

Cluster Formation of Wireless Sensor Nodes using Adaptive Particle Swarm Optimisation

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Abstract — Hierarchical based clustering protocol for wireless sensor network is suitable to use in energy efficient environmental monitoring. In clustering protocol, sensor nodes that are cluster heads (CHs) have to collect information from cluster member and transmit to the base station. Strategic CHs location can significantly affect the network overall energy consumption. Therefore, selecting suitable CHs location becomes a challenging task. In this work, CHs distribution using adaptive particle swarm optimisation (APSO) is proposed. Particle swarm optimization (PSO) is one of the swarm intelligence methods that is designed to search for optimum solution by mimicking the behavior of bird flocking and fish schooling. Introduction of adaptive cognitive and social learning factor can achieve better convergence speed and particles reselection mechanism to reduce the chances of getting trapped at local maximum. The performance of the proposed method is compared with the low energy adaptive cluster hierarchical (LEACH) protocol. Simulation results show that the proposed method outperforms LEACH in terms of first node dies (FND) round, total data received at the base station and energy consumed per round.

Keywords – cluster, particle swarm optimisation, cluster head, cluster member

I. INTRODUCTION

In general, wireless sensor networks consist of numerous sensor nodes that are equipped with sensing capabilities and wireless communication module under limited energy resources. It is challenging to design the wireless sensor nodes. For convenience, the size of the wireless sensor nodes has to be compact [1, 2]. Thus it has limited computation capability, small battery size and small memory storage. In some cases, it is unpractical to frequently change the battery because the nodes are small in size and it may be deployed in hazardous areas. Therefore, it is more practical to save node energy and prolong the network lifetime by improving the algorithm [3]. Computer intelligence can be implemented to prolong the network lifetime [4].

In wireless sensor networks, cluster based hierarchical routing protocol is an energy efficient routing protocol. Typically, sensor nodes are grouped into clusters. Each cluster has a node that acts as the CH. Selected CH will collect sensor data forwarded from the nodes whose belong within the same cluster [5]. Data aggregation on received data by CH will reduce the overall data needed for transmission to the base station, which further reduces the energy consumption and network congestion [6].

The dynamic clustering protocol LEACH is proposed by Heinzelman et al [7]. The protocol aims to achieve load balancing among sensor nodes so it can prolong the network lifetime. In cluster routing protocol, energy consumption is concentrated on CHs. The cluster heads have to collect the data from other neighboring nodes. After data aggregation, it has to forward the aggregated information to the base station. In LEACH protocol each sensor nodes elects itself as a CH

based on a probability model. Each sensor node will become CH in every cycle to evenly distributing the work loads. Since LEACH protocol elect cluster heads based on probability model, elected cluster heads may not guarantee efficient energy distribution. There is possibility that no CHs are elected or too many are elected in a single round. Furthermore, the elected CHs may concentrate in one region or the CH may locate at the edge of the networks.

The improvement of LEACH protocol, LEACH-C uses a centralized algorithm to select the CHs where each node sends information about its current location and energy level to the base station [8]. Base station with overall network view will calculate and select the CHs using the simulated annealing algorithm. Applying PSO into the base station can find better CHs formation, the method consider three parameters which are energy, concentration and centrality of sensor nodes selection parameters [9, 10]. PSO will select the best CHs formation and lead to improvement of network lifetime as compared to LEACH [11].

Adaptive particle swarm optimisation (APSO) conducted PSO at the base station for cluster head selection [12]. The paper proposed an adaptive inertia weight function that will consider not only the iteration cycle, but also the ratio of the particles local best over the global best. Simulation results demonstrate that APSO achieve better network lifetime over LEACH [13].

In the previous work, applying fuzzy logic system into the base station for CH selection has shown improvement over LEACH protocol in terms of the network lifetime [14]. Fuzzy logic system can also provide fast convergence speed [15]. Applying PSO in CHs selection can provide better clusters formation compare to the previous work [16].

This project aims is to prolong the FND period, increase total data sent to the base station before FND and stable energy consumption per round. In this work, a CHs selection mechanism conducted in the base station by using PSO is proposed. PSO can iteratively calculate the best cluster based network formation. Furthermore, introduction of adaptive learning factors and reselection mechanism can prolong the network lifetime.

This paper is organized as follows: In the next section, the overview of cluster based hierarchical routing protocol LEACH and PSO are demonstrated. Section III explains the system model followed by the analysis of the simulation results in section IV. Lastly, section V concludes the findings.

