Amplify-and-Forward SIMO/MISO Distributed Relay Networks under Power Constraints

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Abstract - This paper presents an optimal solution for amplify-and-forward (AF) single-input multiple-output (SIMO) and multiple-input single-output (MISO) wireless distributed relay networks. The optimal solutions are determined based on minimum mean square error (MMSE) criterion for optimum nondiagonal and diagonal relay amplifying matrices under the transmit power constraints at the relays. The bit error rate (BER) and the achievable rate of the AF SIMO/MISO wireless distributed relay network are evaluated for both nondiagonal and diagonal relay amplifying matrices. Additionally, the system performance of the AF MISO wireless distributed relay network are better than that of the AF SIMO one when the optimal nondiagonal relay amplifying matrices are applied. In contrast to this, the AF SIMO wireless distributed relay network shows a better system performance than the AF MISO one when the optimal diagonal relay amplifying matrices are applied.

Keywords - Single-input multiple-output (SIMO), multiple-output single-input (MISO), minimum mean square error (MMSE), distributed relays.

1. INTRODUCTION

A wireless distributed relay network consists of small and geographically distributed sensors in order for them to transmit their signals received from the sources to the destinations [1]. According to the role of relays, relay schemes can be generally classified as amplify-and-forward (AF), decode-and-forward (DF), compute-and-forward, and compress-and-forward relay scheme [1]–[7]. In [4], the DF wireless distributed relay network was investigated for the sake of enhancing the system performance. It required a signal processing at the relays for decoding and forwarding in [4]. In contrast to the DF distributed relaying scheme, the AF relay scheme proposed in this paper only forwards an amplified version of theirs received signals from sources to destinations. The authors in [5], [6] studied the AF wireless distributed relay system. In [5], the single-input single-output relaying system was studied, while the multiple-input multiple-output relaying system was investigated [6]. The authors in [5], [6] presented the relay amplifying matrices based on the maximization signal-to-noise ratio (SNR) and minimum mean square error (MMSE) with the transmit power constraints at the relays, respectively.

The authors proposed the cooperative AF single-input multiple-output (SIMO) systems [7], [8] and multiple-input single-output (MISO) systems [9], [10]. Additionally, the capacity region of the symmetric MISO and SIMO Gaussian interference channels was studied in [11]. The authors of this paper not related to the relay power constraint during data transmission. The current paper focuses on the AF SIMO/MISO wireless distributed relay systems under the relay power constraint. The MMSE criterion is used for the benefit of determining the relay amplifying matrices. Depending on the distribution of relay, the relay amplifying matrices can be either nondiagonal or diagonal. Hence, both nondiagonal and diagonal relay amplifying matrices will be derived. A positive scalar factor and an equalizing gain factor will be used for determining them. By adopting them, the system performances will be evaluated from the bit error rate (BER) and achievable rate. The BER performance and achievable rate of the AF MISO wireless distributed relay system are better than that of the AF SIMO one when the optimal non-diagonal relay amplifying matrices are applied. On the contrary to this, the AF SIMO wireless distributed relay system shows better BER performance and achievable rate than...
the AF MISO one when the optimal diagonal relay amplifying matrices are used.

The rest of this paper is organized into five sections. Section II describes the system models and data transmission strategies applied. Section III presents MMSE wireless distributed relay schemes based on the AF SIMO/MISO strategies. Section IV studies the achievable rates by adopting both nondiagonal and diagonal relay amplifying matrices derived in Section III. Section V provides the simulation results. Finally Section VI concludes the paper.

Notation: Matrices, vectors, and scalars are denoted, respectively, by uppercase boldface, lowercase boldface, and italic characters (e.g., \( \mathbf{A} \), \( \mathbf{a} \), and \( a \)). The transpose, pseudo-inverse, trace, complex conjugate, inverse, and Hermitian of \( \mathbf{A} \) are denoted, respectively, by \( \mathbf{A}^T \), \( \mathbf{A}^+ \), \( tr(\mathbf{A}) \), \( \mathbf{A}^* \), \( \mathbf{A}^{-1} \), and \( \mathbf{A}^H \). An \( N \times N \) identity and diagonal matrix are denoted, respectively, by \( \mathbf{I}_N \) and \( diag(a_1, \cdots, a_N) \). The expectation operator, the real operator, and the Hadamard product operator are denoted by \( E[\cdot] \), \( \text{Re}(\mathbf{A}) = (\mathbf{A} + \mathbf{A}^*)/2 \), and \( \circ \), respectively. Notations \( |a| \), \( |\mathbf{a}| \), and \( |\mathbf{A}|_F \) denote the absolute value of \( a \) for any scalar, 2-norm of \( \mathbf{a} \), and Frobenius-norm of \( \mathbf{A} \), respectively.

