Abstract—This paper demonstrates the signal combining by use of adaptive algorithms for wireless communication networks. The proposed adaptive combiner operates under different communication channel noise variances on each branch of the multi-antenna receiver. The used algorithm for adaptive signal combining is least mean square (LMS) algorithm based on Newton’s Recursion Method. The proposed algorithm uses inverse of noise variances estimate in step size of adaptive algorithm. It is shown that the adaptive combining filter with LMS converges with respect to signal to noise ratio (SNR) and not to the received power. Simulation results in Gaussian channels with different channel noise variances shows that proposed scheme provide performance very close to Wiener’s solution of signal combining. It provide $10^{-4}$ bit error rate (BER) at 10 dB SNR. The performance of system in Flat Fading Raleigh channels is about $10^{-5}$ at 20 dB SNR. Where as the classical maximum ratio combining, adaptive combining with classical LMS and recursive least square algorithm provide $10^{-3}$ bit error rate at 20 dB SNR. The improvement in MSE and BER performance with proposed algorithm is more obvious when two independent signals arrive at the receiver of communication terminal (on multiple antennas) with 10dB of SNR difference, which is very common situation in wireless communication systems.

Keywords- Adaptive Signal Combining, Unequal noise variances, RLS, LMS, Newton’s Recursion base LMS, Optimum signal combining, Optimum Adaptive signal combining.

I. INTRODUCTION

Space diversity provides an attractive method for the bit error rate (BER) performance improvement of wireless communication networks. Receive spatial diversity can be achieved by multiple antennas receiving different versions of the same original transmit signal. In other words, various received signal sequences are subjected to different statistical corruption that may be additive and/or multiplicative due to thermal noise/impulsive noise (Gaussian noise due to multiple electronic circuitry) and signal fading. We consider additive corruption of received signals which is due to random Gaussian distribution of noise. It has been generally neglected in the past literature. In literature an extensive work has been performed on adaptive algorithms, for channel equalisation, noise cancellation and signal combining. Theory on the speed of convergence and ensemble average mean square error performance of adaptive filter with Least Mean Square (LMS) and Recursive Least Square (RLS) algorithm is given by G. Ungerboeck in [1]. The classical scheme of adaptive combining was discussed by B. Widrow [2] and Jack H Winters in [3] where they managed to show BER performance arbitrarily close to the Wiener solution (optimum solution) under the common classical assumption of zero mean and unit variance channels noise. Effects on correlation between two mobile radio base-station antenna is given by [4]. Later adaptive combining presented for various applications [5][6][7] [8]. In particular under water signal combining presented in [9] The capacity of adaptive combining receivers discussed in [10] [11][12]. And the performance of different adaptive receiver structures (adaptive multiuser detection) presented by Rapajic in [13]. Frequency domain adaptive equalisation and combining presented in [14][15], adaptive combining for mobile communication 3GPP system in [16] and antenna selection diversity presented in [17]. All the research work for the theory of adaptive filters is summarised in the books of S Hykin [18] and Ali H Saeed [19]. Most of perious research work neglects the unequal/different channels noise variance. And they assumed the Gaussian noise with zero mean and unit variance for communication channels.

Newton’s Recursion Method base LMS algorithm previously mentioned in [18][19], but with out realizing its importance related to adaptive combining of signals. In modern wireless communication system we are using multiple input and multiple output antenna systems, cooperative wireless networks, orthogonal frequency division multiplexing access scheme and complex modulation schemes other then binary phase shift keying . Due to these advancements in wireless communication the probability of unequal/different channel noise variance increased, consequently one of signal to combine is of very high Signal to Noise Ratio (SNR) and other is at low SNR received. We also statistically treat channel interference as additive Gaussian thermal noise, which also effect the noise variance level and receiver communication terminal receives different (unequal) noise variance.