II. RELATED WORK

In this section, the overview of cluster based hierarchical routing protocol LEACH and basic PSO algorithm will be illustrated. The discussion to the theory underline in LEACH protocol is carried out followed by the discussion of PSO that includes the basic framework of the PSO.

A. LEACH

LEACH is a classical clustering mechanism aims to distribute the energy concentration in the randomly self-elected CHs. Every single sensor node in LEACH randomly elects itself as CH according to a threshold defined as (1):

$$T(n) = \begin{cases} \frac{p}{1 - p \times (r \bmod \frac{1}{p})} & , \text{if } n \in G \\ 0 & , \text{otherwise} \end{cases} \quad (1)$$

where p is the proportion of the nodes to CH; r is the number of rounds that have been ended; G is a set of nodes which have never been CH in the last $1/p$ rounds. This algorithm ensures that every sensor node becomes a CH exactly within $1/p$ rounds.

LEACH avoids the battery depletion of an individual sensor by randomly rotate of CH location. This protocol operates in rounds; each round consists of two phases which are set-up phase and steady phase.

Set-up phase includes advertisement phase and cluster set-up phase while steady phase includes schedule creation and data transmission. In each round, each node independently generates the random number between 0 and 1. If the generated number is less than the threshold value $T(n)$ which is defined by (1), the node will self-elect to become the CH in the current round.

Although LEACH protocol distributes the energy concentration in the CH but it has some disadvantages due to its probability CHs election model:

- LEACH depends only on the probability model to elect the CH, elected CH may be very close to each other.

- In each round the number of the CH is dynamic and it cannot guarantee the data stream received at the base station.
- It is possible that no CH will be elected in the current round.
- CH will appear at the edge of the network or the place where the node density is very low.

The disadvantages of LEACH protocol are mainly due to using the local information. Centralized algorithm conducted in the base station provide network with global information. Therefore, suitable CHs candidates are selected based on collected global information.

B. Overview of PSO

PSO is inspired by observing behavior of bird flocking or fish schooling. PSO begins with a group of random particle or random solution defined as (2), every particle aims to find the optimum solution through iterative process. Each particle interacts with one another while learning from their best experience (local best position) and is defined as (3).

The particle members tend to move towards a better region or solution found by other particle members (global best position) and is defined as (3). In each iterative process, the velocity (4) and position of each particle is updated using (5) and (6). The process will terminate when maximum number of iteration is fulfilled.

Assume in D-dimensional space, each particle represent as potential solution. The position x of particle i is represented by (2):

$$x_i = (x_{i1}, x_{i2}, \dots, x_{iD}) \quad (2)$$

Each particle also maintains a memory of its previous best position P_i and refers to the same global best position P_G , represent by (3):

$$P_i = (P_{i1}, P_{i2}, \dots, P_{iD}), P_G \quad (3)$$

Velocity for each particle can be represented by (4):

$$V_i = (V_{i1}, V_{i2}, \dots, V_{iD}) \quad (4)$$

Each particle updates its velocity and position by using the distance between the current positions and local best location, as well as the distance between the current position and the global best location. Velocity and position can be updated using these relations:

$$v_{k+1} = wv_k + c_1r(P_k - x_k) + c_2r(P_G - x_k) \quad (5)$$

$$x_{k+1} = x_k + v_{k+1} \quad (6)$$

where, w is the inertia weight for particle; c_1 is the cognition learning factor; c_2 is the social learning factor; r_1

and r_2 are random numbers that distributed between [0,1]. Inertia weight plays an important role to explore the searching field and converge on the searching result. A larger weight value can provide particle with better chance to explore. On the other hand, the smaller weight value can provide particle for better convergence. Eq. (7) is implemented in (5) to balance the particle exploitation and exploration ability [17].

$$w_{iter} = w_{max} - \left(iteration \times \frac{w_{max} - w_{min}}{iteration_{max}} \right) \quad (7)$$

In this work, learning factor such as cognitive learning factor and social learning factor will be altered to adapt to the changing environment. Previously, most authors fix the learning factor between 2 to 4. This makes it hard to converge to optimum solution. To overcome the problem, adaptive learning factors are proposed in this paper. The defined adaptive learning function is as follows:

$$c_1 = c_2 = 2 - \exp\left(-\frac{P_i/P_G}{iteration}\right) \quad (8)$$

Although adaptive learning function can provide very fast convergence speed, it may easily trap at local maximum if most of the particles stay in the same region. Therefore, to reduce the chances of getting trapped, particles reselect mechanism with hybrid will be applied to PSO. By using the concept of particles or birds getting tired with the current achievement, old particle will be replaced by the new particle with higher solution exploration energy.