II. SYSTEM MODEL

A. SIMO Distributed Relay System

![AF SIMO wireless distributed relay system](image)

Figure 1 shows an AF wireless distributed relay network with \( N \) distributed relays between a source and \( M \) destinations (\( N \geq M \)). For data transmission, there are two stages where a source transmits a signal symbol \( s \) in Stage I and the relays retransmit their received signals to destinations in Stage II by employing the relay amplifying matrix. Assume that all nodes have only a single antenna. However, all relays are shown with separate receive and transmit antennas for the convenience of illustration. Here, a relay amplifying matrix can be either nondiagonal or diagonal according to the assumption that they are arbitrarily located to be in close proximity with high SNR links among them so that received signals from the source and channel coefficients can be passed between relays with negligible error. Assume that all nodes can have a perfect knowledge of the forward and backward channels. This is a feasible assumption through feedbacks from the destinations.

Let \( \mathbf{h}_s \in \mathbb{C}^{N \times 1} \) denote the perfectly known channel coefficients complex column vector from the source to the \( i \)-th relays as

\[
\mathbf{h}_s = [h_{s,1}, h_{s,2}, \cdots, h_{s,N}]^T
\]

where \( h_{s,i}, i = 1, \cdots, N \), is the \( i \)-th entry of \( \mathbf{h}_s \), representing the channel coefficient from the source to the \( i \)-th relay. Using (1), the received signal complex column vector \( \mathbf{r}_1 \in \mathbb{C}^{N \times 1} \) at the relay outputs is written as

\[
\mathbf{r}_1 = \mathbf{h}_s + \mathbf{v}_s
\]

where \( \mathbf{v}_s \in \mathbb{C}^{N \times 1} \) is a zero mean complex thermal additive white Gaussian noise (AWGN) column vector with covariance matrix \( \sigma_v^2 \mathbf{I}_N \). At the relay outputs, the amplified signal complex column vector \( \mathbf{x}_r \in \mathbb{C}^{N \times 1} \) is written as

\[
\mathbf{x}_r = \mathbf{F}_r \mathbf{r}_1
\]

where \( \mathbf{F}_r \in \mathbb{C}^{N \times N} \) is a nondiagonal relay amplifying matrix at the relays to enhance performance at the destinations. Let \( \mathbf{H}_y \in \mathbb{C}^{M \times N} \) denote the perfectly known channel coefficient complex matrix from the relays to the destinations as

\[
\mathbf{H}_y = [h_{y,1}, h_{y,2}, \cdots, h_{y,M}]^T
\]

where \( h_{y,m} = [h_{y,m,1}, \cdots, h_{y,m,N}], m = 1, \cdots, M \), is a row vector, representing the channel coefficient from all relays to the \( m \)-th destination. Additionally, \( h_{y,m} \) and \( h_{y,m,N} \) are assumed, respectively, to be independent identically distributed (i.i.d.) with a zero mean circular complex Gaussian of unit variance and quasi-static Rayleigh fading so that they stay fixed during data transmission. Finally, at the destinations, the received complex signal column vector \( \mathbf{y}_1 \in \mathbb{C}^{M \times 1} \) can be represented as

\[
\mathbf{y}_1 = \mathbf{H}_y \mathbf{x} + \mathbf{v}_y
\]

where \( \mathbf{v}_y \in \mathbb{C}^{M \times 1} \) is zero-mean complex thermal AWGN column vector with covariance \( \sigma_v^2 \mathbf{I}_N \). Substituting (2) and (3) into (5), the received complex signal column vector \( \mathbf{y} \) can be rewritten as vector \( \mathbf{y}_1 \in \mathbb{C}^{M \times 1} \) at the destinations can be rewritten as

\[
\mathbf{y}_1 = \mathbf{H}_y \mathbf{F}_r \mathbf{r}_1 + \mathbf{v}_y.
\]
arbitrarily far from each other. In this case, a relay amplifying matrix at the distributed relay is assumed to be diagonal, i.e., \( F_i = \text{diag} (f_i) = \text{diag} (f_{i1}, \ldots, f_{iN}) \). Hence, let the \( N \times 1 \) vector \( a = \text{diag}(A) \) denote the diagonal elements of \( A \). Hence, using this vector notation, \( F_i \) can be expressed by \( f_i = \text{diag}(F_i) \).

**B. MISO Distributed Relay System**

An AF wireless distributed relay network with \( N \) distributed relays between \( M \) sources and a destination is shown in Fig. 2. Similarly, there are two stages for data transmission in an entire network. In stage I, sources transmit signal symbols \( s_m, m = 1, \cdots, M \), while relays multiplied by the relay amplifying matrix retransmit their received signals to the destination in Stage II. Additionally, all nodes are also assumed to have only a single antenna, similar to the SIMO system case. Furthermore, all relays are assumed to be close each other during data transmission. Due to this assumption, the relay amplifying matrix is nondiagonal.

Let \( \mathbf{H}_s \in \mathbb{C}^{N \times M} \) denote the perfectly known channel coefficient complex matrix from sources to relays as

\[
\mathbf{H}_s = \left[ h_{s1}, h_{s2}, \cdots, h_{sM} \right]
\]

where \( h_{sm} \in \mathbb{C}^{N \times 1} = \left[ h_{s1m}, \cdots, h_{sNm} \right] \), \( m = 1, \cdots, M \), is a column vector, which representing the channel coefficient from the \( m \)-th source to all relays. While, let \( \mathbf{h}_r \in \mathbb{C}^{N \times 1} \) denote the perfect channel coefficient row vector from the relays to the destination as

\[
\mathbf{h}_r = \left[ h_{r1}, h_{r2}, \cdots, h_{rN} \right]
\]

where \( h_{rm}, i = 1, \cdots, N, i \)-th element of \( \mathbf{h}_r \), which stands for the channel coefficient from the \( i \)-th relay to the destination. It is assumed that \( h_{sm} \) and \( h_{rm} \) are i.i.d. with a zero-mean circular complex Gaussian of the unit variance and quasi-static Rayleigh fading.

