upon a dynamic fitting of the handwriting scripts, both in the static and the kinematic domains, using the Beta-elliptic model.

To represent a simple movement called stroke the model requires a set of ten parameters that describes the movement in both domains.

Complex movements such as handwriting are described by the model as an algebraic summation of time overlapped single strokes. These strokes are not apparent directly in the image of a handwriting script. They are partially hidden in the trajectory as a consequence of superimposition process [28, 66].

(A)



(B)

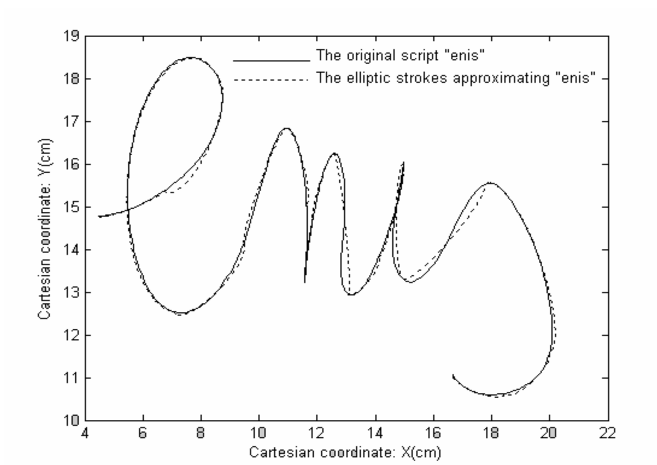


Figure 19: (A) The French handwriting script “enis” approximated by elliptic portions-The different ellipses are designed.
(B) The French handwriting script “enis” approximated by elliptic portions-The different elliptic arcs are designed.

As one can see from the previous examples, different type of realistic patterns can be produced by the Beta-elliptic model. Using the decomposition scheme of handwriting scripts into strokes, depending on the tuning between two strokes, different visual patterns can be generated with specific curvilinear velocity components.

Many models have been applied to French handwriting, but unfortunately, only a few publications have been interested to the modelling of Arabic handwritten scripts so far. It has been shown that the Beta-elliptic model can be applied not only to French handwriting but also to Arabic handwriting words. Nevertheless, it is likely, that we will see more applications to our Beta-elliptic model in the future.

A method for parameters estimation and optimization is proposed in this paper. A two-step approach is proposed, one step to approximate the parameters and a second to optimize the solution by means of non linear regression technique. Despite the fact that the optimal tuning functions provide better explanation of the experimental data, still there is a fraction of difference between the two curvilinear velocity profiles.

So, our model can be employed as a preliminary state of a dynamic signature verification system, as well as, for an on-line handwriting recognizer system. As we have shown that we can generate human handwriting, not only Latin handwriting scripts but also Arabic ones using the Beta-elliptic model. Then as a future work, we have firstly to test our model for multi signatures: European signatures and Arabic ones. Assuming that, European signatures have more a resemblance for shapes, usually displaying a number of loops lines and curves surrounding the main body of the signature. Furthermore, Arabic signatures are written in the opposite direction from right-to-left. Secondly the Beta-elliptic model can be investigated for the segmentation state of an automatic signature verification system, it may incorporate design considerations covering a wide range of signatures taken into account the style of these signatures, that can be universally successful.

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AUTHORS' BIOGRAPHIES

Hala Bezine was born in Sfax (Tunisia) in 1973. She received Engineering Diploma from the National School of Engineering of Sfax in 1996, the Diplôme d'Etudes approffondies in Automatics in 1998 and a Ph.D. in Electrical Engineering in 2006. Currently, she is an associate professor of Electrical and Computer Engineering at the University of Sfax. Her interests include cursive handwriting modelling, robotic systems and control of complex human movements. She is an IEEE member.

Adel M. Alimi was born in Sfax (Tunisia) in 1966. He graduated in Electrical Engineering in 1990, obtained a Ph.D. and then an HDR both in Electrical Engineering in 1995 and 2000, respectively. He is now an associate professor in Electrical and Computer Engineering at the University of Sfax.

His research interest includes of intelligent methods (neural networks, fuzzy logic, genetic algorithms) to pattern recognition, robotic systems, vision systems and industrial processes. He focuses his research on intelligent pattern recognition, learning, analysis and intelligent control of large scale complex systems. He is the associate editor of the international journal Pattern Recognition Letters. He is IEEE senior member of IAPR, INNS.

Nasser Sherkat received a B.Sc Honours degree in Mechanical Engineering from the University of Nottingham in 1985. He received a Ph.D. in High Speed Geometric Processing for Continuous Path Generation, from the Nottingham Trent University in 1989. He is now a Reader in Computing in the School of Computing and Informatics at The Nottingham Trent University. His interests include cursive handwriting recognition, poor quality optical character recognition and real-time object tracking and he has authored numerous articles on these subjects in various international journals and conferences. Dr. Sherkat is the leader of the Intelligent Systems Team of the School, a chartered engineer and a Fellow of the Institution of Electrical and Electronic Engineers.