The basic flows of particle reselect mechanism algorithms: if the global best value is continuously same for 7 iterations, 25% of the worst particles will be reinitialized and given with new life. The new created particles may use the same adaptive learning factor show in (8), but with the iteration count starting from 1 on the cycle it is being created.

III. SYSTEM MODEL

In this section, the structures of radio energy model and PSO based CHs selection will be illustrated. Fig. 1 is the radio energy model used for data communication including the wireless transmitter and receiver. The section continue with the framework illustration of PSO based CHs selection.

A. Radio Energy Model

The expended energy during transmission and reception for bits data to a distance between transmitter and receiver is given by (9). In this radio energy model, sensor nodes are assumed to have the ability to control the transmission power. It uses minimum transmit power to transmit the data to its destination with acceptable signal to noise ratio (SNR).

$$E_{Tx}(k, d) = E_{Tx-elec}(k) + E_{Tx-amp}(k, d) \quad (9)$$

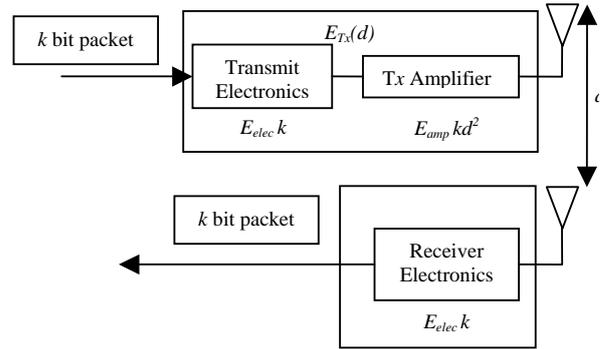


Figure 1. Radio energy model.

Two channel amplification models, the free space and the multipath fading is in the energy model (10). The distance d_0 threshold for swapping amplification model can be expressed as (11). If the distance d_0 between transmitter and receiver is bigger than threshold d_0 , the ϵ_{mp} model is used. Otherwise, the ϵ_{fs} model is used.

$$= \begin{cases} kE_{elec} + k\epsilon_{fs}d^2 & \text{if } d < d_0 \\ kE_{elec} + k\epsilon_{mp}d^4 & \text{if } d \geq d_0 \end{cases} \quad (10)$$

$$d_0 = \sqrt{\epsilon_{fs}/\epsilon_{mp}} \quad (11)$$

To receive k bit of data, the radio model express as in (12).

$$E_{Rx}(k) = E_{elec}k \quad (12)$$

B. PSO Based Cluster Heads Selection

This work assumes base station have unlimited power supply to run the algorithm for sensor nodes. PSO based CH selection is a central control that is conducted in the base station. Base station obtains the following information from sensor nodes:

- Sensor node location: Sensor node may acquire location information from global positioning system (GPS) or predefined location.
Note: Node location does not change after deployment.
- Initial energy level: Sensor node may acquire battery level through analog to digital converter (ADC) module.

Since base station collects initial energy level form all the sensor nodes during initial state, it can compute and simulate the remaining energy for each node after each round. Therefore, to save energy, sensor nodes do not need to transmit energy information to the base station.

Generally, PSO for CHs selects M number of CHs from network with N number of sensor nodes. Therefore, there are C_N^M number of combination. The performance for each set of combination can be evaluated by following formula which the overall output is described as fitness (13). Smallest fitness value represents the best combination among others.

$$fitness = \alpha f_1 + (1 - \alpha) f_2 \quad (13)$$

$$f_1 = \frac{\sum_{i=1}^N E(n_i)}{\sum_{m=1}^M E(CH_m)} \quad (14)$$

$$f_2 = \max_{m=1,2,\dots,M} \left\{ \frac{\sum_{i \in m} d(n_i, CH_m)}{|C_m|} \right\} \quad (15)$$

where f_1 (14) is the energy representative part, and it is equal to the sum of all member node energy $E(n_i)$ (not including CH) divided by the sum of all CH energy $E(CH_m)$. Referring to Eq. (14), f_2 represents the density and it is equal to the cluster with highest average distance between CH and joined member nodes $d(n_i, CH_m)$ divided by the total member nodes in the same cluster $|C_m|$.

