Hybrid iterative learning control design: Application to inverted pendulum

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Abstract — This paper deals with the design of hybrid iterative learning control (ILC) strategy. The design strategy is based upon solving the robust performance to obtain feedback controller. In order to improve the dynamic performance of the closed loop system a first order filter is designed based on bode plot response of the system. The hybrid ILC is added outside the existing feedback control loop through a modified reference signal without reconfiguring the control signal. The hybrid ILC strategy is tested on inverted pendulum system and the experimental results show that the proposed control strategy is easy to design, implement and also effective in controlling the angle of inverted pendulum system.

Keywords - hybrid ILC, zero phase shift filter, inverted pendulum

I. INTRODUCTION

Industrial Automation plays an important role in the global economy. The correct incentive for applying automation in industries is to increase productivity and quality. It has a notable impact in a wide range of industries. Mechatronics system and robots have replaced humans in the assistance of performing repetitive tasks. It is well known that Proportional, Integral and Derivative (PID) control is a generic feedback control mechanism widely used in industrial control systems [1]. The finite-time tracking control is difficult with the use of standard PID controller. To achieve a better tracking performance with every new operation, a learning component is added to the PID controller. This learning component allows the PID controller to learn from the tracking errors of the previous operations. This technique is referred to as ILC [2].

ILC is a feedforward control strategy that updates control signals at every repeated operation through iterative learning. As the number of iterations increase, the system tracking error over the entire operation time period including the transient portion will decrease and eventually vanish. ILC is capable of eliminating the effects of uncertainties on the system output and in turn improves the control performance by operating the system repeatedly. After each trial, the system is reset and the learning controller makes a correction to the input which is to be used in the next trial [3].

In 1984, the method of iterative learning control was introduced which is considered to be the origin of ILC [4]. ILC algorithms achieve exact tracking of the specified task that repeats in a batch process [5]. Mikael Norrloof and Svante Gunnarsson investigated the influence of the disturbances in the frequency domain with the help of linear framework. They showed that the choice of design of filters, in particular the filter Q, affected the way load and measurement disturbances affected the performance of ILC algorithms [6].

Most of the ILC approaches proposed in the literature focuses on the determination of the convergence conditions and only a very few results deal with solving these conditions to obtain the ILC filters. In the literatures, [7-12] feedback-based ILC schemes are introduced and in these schemes the design of the ILC filters and the feedback controller are carried out separately. To the best of our knowledge, all of the existing feedback-based ILC schemes in the literature are based upon the design of the ILC filters and the feedback controller separately. The main advantage of designing the feedback controller and ILC filters in two separate steps permits to assign the desired performance at the first iteration through the feedback controller, and the performance improvement of the iterative process through ILC. Hence, there is no need to design the learning filter if a feedback controller is designed to satisfy the robust performance condition.

One of the most important reasons why ILC has not been widely in industry till now is that when a learning controller is implemented over a large number of trials, the tracking error will begin to grow up and results in the instability of the system. When ILC begins to operate, low frequency error becomes negligible, the effect of the high frequencies becomes evident and the overall tracking error...
noticeably grows. To overcome this, many filtering techniques have been introduced and designed in order to cut off unnecessary frequencies in the learning process. The design of the learning function involves complex mathematical calculations which are tedious to calculate and implement in real time. Another drawback is that the replacement of the available control structure with ILC results in wastage of money. The strategy which provides an addition of learning block that could be appended with the available industrial controller would be a better solution to this problem.

The proposed Hybrid ILC strategy modifies the reference signal in order to achieve perfect tracking without reconfiguring the existing PID control loop. The use of Zero phase shift filter provides opportunities for advanced filtering, signal processing and high frequency attenuation thereby avoiding the phase lag in the output. In this paper, a hybrid ILC design procedure guarantees robust performance for the feedback system and the convergence of the iterative process, for both stable and unstable uncertain linear time invariant systems. For the sake of simplicity, single-input single-output plant is considered, but the results can be generalized to multivariable systems. Finally, the approach is validated experimentally on an inverted pendulum.

This paper is organized as follows. Section 2 describes the configuration of Hybrid ILC strategy. In section 3, dynamics of inverted pendulum system is presented. In Section 4, the implementation of hybrid ILC scheme for an inverted pendulum system is demonstrated with experimental results. Finally, conclusions are given in Section 5.