This problem was discus and its wiener solution described in [19] but adaptive solution with LMS not given in previous literature. And various researchers have been presenting sub optimum version of LMS algorithms which usually consider the signal power for adaptive signal
combining. In [20], based on Newton’s Recursion Method, we present optimal adaptive combining with LMS. We propose the use of inverse noise variance in the step size (correction factor) to deal the respective channels noise differently. Our presented scheme makes use of unequal variable step sizes for multiple reception branches. Our proposed receiver weights the signal sequences according to its respective channel noise to provide the BER performance arbitrarily close to Wiener solution. Our proposed scheme is been verified by simulation results in Gaussian and Raleigh communication channels. Results demonstrate that the proposed scheme minimises MSE performance and consequently improves BER performance, especially when two independent signals arrived at receiver with 10dB SNR difference which is very common situation in wireless communications. This claim is confirmed by analysing the ensemble average mean square error and bit error rate performance of the classical and proposed combiners. In previous conference proceedings we presented adaptive multiuser detection for cooperative communication and adaptive signal combining systems [21][22]. This paper is extension and further verification of our conference proceeding work by examining the performance of the proposed algorithm in flat fading Raleigh wireless channel. The main contributions of this paper and conference proceeding are

We shown by mathematical analysis that proposed algorithm provide performance close to wiener solution of signal combining. We verified by computer simulation that the ensemble average mean square error performance of proposed algorithm with factor of inverse of variance of respective channel noise, provide significant gain in minimisation of error.

It is also shown mathematically for signal combining adaptive algorithm actually converge according to signal to noise ratio(SNR) not to the signal power. We confirmed further by series of computer experiments that the adaptive combiner with the factor of inverse noise variance of the respective channel in algorithm provides bit error rate (BER) gain. The BER gain is more significant when there is difference of signal to noise ratio is 10 dB of the signals to be combined.

Simulation results in Gaussian channels with different channel noise variances shows that proposed algorithm provide performance very close to wiener’s solution of signal combining. It provide $10^{-3}$ bit error rate at 10 dB SNR. The performance of system in Flat Fading Raleigh channels is about $10^{-3}$ at 20 dB SNR. Where as the classical maximum ratio combining, adaptive combining with classical LMS and recursive least square algorithm provide $10^{-1}$ bit error rate at 20 dB SNR.

The rest of the paper is organised as follows: In section II we present system models, in section III we describe the Wiener solution, section IV is about classical adaptive combining with use of LMS algorithm, section V describe the proposed modification of multiplying the respective channel inverse of noise variance in LMS algorithm correction factor., section VI presents adaptive combining with classical RLS algorithm, section VII describe computer simulation results and VIII is for conclusion.

II. SYSTEM MODEL

In classical wiener solution of combining (optimum combining), multiple signal received and the noise variance is estimated on each branch. The reciprocal of estimated noise is multiplied as weighing coefficient by each combining branch to achieve optimum combining. In classical Adaptive Signal Combining which is shown in Fig.1 represents the M-branch adaptive signal combining system where M is the number of antennas of the receiver communication terminal. Signal received ($r_m$) on antenna is down converted (DC) and fed into low pass filter (LPF). The output of the LPF is fed into an analogue to digital converter (ADC). Adaptive filter combines by adaptive algorithm with a step size regulated by the signal received power. A training operation coordinated by the transmitter communication terminal is been used to adjust the mth weight coefficient ($c_m$) of the adaptive combiner.

In real communication systems, receiver channel noise variance fluctuates due to various channel environments and electronic circuitry. Therefore, non-equal step sizes with the multiplicative factor of inverse of noise variances is used in proposed adaptive signal combining system. It enables receiver to obtain BER performance arbitrarily close to Wiener solution. The original signal at time n is denoted by $x(n)$ while by $x^*$ or $x$ in Fig.1 denotes the signal at the output of the combiner. $x^*(n)$ denotes the output of estimated signal in time n. The term ($J^*$) represent complex conjugate. Fig.2 shows the proposed adaptive combiner. The main difference between the two combiner systems is the use of a set of Noise Estimators as shown. This noise estimators are responsible to return a noise variance coefficient that is to be used by the adaptive combiner to setup a suitable step size. The rest of its operation is identical to that of a classic combiner. Our purpose system performance depend upon the correct estimate of noise/interference. This device can be a filter (adaptive or non adaptive) for noise estimation.
III. WIENER SOLUTION (OPTIMUM COMBINING)

The symbol estimate at combiner is given by

\[ \hat{x}(n) = c_{H_m}^H r_m(n) \]  

(1)

where \( c_{H_m} \) is a M-dimensional complex value weight.