A particle represents a possible cluster based network formation. The particle consists of a set of CHs which are randomly selected from the networks. Fig. 2 shows the concept of velocity and position update for the node. The sum of total vector from previous inertia velocity wV_i , cognitive learning $c_1r_1(P_i - X_i)$ and social learning $c_2r_2(P_g - X_i)$ will produce a new location which is labeled as virtual point in Fig. 2. Since there are no existing nodes on virtual point, therefore nodes (can be CH) who have the shortest distance to virtual point will be selected as new CH. In this case, node A will be selected as the new cluster head.

The flow for PSO CHs selection can be described as follow and shows in Fig. 3:

- A) *Stage 1:* Initiation of network topology, it includes the deployment of sensor nodes in the sensing field and the BS location (coordinate).
- B) *Stage 2:* CHs selection via PSO, initiate particles (a set of CHs formation) velocity and position.
- C) *Stage 3:* Calculate the fitness value of each particle and update P_i and P_g based on the calculated results.
- D) *Stage 4:* Update the particle velocity and particle position based on P_i and P_g in the PSO equations
- E) *Stage 5:* Calculate the fitness value of each particle and update P_i and P_g based on calculated results.

- F) *Stage 6:* Repeat step (d) to step (e) until all the particles have gone through these steps.
- G) *Stage 7:* Repeat step (d) to step (f) until the iteration count reaches the maximum iteration.
- H) *Stage 8:* BS loaded with PSO algorithm broadcasts the calculated CHs ID to the network. The process of cluster forming is similar to the LEACH protocol.
- I) *Stage 9:* Repeat the step (b) to step (h) until the iteration count reached the maximum iteration (round).

Set-up phase from LEACH will be implemented after base station assigns the CH candidates. Selected CHs will broadcast advertisement message with same signal strength to other nodes. Nodes will join CH with the strongest signal strength because strong signal strength means the CH is nearer so the transmission energy is minimal.

Fig. 4 shows the example of particle representation with five CHs in each particle. As shown in the figure, the particle represents a set of randomly selected CHs during initial state. There is no duplicated sensor node ID in the same particle so the same ID can exist few times in different particle. Generally, particles in PSO communicate with one another by exchanging information about the success of each particle. In this work, since the particle consists of a set of CHs, the pairing problem among CHs in different particles will affect the accuracy of social interaction among the particles, therefore it is important to assign proper CH in particle 'A' refer to particular CH in particle 'B' as shown in Fig. 5. The fact is, better assigning solution will lead to faster convergence of solution in the search space.

Fig. 5 shows the example of CH pairing of particle 'A' and particle 'B'. This section will demonstrate the CH pairing operation of both particles where CHs ID are randomly generated into each particle. During the pairing operation, each CH in the particle will refer to a particular CH in another particle based on the minimum distance between them. In the pairing algorithm, the distances between all the CH have to be calculated. Then, it will pair up with the CHs with the minimum distance followed by the second lower minimum distance and so on. Paired CHs will not be reconsidered again after the pairing operation.

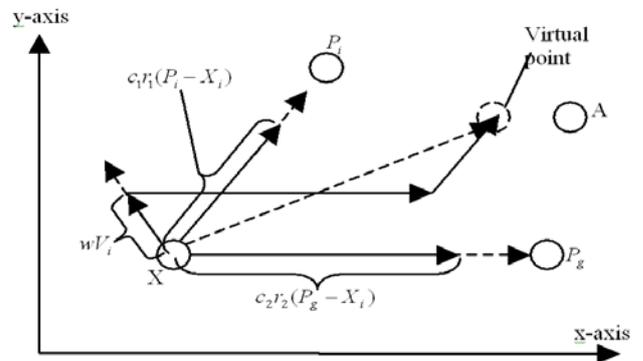


Figure 2. Velocity and position update model.