**II. HYBRID ITERATIVE LEARNING CONTROLLER**

The block diagram of proposed Hybrid ILC is presented in Fig. 1, where \( G_P(s) \) denotes the dynamics model of the process, \( G_C(s) \) denotes the feedback controller, \( G_F(s) \) and \( G_Z(s) \) are the low pass filter and zero phase shift filter respectively, \( y_d(t) \) is the reference (or) desired output signal, \( y_i(t) \) is the actual output signal, \( y_{d,i}(t) \) is the modified reference signal, \( e_i(t) \) is the error signal and \( d(t) \) is the disturbance. The subscript \( i \) in the above notations indicates the iteration index (or) trail index. The blocks labeled MEM denote the memory that store signals of the current iteration for use in the next iteration. Here, the Hybrid ILC is added outside the existing feedback control loop through a modified reference signal without reconfiguring the control signal. The proposed controller can be appended to the controller available in the industry which improves the performance of the existing industrial controller.

The updating control law of the Hybrid ILC is given in (1), (2) and (3).

\[
\begin{align*}
e_i &= y_d - y_i, \quad (1) \\
y_1 &= \frac{G_C G_D}{1 + G_C G_D} y_{d,i+1}, \quad (2) \\
y_{d,i+1} &= G_Z y_d + G_e e_i. \quad (3)
\end{align*}
\]

Where \( G_z \) is the transfer function of the zero phase shift filter and is given in (4) as

\[
G_z = \frac{a}{s + a} \quad (4)
\]

**III. EXPERIMENTAL STUDIES**

The mechanical model of Inverted Pendulum system is shown in Fig. 2.

An inverted pendulum is a popular mechatronics application which is popular for its non-linear and unstable characteristics, uncertainty in friction terms, lack of state variable measurements, and the easy way disturbances are introduced in the process. It was developed on the theory of rocket propeller. It simulates the flight control of rocket or missile during initial stages of flight and stabilizing control in
walking robots. The largest implemented use is on huge lifting cranes on shipyards. When moving the shipping containers back and forth, the cranes move the box accordingly so that it never swings or sways. It always stays perfectly positioned under the operator even when moving or stopping quickly.

The effectiveness of the proposed Hybrid ILC strategy is demonstrated by applying it to the inverted pendulum system. The system under consideration consists of a cart that moves on a sliding shaft horizontally. The pendulum rod is pivoted on the cart and it swings freely in the vertical plane. Rotary encoder mounted on the axis of the rotation allows the measurement of the vertical angle of the rod. The translation of the cart is enabled by AC servo motor and toothed belt. The pendulum rod starts to swing when the motor drives a horizontal force to move the cart back and forth. The control objective is to swing up a pendulum from its stable equilibrium point (vertical down point) to the unstable equilibrium point (vertical up point) and maintain it at that position by regulating the force applied to the cart. Control algorithm is realized with Mathworks Matlab, Simulink and xPC target products on a personal computer (PC).

Fig. 3 shows the real time interface of inverted pendulum system. The control box of inverted pendulum system houses a power supply, a servo amplifier and circuit protection electronics. The control hardware consists of a PC for program development and user interface, a Digital Signal Processing based motion control card with advanced data I/O and storage capability. Simulink is used as an interactive tool for analyzing inverted pendulum system. The control box acquires the signals from the sensors mounted on the inverted pendulum plant and routes them to PC through the motion controller. PC processes the input signals according to the control algorithms to calculate the control signals. Then the control signal is sent to the plant via the motion controller and the control box.

IV. MATHEMATICAL MODELING OF INVERTED PENDULUM SYSTEM

The parameters associated with Fig. 2 are described in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description of the parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Cart mass</td>
<td>1.096 kg</td>
</tr>
<tr>
<td>m</td>
<td>Rod mass</td>
<td>0.109 kg</td>
</tr>
<tr>
<td>b</td>
<td>Friction coefficient of the cart</td>
<td>0.1 N/m/sec</td>
</tr>
<tr>
<td>l</td>
<td>Distance from the rod axis rotation center to the rod mass center</td>
<td>0.2 m</td>
</tr>
<tr>
<td>I</td>
<td>Rod inertia</td>
<td>0.0034 kgm²</td>
</tr>
<tr>
<td>F</td>
<td>Force acting on the cart</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>Cart position</td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>Angle between the rod and vertically upward direction</td>
<td></td>
</tr>
<tr>
<td>φ</td>
<td>Angle between the rod and vertically downward direction</td>
<td></td>
</tr>
</tbody>
</table>