And \( r_{H_m} \) is received complex valued vector

\[ [r_1(n), r_2(n), \ldots, r_M(n)] \]

The received signal itself is represented by

\[ r_m(n) = b_m x(n) + v_m(n) \]  

(2)

Where \( b_m \) and \( v_m \) are the channel and noise vector given by

\[ b_m(n) = [h_1(n), h_2(n), \ldots, h_M(n)] \]

\[ v_m(n) = [v_1(n), v_2(n), \ldots, v_M(n)] \]

By assuming channel and noise constant for the whole block of transmitted data, the error \( e(n) \) between the reference signal and the output of the adaptive filter for the \( n \)th symbol is given by

\[ e(n) = (x(n) - \hat{x}(n)) \]  

(3)

Here \( x(n) \) is a digital reference training sequence known at the receiver filter. From equation (1) and equation (2)
The mean square error (MSE) is represented by
\[ J(c_m(n)) = \mathcal{E}[e(n)^* e(n)] \]  
(5)

From equation (4) and equation (5) we get
\[ J(c_m(n)) = \mathcal{E}[(x(n) - c_m(n)r_m(n))^2] + c_m^H E[r_m(n)r_m(n)^*]c_m(n) - c_m^H E[r_m(n)x(n)^*] - \mathcal{E}[x(n)r_m(n)^*]r_m(n) \]  
(7)

The first term \( \mathcal{E}[x(n)x^*(n)] \) in equation (7), represents the variance of the desired signal. The expectation \( E[r_m(n)r_m(n)^*] \) denotes M by M correlation matrix \( R \) of the received signal. The third term \( z \) is M by 1 cross-correlation matrix between the received components and the reference sequence given by
\[ z = \mathcal{E}[r_m(n)x(n)^*] \]  
(3)

And its hermitian transpose is represented by
\[ \mathcal{E}[x(n)r_m(n)^*] = z^H. \]

Differencing the mean squared error function \( J(c_m(n)) \) with respect to each coefficient of weight vector \( c_m(n) \) yeilds the gradient is given by the following equation.
\[ \nabla c_m(n) = \frac{\partial J(c_m(n))}{\partial c_m(n)} \]  
(9)

Here \( [c_m(n)]_{opt} \) is optimal weight vector and it can be determined by setting the gradient equal to zero, therefore,
\[ \nabla c_m(n) = -2z + 2Rc_m(n) = 0 \]  
(10)

where 0 is an M by 1 null vector at the minimum point of error space.
\[ R[c_m(n)]_{opt} = z \]  
(11)

It is wiener equation or the normal equation(optimum combining)[3][7]. One possible solution of this equation is matrix inversion of correlation matrix \( R \), mathematically
\[ c_m(n)_{opt} = R^{-1}z \]  
(12)

We earlier mentioned \( R \) as
\[ R = \mathcal{E}[r_m(n)r_m(n)^*] \]  
(13)

For two receive antenna, M =2, it is given by
\[ R = \mathcal{E}[r_1(n)r_1(n)^*][\mathcal{E}[r_2(n)r_2(n)^*]] \]  
(14)

The above matrix equation mentioned by Ali H Saeed in [19] for linear least mean squares estimate of \( x(n) \) for given \( r_1(n) \) and \( r_2(n) \) when two receive signals with different noise variance, and the optimal linear receiver solution \( [c_m(n)]_{opt} \) depend upon the variances of noise of receive signals. Hence for uncorrelated signals, the above equation becomes,
\[ R = \mathcal{E}[r_m(n)r_m(n)^*] = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \]  
(15)