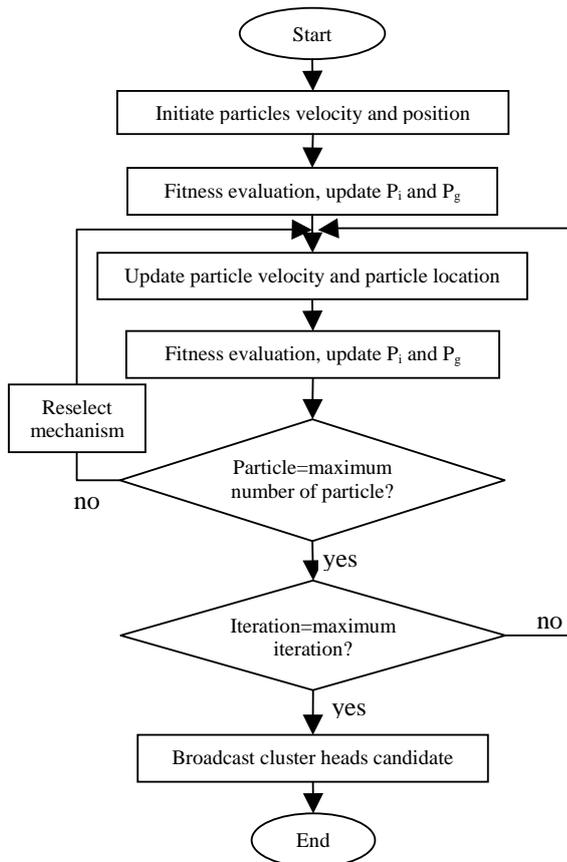


Figure 3. General flow chart for proposed PSO.

Sensor Node ID

Particle 1:	32	64	45	8	17
Particle 2:	8	43	27	10	12
Particle 3:	24	56	35	12	51
⋮					
Particle 17:	42	25	23	8	42

Figure 4. Particles representation of five cluster heads in each particle.

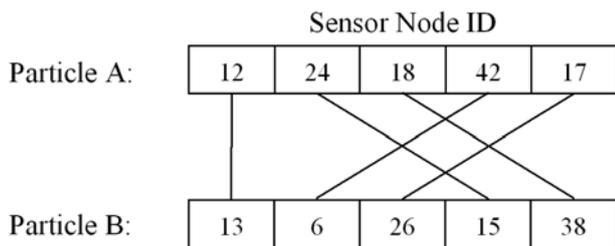


Figure 5. Cluster heads pairing of particle A and particle B.

IV. RESULTS AND DISCUSSIONS

The simulation experiments are carried out in MATLAB to evaluate the performance of proposed method compared to LEACH protocol. The simulation parameters for radio model and network are given in Table I whereas simulation parameters for PSO algorithm are given in Table II. The performance of the proposed method is compared with LEACH in terms of network lifetime, energy use per round and data receive at base station.

The same simulation topology is used for fair comparisons. The simulation contains 100 nodes with equal initial energy. Sensor nodes are distributed in a 100 m × 100 m network area and the base station is located at coordinates x - 50 m, y - 200 m.

In Fig. 6, the simulation result shows that the proposed method outperforms LEACH protocol in terms of FND round. The FND round for the proposed method is 93 rounds, while LEACH can only achieve 53 rounds. Around 75% of lifetime improvement compare to LEACH protocol. The significant improvement is mainly due to a better CH selection approach. LEACH elect CHs based on probability model, therefore it may cause inefficient CHs formation which will drain more energy from the nodes. Meanwhile the proposed method considers critical factors such as energy and node density to select exactly 5% of suitable CHs among other nodes.

Fig. 7 shows the data received at the base station. It is important to prolong the network lifetime while maintaining continuous data from sensor nodes. Data sent to the base station before FND will be more desirable, it is because more nodes can provide accurate data for data aggregation in CH. Simulation result shows that proposed method outperformed the LEACH.

TABLE I. SIMULATION PARAMETERS I

Parameter	Value
Network size	100 x 100 m ²
Base station location	x = 50 m, y = 200 m
Simulation round	200
Number of node, <i>n</i>	100
Cluster head probability, <i>p</i>	0.05
Initial energy, <i>E₀</i>	0.05 J
Packet Size, <i>k</i>	4000 bit
Transceiver energy, <i>E_{elec}</i>	50 nJ/bit
Aggregation energy per bit, <i>E_{DA}</i>	5 nJ/bit
Free space amplifier energy, <i>E_{fs}</i>	10 pJ/bit/m ²
Free space amplifier energy, <i>E_{fs}</i>	0.0013 pJ/bit/m ⁴

