Relay Precoder Design for Amplify-and-Forward SIMO/MISO Wireless Relay Networks under Jamming Environment

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Abstract - This current paper addresses the noncooperative distributed wireless relay networks under a jamming environment. The amplify-and-forward single-input multiple-output (SIMO) and multiple-input single-output (MISO) schemes are considered with multiple N-relay nodes. The main contribution of this paper is to determine the optimal relay precoder vectors by minimizing the minimum mean square error (MSE) criterion. Using the derived optimal relay precoder vectors, the impacts on the jamming environment in the AF SIMO/MISO wireless relay systems are investigated using bit error rate (BER). Results show that the diversity order of a system decreases under a jamming environment. It is also seen that the system BER performance enhances as the number of relay nodes increases.

Keywords - Amplify-and-forward (AF), broadband noise jamming (BNJ), minimum mean square error (MMSE), single-input multiple-output (SIMO), multiple-input single-output (MISO), relay precoder vector

I. INTRODUCTION

In the recent years, relay nodes in wireless networks have played an important role in communication area [1]. Hence, various relay schemes have been studied over the past years. Relay schemes can be typically classified as amplify-and-forward (AF), decode-and-forward, compress-and-forward, and compute-and-forward [2]–[10]. The AF relay scheme employed in this paper forwards an amplified version of its received signals from a source node to a destination node [2]–[5], [10]. Since there is no required signal processing at the relay nodes for decoding, encoding, compressing, and computing, the AF relay scheme in [2]–[5], [10] has lower complexity compared to other schemes [6]–[9].

In reality, due to wireless communication features during data transmission, all communication nodes in wireless relay networks are exposed at the threat of jamming signals. Hence, partial-band noise jamming and broadband noise jamming (BNJ) are investigated in [11]–[13], but for non-relay systems. In addition, the authors considered the cooperative AF single-input multiple-output (SIMO) system [14], [15] and multiple-input single-output (MISO) system [16], [17] under no-jamming environment.

However, the authors of this paper in [10] investigated the noncooperative AF SIMO wireless relay network under a jamming environment without power constraint, but noncooperative AF MISO wireless relay network. Namely, power constraint was intentionally not considered [10]. Therefore, to make wireless relay systems more desirable, this paper considers a positive scaling factor in the minimum mean square error (MMSE) cost function to meet the target signal-to-noise ratio (SNR) at the destination. In other words, this current paper focuses on the noncooperative distributed AF SIMO wireless relay system under a jamming environment based on the MMSE criterion with a positive scaling factor during data transmission. In particular, BNJ is used for a jamming environment due to the fact that partial-band noise jamming affects more negatively on bit error rate (BER) than BNJ. In addition, the noncooperative distributed AF MISO wireless relay system under a jamming environment with the positive scaling factor is also investigated. Furthermore, this current paper presents how to determine the optimal relay precoder vectors for the noncooperative distributed AF SIMO/MISO wireless relay systems under a jamming environment.

Section II addresses the system models and data transmission strategies applied. Section III presents the noncooperative AF SIMO/MISO wireless relay schemes under a jamming environment with a positive scaling factor based on the MMSE criterion. Section IV presents the simulation results. Finally, Section V concludes the paper.

Notation: Matrices, vectors, and scalars are denoted, respectively, by uppercase boldface, lowercase boldface, and italic characters (e.g., A, a, and a). Notations $A^{-1}$, $A^{H}$, and $A^{T}$ denote inverse, Hermitian, and transpose of A, respectively. The $N \times N$ identity matrix and diagonal matrix are, respectively, denoted by $\textbf{I}_N$ and dial (a<sub>1</sub>,⋯,a<sub>κ</sub>). The expectation operator is $E[\cdot]$. Notation $\lfloor a \rfloor$ denotes the absolute value of a for any scalar.

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II. SYSTEM MODEL

A. SIMO Wireless Relay Scheme

The BNJ is defined as a complex Gaussian noise signal with zero-mean and power \( \sigma_J^2 = N_J \). Namely, the BNJ column \( j_{\text{BNJ}} \in \mathbb{C}^{N \times 1} \) at the relay nodes and \( j_{\text{BNJ}} \in \mathbb{C}^{N \times 1} \) at the destination nodes are zero-mean complex additive white Gaussian noise (AWGN) vectors with covariance matrices \( N_j \text{I}_N \) and \( N_j \text{I}_M \). Here, the subscript \( \text{SM} \) denotes the case of the BNJ environment of the AF SIMO wireless relay system.