IV. CLASSICAL ADAPTIVE COMBINING SCHEME WITH LEAST MEAN SQUARE(LMS) ALGORITHM

Fig.1 represent a classical adaptive combining. Adaptive solution does not require matrix inversion. Explicit calculations of the correlation coefficient is the steepest decent method (SDM). The SDM is recursive procedure that can be used to calculate the optimal weight vector \( [c_m(n)]_{opt} \). Let \( c_m(n) \) and \( \mu \nabla_m(n) \) denote the values of the weight vector, and the gradient vector , respectively. Then succeeding values of the weight vector are obtained by the recursive relation. After obtaining the weight vector, adaptive filter operate in decision directed mode. Therefore,
\[ c_m(n + 1) = c_m(n) - \mu \nabla_m(n) \]  
(16)

Where \( \mu \) is step size constant that controls stability and the rate of adaptation. Various algorithms have been proposed by researchers for the value of the step size, but most of them focused the signal power for their algorithm. Our presented mathematical derivations shows that the step size selection only depend upon the noise variances of the signals. Therefore, for the optimum adaptive combining we use the inverse of the variance in the step size of adaptive algorithm. On this principle the other sub optimum algorithms can be developed for the future research.

Putting the value of equation (10) in above equation
\[ c_m(n + 1) = c_m(n) - \mu(-2z + 2Rc_m(n)) \]  
(17)

Further simplification yeilds
\[ c_m(n + 1) = c_m(n) + \mu(2z - 2Rc_m(n)) \]  
(18)

If we express \( \mu \nabla_m(n) \) in term of instantaneous estimates
\[ \mu z = r_m(n)x(n)^* \text{ and } R = r_m(n)r_m(n)^* \]

Then the equation can be simplified as
\[ c_m(n+1) = c_m(n) + 2\mu r_m(n)(x(n)^*-r_m(n)^*c_m(n)) \]  
(19)
Here \( n \) represent the iteration number which can be expressed in term of \( e^*(n) \) as

\[
e_{n}(n+1) = e_{n}(n) + 2\mu e_{n}(n)e^*(n) \tag{20}
\]

The term \( 2\mu e_{n}(n)e^*(n) \) is called correction factor. Here \( \mu \) control the size of correction. In classical adaptive signal combining, it is usually selected by multiple of number of taps and signal power, therefore in classical adaptive combining adaptive filter converge to the power of signal. Classical system provide insignificant performance, specially when the systems are installed for practical use. Some engineers use RLS algorithm, but RLS is also derived from Newton’s recursive relation. The higher computational complexity is major disadvantage of RLS. Where as, our proposed algorithm only dependant upon the accuracy estimate of channel noise. Hence, for two receive antennas, the recursive relations are

\[
c_{1}(n+1) = c_{1}(n) + 2\mu r_{1}(n)e^*(n) \tag{21}
\]

\[
c_{2}(n+1) = c_{2}(n) + 2\mu r_{2}(n)e^*(n) \tag{22}
\]

V. PROPOSED ADAPTIVE COMBINING SCHEME WITH LEAST MEAN SQUARE (LMS) ALGORITHM

The well known Newton’s recursive formula is given by

\[
e_{n}(n+1) = e_{n}(n) + \mu R^{-1}(2x - 2Re_{n}(n)) \tag{23}
\]

In our scheme we propose to multiply estimate of inverse of noise from the noise estimator filters of receiving signal in combiner LMS algorithm to get unequal step sizes in the correction factor of adaptive algorithm. The term correction factor of equation (20) becomes \( 2\mu R^{-1}r_{n}(n)e^*(n) \). If we assume noise is un correlated, then the Inverse of noise variance matrix for M antenna is given by

\[
R^{-1} = \begin{bmatrix}
1/\sigma_{v1}^2 & 0 & \ldots & 0 \\
0 & 1/\sigma_{v2}^2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 1/\sigma_{vM}^2
\end{bmatrix} \tag{24}
\]