Let N and P be the interactive force of cart and rod in the horizontal and vertical direction respectively. With the knowledge of the Newton’s law [13], the equations describing the system are obtained as

\[ F = M \ddot{x} + b \dot{x} + N. \]  
\[ N = m \frac{d^2\theta}{ds^2} [x + l \sin \theta]. \]

Combining (5) and (6), the dynamic equation can be reformulated as in (7).

\[ (M + m) \ddot{x} + b \dot{x} + mgl \cos \theta - ml \theta \dot{\theta} \sin \theta = F. \]  

The dynamic equation of Inverted Pendulum System in the vertical direction is

\[ F' - mg = m \frac{d^2 \theta}{ds^2} (l \cos \theta). \]

By moment conservation,

\[ -P' \sin \theta - Nl \cos \theta = l \dot{\theta}. \]

Let \( \theta = \pi + \phi \) and assuming \( \phi << 1 \), the approximation obtained are \( \cos \theta = -1 \), \( \sin \theta = -\phi \), \( \left( \frac{d^2 \theta}{ds^2} \right)^2 = 0 \). \( u \) denote the input force of the object and by substituting (6) and (8) in (9) and linearizing

\[ \left( (l + ml^2) \ddot{\theta} - mg l \dot{\theta} = ml \ddot{x} \right) \]

\[ \left( (M + ml) \ddot{x} + b \dot{x} - ml \ddot{\theta} = u \right). \]

Taking laplace transform of (10) and rearranging, the transfer function of the system is obtained in (11) and (12).

\[ \frac{\theta(s)}{X(s)} = \frac{mls}{s^2 + \frac{b(l + ml^2)}{q} s + \frac{(M + m)gl}{q} - \frac{bml}{q}}. \]

\[ \frac{\theta(s)}{U(s)} = \frac{\frac{ml}{q}}{s^2 + \frac{b(l + ml^2)}{q} s + \frac{(M + m)gl}{q} - \frac{bml}{q}}. \]

As \( v = \dot{x} \), the transfer function of pendulum rod and cart acceleration is obtained as in (13).

\[ \frac{\dot{\theta}(s)}{\dot{v}(s)} = \frac{ml}{(l + ml)^2 s^2 - mgl}. \]
With the parameters of actual system given in table 1, the model is obtained is given in (13) as
\[
\frac{0.02725}{0.0102125s^2 - 0.26705}.
\] (14)

The above plant transfer function in (14) is a second order unstable system.

V. EXPERIMENTAL RESULTS AND OBSERVATIONS

In order to verify the effectiveness of the hybrid ILC for balancing the inverted pendulum, a series of experiments were carried out. In this paper, the parameters of PID controller are selected as proportional gain $K_p = 19.55$, integral gain $K_I = 28.5$ and derivative gain $K_d = 3.82$ to show that the proposed strategy is suitable for balancing tasks [14]. The cutoff frequency ‘a’ of the zero phase shift filter $G_z$ is determined as 4 rad/sec based on bode plot response of the inverted pendulum system.

Fig. 5 shows the experimental results of inverted pendulum with hybrid ILC.

The stabilization of the pendulum rod in the upright position and smoothness of the trajectory with faster approaching rate shows that hybrid ILC effective in the control of inverted pendulum. Even in steady state there are no oscillations as the filter in the forward path $G_F$ attenuates the effect of high frequency noise. The phase shift produced by the filter $G_F$ is removed by the zero phase shift filter in the feedback path.

To verify the disturbance rejection property of the proposed hybrid ILC, the pendulum is hit at the time 15 seconds and the response is shown in figure 6.

The cart moves quickly in response to the external force applied to the inverted pendulum and this fast motion reduces the influence of external disturbance on the pendulum. Overall, the proposed controller controls the motion of inverted pendulum and maintains stability and robustness. The results thus demonstrate the usefulness of the proposed controller in handling the unstable system.

![Figure 5. Experimental results of inverted pendulum system with hybrid ILC](image1)

![Figure 6. Disturbance rejection response inverted pendulum system with hybrid ILC](image2)
VI. CONCLUSIONS

In this paper, the hybrid ILC strategy to solve the dynamics of an inverted pendulum has been proposed. The hybrid ILC strategy has the capability of achieving the control objective smoothly at faster rate. It also has an excellent stabilizing ability for suffering an external disturbance applied to the pendulum proving the robustness of the control system. Validity of Hybrid ILC on the inverted pendulum dynamics have been verified by the experimental results. The proposed control strategy can also be used to control a wide class of nonlinear control systems.

REFERENCES