Fig. 1 shows an AF SIMO wireless relay network with \( N \) noncooperative distributed relay nodes between a source node and \( M \) destination nodes under a BNJ environment. \(^2\) It is assumed that relay nodes cannot communicate their received signal information with each other due to relay noncooperation. The BNJ is placed to the data flow in all node links, as shown in Fig. 1. All relay nodes receive the transmitted signal \( s \) from the source node with the jamming signal in Stage I. They retransmit an amplified version of the jammed signal to the destination nodes under a jamming environment in Stage II. Then, the destination nodes receive the jammed signal from all relay nodes.

Let \( h \in \mathbb{C}^{N \times 1} \) denote the complex channel coefficient column vector from the source node to the relay nodes as

\[
 h_y = \begin{bmatrix} h_{y,1} \; h_{y,2} \; \cdots \; h_{y,N} \end{bmatrix}^T
\]

(1)

where \( h_{y,i} \), \( i = 1, \ldots, N \), is the \( i \)-th entry of \( h_y \), representing the complex channel coefficient from the source to the \( i \)-th relays. The received signal complex column vector \( r_{\text{BNJ}} \in \mathbb{C}^{N \times 1} \) at the relay node inputs under a BNJ environment is written as

\[
r_{\text{BNJ}} = h_s + v_j + j_{\text{BNJ}}
\]

(2)

where \( v_j \in \mathbb{C}^{N \times 1} \) is a zero-mean complex AWGN vector with covariance matrix \( \sigma_J^2 \text{I}_N \) and the subscript \( \text{SIMO} \) refers to the AF SIMO wireless relay system. Each channel \( h_{y,i} \) is assumed to be independent identically distributed (i.i.d.) with a zero-mean circular complex Gaussian of unit variance and quasi-static. It is assumed that complex channel coefficients are fixed during data transmission. The amplified signal complex column vector \( x_{\text{BNJ}} \in \mathbb{C}^{N \times 1} \) at the relay node outputs is written as

\[
x_{\text{BNJ}} = F_{\text{BNJ}} r_{\text{BNJ}}
\]

(3)

where \( F_{\text{BNJ}} \in \mathbb{C}^{N \times N} \) is called a relay precoder matrix at the relay nodes to minimize the mean square error (MSE) between the originally transmitted signal from the source node and the received signals at the destination nodes. In addition, as assumed, due to relay noncooperation, \( F_{\text{BNJ}} \) in this paper is diagonal, i.e., \( F_{\text{BNJ}} = \text{diag}(f_{\text{BNJ},1}, \ldots, f_{\text{BNJ},N}) \).

Therefore, to apply the noncooperative MMSE relay scheme in the next section, the relay precoder column vector \( f_{\text{BNJ}} \subset \mathbb{C}^{N \times 1} \) is defined as a column vector whose entries are the diagonal entries in \( F_{\text{BNJ}} \). Let \( H_y \in \mathbb{C}^{M \times N} \) denote the complex channel coefficient matrix from the relay nodes to the destination nodes as

\[
 H_y = \begin{bmatrix} h_{y,1} & h_{y,2} & \cdots & h_{y,M} \end{bmatrix}
\]

(4)

where \( h_{y,m} \in \mathbb{C}^{N \times 1} \) is a row vector, representing the complex channel coefficient from all relay nodes to the \( m \)-th destination node. Each channel \( h_{y,m} \) is also assumed to be i.i.d. with a zero-mean circular complex Gaussian of unit variance and quasi-static Rayleigh fading. The received complex signal column vector \( y \in \mathbb{C}^{M \times 1} \) at the destination nodes under a BNJ nodes to the destination nodes as

\[
y = H_y x + v_j + j_{\text{BNJ}}
\]

(5)

where \( v_j \in \mathbb{C}^{N \times 1} \) is zero-mean complex AWGN column vector with covariance \( \sigma_J^2 \text{I}_M \).