Hence equation (20) of adaptive algorithm becomes

\[
e_{n}(n+1) = e_{n}(n) + 2\mu R^{-1}r_{n}(n)e^*(n) \tag{25}
\]

For two receive singal combining

\[
c_{1}(n+1) = c_{1}(n) + 2\mu[1/\sigma_{v1}^2]r_{1}(n)e^*(n) \tag{26}
\]

\[
c_{2}(n+1) = c_{2}(n) + 2\mu[1/\sigma_{v2}^2]r_{2}(n)e^*(n) \tag{27}
\]

Where \( R \) matrix converge the algorithm according to SNR and its peformance is very close to the weiner solution (optimum combining).

VI. RECURSIVE LEAST SQUARE (RLS) ALGORITHM

RLS algorithm Newton’s Recursive Method with regularization employed in step size therefore Newton’s Method is replaced by

\[
e_{n}(n+1) = e_{n}(n) + \rho\phi_{1}(n)[R]^{-1}(2x - 2Re_{n}(n)) \tag{28}
\]

Where \( \rho \) is a constant called iteration dependent regularization parameter and \( I \) is identity matrix of same dimension of \( R \). By instantaneous approximation of above equation and setting parameter of RLS algorithm for initialization following by the sets of equations for RLS [19].

Important aspect of the RLS algorithm is it also converge with respect to SNR but with higher computational complexity. In the RLS algorithm the \( \lambda \) forgetting factor is taken as \( \lambda = 0.99 \).

We initially set \( P_1(n-1) = 0.3 \). These are defined as complex matrix approximately equal to inverse of covariance matrix. The term \( \phi_{1} \) and \( \phi_{2} \) is a gain term applied to weight update and it is function of \( \lambda \). Following RLS algorithm equations, for two received signal combining.

\[
\phi_{1}(n) = \lambda\phi_{1}(n-1) + r_{1}^*(n)r_{1}(n) \tag{30}
\]

\[
\phi_{2}(n) = \lambda\phi_{2}(n-1) + r_{2}^*(n)r_{2}(n) \tag{31}
\]

\[
P_{1}(n) = \phi_{1}^{-1}(n) \tag{32}
\]

\[
P_{2}(n) = \phi_{2}^{-1}(n) \tag{33}
\]

\[
= \frac{\lambda^{-1}[P_{2}(n-1) - (\lambda^{-1}P_{2}^2(n-1)\phi_{1}(n)r_{1}(n)P_{1}(n-1))]}{(1 + \lambda^{-1}r_{1}(n)P_{1}(n-1)\phi_{1}(n))} \tag{34}
\]

\[
P_{2}(n) = \phi_{2}^{-1}(n) \tag{35}
\]

\[
= \frac{\lambda^{-1}[P_{2}(n-1) - (\lambda^{-1}P_{2}^2(n-1)\phi_{2}(n)r_{2}(n)P_{2}(n-1))]}{(1 + \lambda^{-1}r_{2}(n)P_{2}(n-1)\phi_{2}(n))} \tag{36}
\]

\[
c_{1}(n) = c_{1}(n-1) + (P_{1}(n)r_{1}^*(n))(e(n)) \tag{37}
\]

\[
c_{2}(n) = c_{2}(n-1) + (P_{2}(n)r_{2}^*(n))(e(n)) \tag{38}
\]

Same set of equation used for computer simulation in section VII.

The general form of the equation (38).

\[
e_{n}(n) = e_{n}(n-1) + (P_{eq}(n)(e_{n}(n)))(e(n)) \tag{39}
\]
VII. PERFORMANCE COMPARISON AND NUMERICAL SIMULATIONS

A. Ensemble Average Mean Square Error

a) Simulation System Model:

In our first simulation 500 iteration of ensemble average mean square error taken. We analyzed the convergence of combining [1] of two un equal signal to noise ratio signals at the branches of combiner, signal 1 is at 1.7051 dB and signal 2 of 11.24 dB. a) 300 bits of training sequences are generated independently. b) We used adaptive transversal finite impulse response filters on each branch with LMS algorithm to estimate the noise. We kept in memory device for the further use in LMS algorithm minimize error. c) For classical LMS, presented LMS and RLS we use the algorithm from Section III, IV and V respectively.