TABLE II. SIMULATION PARAMETERS II

PSO	
Number of particle	20
Number of Cluster	5
Simulation round	50
α	0.5

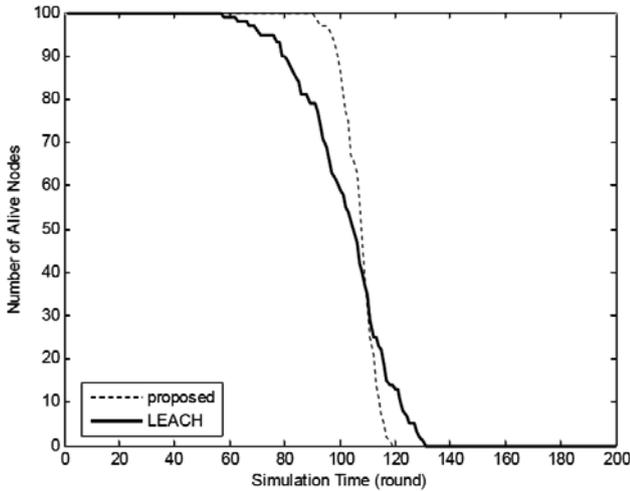


Figure 6. Network lifetime comparison.

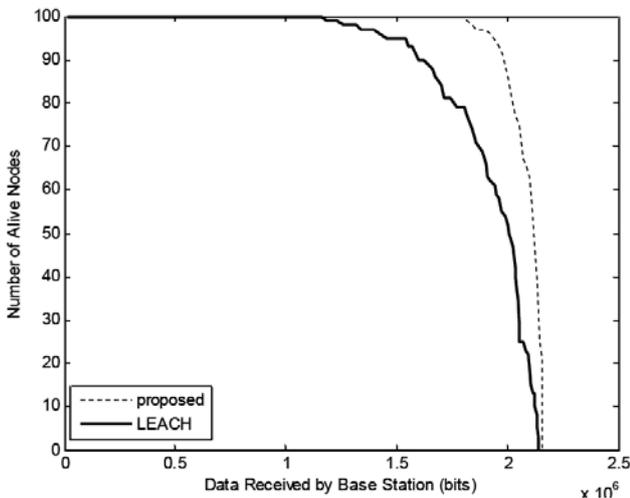


Figure 7. Data received at the base station (bits).

Fig. 8 shows the energy consumption per round. It clearly shows that energy consumption in every round before FND for proposed method is very stable. The proper CHs location will result in less energy consumed per round. On the other hand, unpredictable numbers of elected CHs results in unstable energy consumption per round to LEACH protocol. Table III shows the average global best fitness for 30 networks cycles in different sample numbers (network topologies).

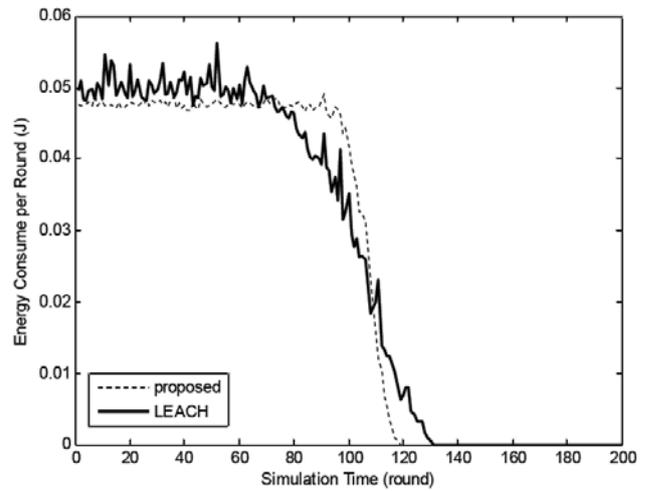


Figure 8. Energy consumption per round.

TABLE III. AVERAGE FITNESS COMPARISONS

Sample	Average Global Best fitness for 30 network cycles	
	Standard PSO	Proposed PSO
1	11.3667	8.8667
2	11.0333	7.8667
3	9.5333	10.4333
4	9.4667	9.06333
5	11.0667	7.9000

Standard PSO integrated with inertia weight function is compared to the proposed PSO integrated with the adaptive learning factor and particles reselect mechanisms. Proposed PSO can obtain better fitness value compared to standard PSO. For sample 4, standard PSO performs better than proposed PSO. It is due to the random characteristic of PSO. It cannot guarantee that the proposed method will deliver better result but the chances to achieve better result will be higher.

V. CONCLUSION

In conclusion, proper selection of CHs formation using PSO shown improvement over LEACH protocol in terms of network lifetime, total data received at the base station before FND and energy consumption per round.

In future, CHs set in particles can share more information to avoid PSO common problems such as trapped at local maximum. By sharing information and hybrid with fuzzy logic, inter particles may have group global view to achieve better combination.

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