Substituting (2) and (3) into (5), the received complex signal column vector \( y \) can be rewritten as

\[
y = H_y ( F_{\text{BNJ}} h_s + h_{\text{BNJ}} v_j + j_{\text{BNJ}} ) + v_j + j_{\text{BNJ}}
\]

(6)

Using the relay precoder column vector \( f_{\text{BNJ}} \), the received complex signal column vector \( y \) in (6) can be also rewritten as

\[
y = H_y ( h_{\text{BNJ}} s + H_y F_{\text{BNJ}} v_j + H_y j_{\text{BNJ}} ) + v_j + j_{\text{BNJ}}
\]

(7)
Let $\mathbf{h}_m = [h_{m,1}, h_{m,2}, \ldots, h_{m,J}]$ denote the complex channel coefficient row vector from source nodes to relay nodes as

$$\mathbf{H}_r = [\mathbf{h}_{r,1}, \mathbf{h}_{r,2}, \ldots, \mathbf{h}_{r,M}]$$

where $\mathbf{h}_{r,m} \sim \mathcal{CN}(0, \mathbf{I})$ is a column vector, which represents the complex channel coefficient from the $m$-th source node to all relay nodes. Each channel $h_{r,m}$ is assumed to be i.i.d. with zero-mean complex Gaussian of unit variance and quasi-static Rayleigh fading. While, let $\mathbf{h}_i \sim \mathcal{CN}(0, \mathbf{I})$ denote the complex channel coefficient row vector from the relay nodes to the destination node

$$\mathbf{h}_i = [h_{i,1}, h_{i,2}, \ldots, h_{i,J}]$$

where $h_{i,j}$, $i = 1, \ldots, N$, $j$-th element of $\mathbf{h}_i$, which represents the complex channel coefficient from the $i$-th relay node to the destination node. It is also assumed that each channel $h_{i,j}$ is i.i.d. with zero-mean circularly symmetric complex Gaussian of unit variance and quasi-static Rayleigh fading.

An AF MISO wireless relay network with $N$ noncooperative distributed relay nodes between $M$ source nodes and a destination node under a BNJ environment with no-power constraint is shown in Fig. 2.\textsuperscript{5} Hence, the received signal complex column vector $\mathbf{r}_{\text{ms}} \sim \mathcal{CN}(0, \mathbf{I})$ at the relay node inputs under a BNJ environment, the amplified signal complex column vector $\mathbf{x}_{\text{ms}} \sim \mathcal{CN}(0, \mathbf{I})$ at the relay node outputs, and the received complex signal $\mathbf{t} \sim \mathcal{CN}(0, \mathbf{I})$ at the destination node under a BNJ environment can be written, respectively, as

$$\mathbf{r}_{\text{ms}} = \mathbf{H}_r \mathbf{s} + \mathbf{v} + \mathbf{j}_{\text{ms}}$$

$$\mathbf{x}_{\text{ms}} = \mathbf{F}_{\text{ms}} \mathbf{r}_{\text{ms}}$$

and

$$\mathbf{t} = \mathbf{h}_i \mathbf{F}_{\text{ms}} \mathbf{H}_r \mathbf{s} + \mathbf{h}_i \mathbf{F}_{\text{ms}} \mathbf{v} + \mathbf{h}_i \mathbf{F}_{\text{ms}} \mathbf{j}_{\text{ms}} + \mathbf{v} + \mathbf{j}_{\text{ms}}$$

where $\mathbf{s} \sim \mathcal{CN}(0, \mathbf{I})$ is a zero-mean complex Gaussian column vector at the source nodes, $\mathbf{v} \sim \mathcal{CN}(0, \mathbf{I})$ is a zero-mean complex AWGN column vector with covariance $\mathbf{I}$, and $\mathbf{j}_{\text{ms}} \sim \mathcal{CN}(0, \mathbf{I})$ is a zero-mean complex AWGN with variance $\sigma^2 \mathbf{I}$ at the destination node, $\mathbf{j}_{\text{ms}} \sim \mathcal{CN}(0, \mathbf{I})$ is a BNJ signal at the destination node with zero-mean and covariance $\mathbf{N}_{\text{ms}} \mathbf{I}$ at the relay nodes, and $\mathbf{j}_{\text{ms}} \sim \mathcal{CN}(0, \mathbf{I})$ is a BNJ signal at the destination node with zero-mean and variance $\mathbf{N}_{\text{ms}}$ at the destination node. Similarly, using the relay precoder row vector $\mathbf{f}_{\text{ms}} \sim \mathcal{CN}(0, \mathbf{I})$, the received complex signal $\mathbf{t}$ in (12) can be rewritten as