Fig.3 demonstrates the ensemble average mean square error performance of classical LMS, presented LMS and RLS algorithms in Gaussian channel. The convergence of presented LMS and RLS algorithm are very fast. In simulation 50 to 60 bits iterations are enough for steady state of adaptive combining. Where as for the classical scheme the convergence into steady state require very long training, where adaptive combiner converge according to signal power. Presented (Newton’s recursion base LMS) and RLS provide gain of about 8 dB for ensemble average mean square error performance even training is very long in that particular case.

b) Simulation Results:

For classical adaptive combining scheme step size usually taken very small or according to the power of received signal which can not be precise according to Wiener solution of signal combining, however inverse of variance factor in step size tends to produce accurate weight estimate. In presented adaptive combining filter convergence is according to signal to noise ratio which already mathematical analysis of Weiner’s filter and Newton’s LMS base adaptive combining.

B. Bit Error Rate

a) Simulation System Model:

To confirm the gain of proposed adaptive combining form ensemble average mean square performance result we simulated the bit error performance of the scheme. The following conditions exist in all simulations: a) 1500 training BPSK bit send we send 105 bits un-coded coherent BPSK signal power is send through first Gaussian channel with noise variance of 1.95 and with second Gaussian channel with noise variance of 12.36. We taken the variable transmit signal power. And signal is normalized when received on combiner branches

b) Simulation Results:
Fig. 4 demonstrates the BER performance results of Wiener solution (optimum combining), classical adaptive combining by LMS, presented adaptive combining by LMS and adaptive combining by RLS algorithm in Gaussian channel. The BER performance results of Wiener solution is benchmark as it maximize the output signal to noise ratio. By RLS algorithm we obtained best adaptive performance which indicate from simulation that RLS algorithm convergence is according to signal to noise ratio, as it overlaps the performance result of Wiener’s solution. The performance of presented scheme (Newton’s recursion base LMS) is very close to Wiener solution of signal combining with a lower computational complexity then RLS. Where as the performance of classical scheme about 3dB lagging behind the presented (Newton’s recursion base LMS) adaptive combining with LMS. Fig. 5 is confirmation of proposed scheme in flat fading Raleigh wireless communication channels.

Figure 5. BER performance for adaptive combining of two variable power of signals with noise variance of 1.95 and 12.36, in flat fading Raleigh channels:  
1) Direct Transmission with Channel Noise variance of 1.95.  
2) Equal step sizes Classical Adaptive Combiner [7].  
3) Presented unequal step sizes with inverse of noise variances in the step sizes.  
4) Adaptive Combining with RLS Algorithm.  
5) Wiener solution (Optimum Combining).  
6) Maximum Ratio Combining (MRC).

It is clear from figure that the only our proposed scheme achieve performance exactly to Wiener solution. Where as other schemes classical adaptive combining, MRC and adaptive combining with RLS algorithm fail to provide the diversity gain. However, these schemes provide some benefit at high signal to noise ratio of more than 25 dB, but could not achieve Wiener solution.

VIII. CONCLUSIONS

We proposed an adaptive signal combining scheme for wireless communication in presence of unequal/different noise variances on multiple receive antennas. We proposed to use Newton’s Recursion base adaptive algorithm to obtain unequal step sizes at each branch of combiner filter. The proposed algorithm use the multiplicative factor of respective channel inverse of noise variance in LMS algorithm step size. The presented scheme is optimum and achieve Wiener solution of the signal combining of wireless communication systems. The proposed algorithm is linear and simple in computational complexity. We also shown that the adaptive combiner filter converge according to signal to noise ratio, and not to power of signal. Further work required to design optimum noise estimator for wireless communication channels and to develop adaptive algorithm by considering the channel noise variance.

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REFERENCES