$$\mathbf{t} = \mathbf{f}_{\text{ms}} \mathbf{H}_r \mathbf{s} + \mathbf{f}_{\text{ms}} \mathbf{v} + \mathbf{f}_{\text{ms}} \mathbf{j}_{\text{ms}} + \mathbf{v} + \mathbf{j}_{\text{ms}}$$

where $\mathbf{f}_{\text{ms}} \sim \mathcal{CN}(0, \mathbf{I})$ is a diagonal relay precoder matrix at the relay nodes, $\mathbf{f}_{\text{ms}} = \text{diag}(f_{\text{ms},1}, \ldots, f_{\text{ms},N})$, and $\mathbf{H}_r \mathbf{s}$ is used as $\mathbf{H}_i \mathbf{r}_{\text{ms}}$.

The optimal relay precoder row vector $\mathbf{f}_{\text{ms}}$ and column vector $\mathbf{f}_{\text{ms}}$ under a BNJ environment will be, respectively, determined by using the MMSE criterion in the next section. Here, the superscript $\dagger$ denotes the optimum.

### III. NONCOOPERATIVE MMSE WIRELESS RELAY SCHEME

#### A. SIMO Wireless Relay Scheme

As stated in the beginning, power is intentionally not constrained during data transmission. Instead, a positive scaling factor will be applied during data transmission. Hence, to determine optimal $\mathbf{f}_{\text{ms}}$ in (7), which minimizes MSE between the received signal $\mathbf{y}$ at the destination nodes and the originally transmitted signal $\mathbf{s}$ in (7), the optimization problem can be as

$$\mathbf{f}_{\text{ms}} = \arg \min_{\mathbf{f}_{\text{ms}}} J(\mathbf{f}_{\text{ms}})$$

\textsuperscript{5} To follow the definition of the MISO wireless relay system, the number $M$ of relay nodes $N$ is greater than or equal to the number of source nodes, i.e., $N \geq M$. In addition, $M$ is greater than 2.
where the cost function \( J(f_{\text{simo}}) \) is written using (7) as

\[
J(f_{\text{simo}}) = \frac{1}{M} \sum_{m=1}^{M} E\left[y_m - \alpha^r f^H_{\text{simo}} y_m\right] = \sigma_r^2 \sum_{m=1}^{M} H_{y_m}^H f_{\text{simo}}^H f_{\text{simo}} H_{y_m} + M \sigma_r^2 + M \sigma_r^2
\]

\[
- \alpha_0 \sum_{m=1}^{M} H_{y_m} H_{y_m}^H f_{\text{simo}} + \psi \sum_{m=1}^{M} H_{y_m} H_{y_m}^H f_{\text{simo}}
\]

\[
- \alpha_0 \sum_{m=1}^{M} H_{y_m} H_{y_m}^H f_{\text{simo}} + MN_{\text{fSIMO}}
\]

(15)

where \( E\left[y_m^r\right] = \sigma_r^2, \sum_{m=1}^{M} H_{y_m} H_{y_m}^H f_{\text{simo}} = \alpha_0 \sum_{m=1}^{M} H_{y_m} H_{y_m}^H f_{\text{simo}}
\]

\[
E\left[v_{m}^r\right] = E\left[v_{m}^r\right] = E\left[v_{m}^r\right] = E\left[v_{m}^r\right] = E\left[v_{m}^r\right] = E\left[v_{m}^r\right] = 0, \text{ and } \psi = \sigma_r^2 + N_{\text{fSIMO}}
\]

In (15), \( \alpha \) is a positive scaling factor created by the designer for the noncooperative AF SIMO wireless relay system as

\[
\alpha = \sqrt{\text{SNR}_{\text{fSIMO}}} \frac{\sigma_r^2}{\sigma_r^2}
\]

(16)

where SNR_{\text{fSIMO}} is the target SNR at the destination nodes for the AF SIMO case. Differentiating \( J(f_{\text{simo}}) \) with respect to \( f_{\text{simo}} \) using the cyclic permutation of trace function and the linear/nonlinear properties of the complex vector derivative [18] results in

\[
\frac{\partial J(f_{\text{simo}})}{\partial f_{\text{simo}}} = \sigma_r^2 \sum_{m=1}^{M} H_{y_m}^H f_{\text{simo}} H_{y_m} + \alpha_0 \sum_{m=1}^{M} H_{y_m}^H f_{\text{simo}} + \psi \sum_{m=1}^{M} H_{y_m}^H f_{\text{simo}} = 0
\]

(17)

Implementing some algebra computations to (17) results in

\[
H_{y_m} f_{\text{simo}} = \alpha_0 \sum_{m=1}^{M} H_{y_m}^H f_{\text{simo}} (18)
\]

where \( H_{y_m} \in \mathbb{C}^{N \times M} \) is defined as

\[
H_{y_m} = V \sum U^H
\]

(19)

where \( \sum \in \mathbb{C}^{N \times N} \) is a diagonal matrix with nonnegative diagonal elements consisting of eigenvalues of matrix \( H_{y_m} \) with rank 2M when \( N > 2M \). In other words,

\[
\sum = \text{diag}(\gamma_1, \cdots, \gamma_{2M}, 0, 0, \cdots, 0)
\]

(20)

where \( \gamma_i, i = 1, \cdots, 2M \), is the \( i \)-th positive eigenvalue of \( H_{y_m} \). The matrix \( U \) and \( V \) are eigenvectors of \( H_{y_m}^H U^H \) and \( H_{y_m} V^H \), respectively, i.e., \( U^H U = V^H V = I_N \). Hence, the pseudo inverse of \( H_{y_m} \), i.e., \( H_{y_m}^+ \), using the SVD and least square solution in [19] is defined as

\[
H_{y_m}^+ = U \sum V^H
\]

(21)

where

\[
\sum = \text{diag}(\gamma_1^+, \cdots, \gamma_{2M}^+, 0, 0, \cdots, 0)
\]

(22)

\[
U = U^H \text{ and } V^H = V^H
\]

(23)

Therefore, using \( H_{y_m}^+ \) in (21), the optimal relay precoder column \( f_{\text{simo}}^\dagger \) for the AF SIMO wireless relay network under a BNJ environment with a positive scaling factor can be written as

\[
f_{\text{simo}}^\dagger = \alpha \sigma_r H_{y_m}^+ \left(h_{y_m}^+ \right)^H
\]

(24)

where \( h_{y_m}^+ \in \mathbb{C}^{1 \times N} \), \( m = 1, \cdots, M \). While, a matrix \( H_{y_m} \) has an inverse if \( N < 2M \). then, the optimal \( f_{\text{simo}}^\dagger \) in (24) can be written as

\[
f_{\text{simo}}^\dagger = \alpha \sigma_r H_{y_m}^+ \left(h_{y_m} \right)^H
\]

(25)

Using (24) and (25), the other two special cases for the AF SIMO wireless network system under a BNJ environment with a positive scaling factor can be derived. Namely, BNJ can be added only at the relay inputs in the source-relay links and only at the destination nodes in the relay-destination links.

- **Remark 1**: The optimal \( f_{\text{simo}}^\dagger \) for a special case of the noncooperative AF SIMO wireless relay system under a BNJ environment only at the relay inputs when \( N > 2M \) can be written as

\[
f_{\text{simo}}^\dagger = \alpha \sigma_r H_{y_m}^+ \left(h_{y_m}^+ \right)^H
\]

(26)

where the subscript SIMO,SR refers to the case of a BNJ environment only at the relay inputs in the source-relay links. Note that \( f_{\text{simo}}^\dagger \) refers to the case of a BNJ environment only at the relay inputs in the source-relay links.

- **Remark 2**: The optimal \( f_{\text{simo}}^\dagger \) for a special case of the noncooperative AF SIMO wireless relay system under a BNJ environment only at the destination nodes when \( N > 2M \) can be written as

\[
f_{\text{simo}}^\dagger = \alpha \sigma_r H_{y_m}^+ \left(h_{y_m} \right)^H
\]

(27)

where \( H_{y_m} \in \mathbb{C}^{N \times N} \) is a projection of \( H_{y_m}^H U^H \) and \( H_{y_m} V^H \), respectively, and the subscript SIMO,RD refers to the case of a BNJ environment only at the destination nodes in the relay-destination links. Note also that \( f_{\text{simo}}^\dagger \) refers to the case of a BNJ environment only at the destination nodes in the relay-destination links.

- **Remark 3**: The optimal \( f_{\text{simo}}^\dagger \) for the noncooperative AF SIMO wireless relay system under a no-jamming environment when \( N > 2M \) can be written as

\[
f_{\text{simo}}^\dagger = \alpha \sigma_r H_{y_m}^+ \left(h_{y_m} \right)^H
\]

(28)

where \( f_{\text{simo}}^\dagger \) in (28) has the same expression as \( f_{\text{simo}}^\dagger \) in (26) because \( f_{\text{simo}}^\dagger \) is independent of \( j_{\text{fSIMO}} \), whereas only depends on \( j_{\text{fSIMO}} \).

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\[ \mathbf{f}_{\text{MISO-NON}}^t = \alpha \mathbf{\sigma}^2 \mathbf{H}_y^n \mathbf{H}_y^n (\mathbf{h}_{\text{SM-M}})^H \]  

(28)

where the subscript \( \text{MISO-NON} \) refers to the case of the no-jamming environment. Note that \( \mathbf{f}_t^t \text{MISO-NON} = \sigma^2 \mathbf{H}_y^n \mathbf{H}_y^n (\mathbf{h}_{\text{SM-M}})^H \) when \( N \leq 2M \).

It is observed that the optimal \( \mathbf{f}_{\text{MIMO-RD}}^t \) in (27) under a BNJ environment at the destination nodes is independent of the jamming signals because \( \mathbf{f}_{\text{MIMO-RD}}^t \) in (27) is employed at the relay nodes during data transmission.

B. MIMO Wireless Relay Scheme

As in the case of the SIMO wireless relay network, power is also intentionally not constrained. Instead, the positive scaling factor will be applied during data transmission in the noncooperative AF MISO wireless relay system. As a result, using the MMSE criterion between the received signal \( t \) at the destination node and the originally transmitted signals \( s \) in (12), the optimization problem can be written as

\[ \mathbf{f}_{\text{MISO}}^t = \arg \min_{\mathbf{f}_{\text{MISO}}} J(\mathbf{f}_{\text{MISO}}) \]  

(29)

where the cost function \( J(\mathbf{f}_{\text{MISO}}) = \sum_{m=1}^M \mathbb{E} [v - \beta s_m]^2 \) is written using (12) as

\[
J(\mathbf{f}_{\text{MISO}}) = \sigma^2 \sum_{m=1}^M \mathbb{E} \left[\mathbf{H}_y^n \mathbf{H}_y^n \mathbf{f}_{\text{MISO}}^t + M \beta^2 \mathbf{\sigma}^2 + M \mathbf{\sigma}^2 \right] \\
\quad - \beta \sigma^2 \sum_{m=1}^M \mathbb{E} \left[\mathbf{H}_y^n \mathbf{f}_{\text{MISO}}^t \right] \cdot \mathbf{H}_y^n \mathbf{f}_{\text{MISO}}^t \\
\quad - \beta \sigma^2 \sum_{m=1}^M \mathbb{E} \left[\mathbf{f}_{\text{MISO}}^t \right] \cdot \mathbb{E} \left[\mathbf{H}_y^n \mathbf{H}_y^n \mathbf{f}_{\text{MISO}}^t \right] \\
(30)
\]

where \( \sigma^2 = \ldots = \sigma^2 = \sigma^2 = \sigma^2 \) , \( \sigma^2 = \ldots = \sigma^2 = \sigma^2 \), \( \mathbb{E}[v] = \mathbb{E}[v^*] \) , \( \mathbb{E}[v] = \mathbb{E}[v^*] \) , \( \mathbb{E}[j_{\text{MISO}}] = \mathbb{E}[j_{\text{MISO}}] = 0 \) , and \( \phi = \sigma^2 + N_{\text{tMISO}} \) are used. \( \beta \) in (30) is a positive scaling factor generated by the designer for the noncooperative AF MISO wireless relay system as

\[
\beta = \sqrt{M^t \text{SNR}_{\text{tNON}}} \frac{\sigma^2}{\sigma^2} 
(31)
\]

where \( \text{SNR}_{\text{t NON}} \) is the target SNR at the destination node for the AF MISO case.

Similarly, differentiating \( J(\mathbf{f}_{\text{MISO}}) \) with respect to \( \mathbf{f}_{\text{MISO}} \) results in

\[
\frac{\partial J(\mathbf{f}_{\text{MISO}})}{\partial \mathbf{f}_{\text{MISO}}} = \sigma^2 \mathbf{H}_y^n \mathbf{H}_y^n \mathbf{H}_y^n \mathbf{f}_{\text{MISO}}^t - \beta \sigma^2 \mathbf{H}_y^n \mathbf{f}_{\text{MISO}}^t \\
+ \phi \sigma^2 \mathbf{H}_y^n \mathbf{f}_{\text{MISO}}^t = 0 
(32)
\]

where \( \mathbf{h}_{\text{SM-M}} \in \mathbb{C}^{N_x} \) is defined in (11). Following the similar procedures of the SIMO wireless relay network, the optimal relay precoder column vector \( \mathbf{f}_{\text{MISO}}^t \) for the AF MISO wireless relay network under a BNJ environment with the positive scaling factor can be written as

\[
\mathbf{f}_{\text{MISO}}^t = \beta \sigma^2 (\mathbf{h}_{\text{SM-M}})^H (\mathbf{H}_y^n)^H 
(33)
\]

where \( \mathbf{H}_y (\mathbb{C}^{N_x \times N}) = \sigma^2 \mathbf{H}_y^n \mathbf{H}_y^n + \mathbf{I}_N \) and \( (\phi \mathbf{H}_y^n \mathbf{H}_y^n + \sigma^2 \mathbf{H}_y^n \mathbf{H}_y^n) \) is used.

- **Remark 4**: The optimal \( \mathbf{f}_{\text{MIMO-RD}}^t \) for the noncooperative AF MISO wireless relay system under a BNJ environment only at the relay nodes can be written as

\[
\mathbf{f}_{\text{MIMO-RD}}^t = \beta \sigma^2 (\mathbf{h}_{\text{SM-M}})^H (\mathbf{H}_y^n)^H. 
(34)
\]

- **Remark 5**: The optimal \( \mathbf{f}_{\text{MIMO-RD}}^t \) for the noncooperative AF MISO wireless relay system under a BNJ environment only at the destination node can be written as

\[
\mathbf{f}_{\text{MIMO-RD}}^t = \beta \sigma^2 (\mathbf{h}_{\text{SM-M}})^H (\mathbf{H}_y^n)^H. 
(35)
\]

where \( \mathbf{H}_y (\mathbb{C}^{N_x \times N}) = \sigma^2 \mathbf{H}_y^n + \sigma^2 \mathbf{I} \).

- **Remark 6**: The optimal \( \mathbf{f}_{\text{MISO-NON}}^t \) for the noncooperative AF MISO wireless relay system under a no-jamming environment can be written as

\[
\mathbf{f}_{\text{MISO-NON}}^t = \beta \sigma^2 (\mathbf{H}_y^n)^H (\mathbf{H}_y^n)^H. 
(36)
\]

Similar to the case of the AF SIMO wireless relay system, the optimal \( \mathbf{f}_{\text{MISO-NON}}^t \) in (35) is independent of the jamming signals because \( \mathbf{f}_{\text{MIMO-RD}}^t \) in (35) is employed only at the relay nodes. Additionally, the optimal \( \mathbf{f}_{\text{MISO}}^t \) in (33) also has the same expression as \( \mathbf{f}_{\text{MISO-NON}}^t \) in (34) because \( \mathbf{f}_{\text{MISO-NON}}^t \) in (33) is independent of \( j_{\text{MISO}} \), whereas only depends on \( j_{\text{MISO}} \).

IV. SIMULATION RESULTS

To evaluate the system BER performance for the noncooperative distributed AF SIMO/MISO wireless relay networks under a BNJ environment, Monte-Carlo simulations are performed. The complex channel coefficient column vector \( \mathbf{h}_r \), row vector \( \mathbf{h}_s \), matrix \( \mathbf{H}_r \), and matrix \( \mathbf{H}_s \) are generated from independent Gaussian random variables with zero mean and unity variance. All nodes have only one antenna and the same thermal noise power, i.e., \( \sigma^2 = \sigma^2 = \sigma^2 = \sigma^2 \). The originally transmitted signal at the source is modulated by quadrature phase shift keying with unit power, i.e., \( \sigma^2 = 1 \).

BNJ has one-side power spectral density \( N_J \) equal to 5%, 10%, and 20% of the bit energy, and is generated as an A-
Fig. 3. BER performance of $N = 3, 6$ and $M = 2$ noncooperative distributed AF SIMO wireless relay systems under a BNJ environment for all node links with a positive scaling factor $\alpha = 1$.

Fig. 4. BER performance of $N = 3$ and $M = 2$ noncooperative distributed AF SIMO wireless relay systems under a BNJ environment only at the relays in the source-relay links and only at the destinations in the relay-destination links with a positive scaling factor $\alpha = 1$.

Fig. 5. BER performance of $N = 3, 6$ and $M = 2$ noncooperative distributed AF MISO wireless relay systems under a BNJ environment in both source-relay and relay-destination links with a positive scaling factor $\beta = 1$.

Fig. 6. BER performance of $N = 3$ and $M = 2$ noncooperative distributed AF MISO wireless relay systems under a BNJ environment only at the relays in the source-relay links and only at the destinations in the relay-destination links with a positive scaling factor $\beta = 1$.

WGN. Namely, the variances of the jamming signal are chosen to satisfy $10 \log_{10}(\sigma_j^2 / \sigma_s^2) = 13$ dB, 10 dB and 7 dB, where $\sigma_s^2 = E_s$ and $\sigma_j^2 = N_j$. In addition, it is assumed that $N_{\text{src}} = N_{\text{src}} = N_{\text{des}} = N_{\text{des}} = N_j$. For system performance comparison, simulation results under a no-jamming environment are also included, i.e., $N_j = 0$.

Fig. 3 shows the BER performance of $N = 3, 6$ and $M = 2$ noncooperative distributed AF SIMO wireless relay systems under a BNJ environment for all node links with a positive scaling factor $\alpha = 1$, while Fig. 4 provides the BER performance of the special cases of the AF SIMO wireless relay systems under a BNJ environment only at the relays and only at the destinations with a positive scaling factor $\alpha = 1$. It is observed in Fig. 3 that the BER performance improves as the number of relay nodes $N$ increases. However, it can be seen in Figs. 3 and 4 that BER performance degrades as the variances of the jamming signal increases.

In particular, from Figs. 3 and 4, it is observed that BNJ creates significantly negative effects on the BER performance in the wireless relay networks. Hence, due to the effects of the BNJ environment for all node links in entire network, the worst BER is observed compared to the other two SIMO special cases in Fig. 4. As analyzed, the optimal relay precoder column vector $\mathbf{f}^{\text{SIMO-RD}}$ in (27) is independent of the jamming signal in the wireless relay network. However, $\mathbf{f}^{\text{SIMO-SR}}$ in (26) depends on the jamming signal. As a result, it is observed that the BER performance more slightly degrades if BNJ is added only at the relay node inputs, compared to the case that BNJ is added only at the destination nodes.
Fig. 5 presents the BER performance of $N = 3$, $6$ and $M = 2$ noncooperative distributed AF MISO wireless relay systems under a BNJ environment in both source-relay and relay-destination links with a positive scaling factor $\beta = 1$, while Fig. 6 shows the BER performance of the special cases of the AF MISO wireless relay systems under a BNJ environment only at the relays and only at the destinations with a positive scaling factor $\beta = 1$. Similar to the case of the AF MISO wireless relay system, BER performance improves with increase in the number of relay nodes, as shown in Fig. 5. In addition, as the jamming signal power increases, the BER performance gets worse, as shown in Figs. 5 and 6. It is also observed in Figs. 5 and 6 that the BER performance under a BNJ environment at both the relay node inputs and the destination node is the worst among three cases in the noncooperative AF MISO wireless relay systems due to the impacts on the BNJ environment for all node links in the entire network. Similarly, the optimal relay precoder row vector $\mathbf{f}_{\text{MISO-RD}}^{(1)}$ in (35) is independent of the jamming signal in the wireless relay network. As a result, it is observed that the BER performance under a BNJ environment only at the relay node inputs is slightly better than the one under a BNJ environment only at the destination nodes.

V. CONCLUSION

This paper proposed the noncooperative AF SIMO/MISO relay schemes under a jamming environment with positive scaling factors. The optimal relay precoder column/row vectors derived based on the MMSE criterion were derived.

Under a no-jamming environment, it was observed that the BER performance for the AF SIMO/MISO wireless relay systems improves as the number of relay nodes increases. However, under a jamming environment, it was found that increasing the jamming signal power can cause the diversity order to loss in the AF SIMO/MISO wireless relay schemes. It was also observed that the optimal relay precoder vectors are independent of the jamming signal if jamming is located at the destination. Namely, less harmful jamming location is at the destination in the relay-destination links.

REFERENCES