Therefore, at the relay inputs, the received signal complex column vector \( \mathbf{r}_i \in \mathbb{C}^{N \times 1} \) can be written as

\[
\mathbf{r}_i = \mathbf{H}_s \mathbf{s} + \mathbf{v}_i.
\]

Then, at the relay outputs, the amplified signal complex column vector \( \mathbf{x}_i \in \mathbb{C}^{N \times 1} \) can be represented as

\[
\mathbf{x}_i = F_i \mathbf{r}_i
\]

where \( F_i \in \mathbb{C}^{N \times N} \) is also called a nondiagonal amplifying relay matrix. Finally, at the destination, the received complex signal \( \mathbf{y}_i \in \mathbb{C}^{N \times 1} \) can be written as

\[
\mathbf{y}_i = \mathbf{h}_r \mathbf{x}_i + \mathbf{v}_y.
\]

Here, the subscript \( i \) in \( r_i, x_i, F_i \), and \( y_i \) denotes the AF MISO wireless distributed relay system. Additionally, unlike the above assumption, all relays are arbitrarily far from each other. Hence, similar to the AF SIMO system case, \( F_i \) can be written as or \( f_i = \text{diag}(F_i) \), respectively. In the following two sections, the nondiagonal and diagonal relay amplifying matrices \( F_i \), \( i = 1, 2 \), will be determined by applying the MMSE criterion for AF SIMO and MISO wireless distributed relay networks.

**III. MMSE Distributed Relay Scheme**

**A. SIMO Distributed Relay System**

1) **Nondiagonal Relay Amplifying Matrix Design**: The nondiagonal relay amplifying matrix to minimize mean square error (MSE) between the signal component of the received signal at the destinations and the scaled signal at the source can be found from

\[
\mathbf{F}_i = \arg \min J(\mathbf{F}_i)
\]

\[
s.t. E[|\mathbf{x}_i|^2] = p_i
\]

where the cost function \( J(\mathbf{F}_i) \) with a positive scalar factor \( \alpha \) selected by a designer [15] and the total transmit power usage \( p_i \) at the relays are written, using \( (3) \) and \( (6) \), respectively, as

\[
J(\mathbf{F}_i) = \sum_m E[|y_{im}|^2 - \alpha |\mathbf{f}_i|^2]
\]

\[
= \sigma_r^2 \sum_m \mathbf{h}_s \mathbf{F}_i \mathbf{h}_r \mathbf{h}_r^H + \mathbf{h}_r^H \mathbf{h}_m - \sigma_r^2 \alpha \sum_m \mathbf{h}_s \mathbf{F}_i \mathbf{h}_r \mathbf{h}_m - \sigma_r^2 \alpha \sum_m \mathbf{h}_s \mathbf{F}_i \mathbf{h}_r \mathbf{h}_m
\]

\[
- \sigma_r^2 \alpha \sum_m \mathbf{h}_s \mathbf{F}_i \mathbf{h}_r + M \sigma_r^2 \alpha^2 + M \sigma_r^2
\]

where

\[
\mathbf{H}_s = \mathbf{h}_r \mathbf{h}_r^H + \sigma_r^2 \mathbf{I}_N.
\]

Here, \( \alpha \) is, in fact, used to adjust the total transmit power usage \( p_i \) at the relays. Taking the partial derivative of
Using (17), the optimal $F_1^*$ can be simplified as
\[
F_1^* = \sigma^2 \alpha \mathbf{H}_s \mathbf{H}_s^H
\]  
where
\[
\mathbf{H}_s = \sum_{m=1}^{M} \mathbf{h}_m^H \mathbf{h}_m
\]  
and equalizing gain factor at the destinations
\[
\gamma = \frac{\sigma^2 \alpha}{\sigma^2 \alpha + \sigma^2 \alpha}.
\]  
Additionally, using the KKT conditions [14], the optimal nondiagonal relay amplifying matrix can be derived.

Using (15) and (18), the optimal $\alpha^*$ can be written as
\[
\alpha^* = \frac{\sqrt{p_i}}{\sigma^2 \sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}}.
\]

Substituting (21) into (18), the optimal $F_1^*$ for the AF SIMO distributed relay system under the relay power constraints can be written as
\[
F_1^* = \frac{\mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H}{\sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}} \sigma^2 \alpha \mathbf{H}_s \mathbf{H}_s^H.
\]

Using (15) and (18), the optimal $\alpha^*$ can be written as
\[
\alpha^* = \frac{\sqrt{p_i}}{\sigma^2 \sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}}.
\]

Substituting (21) into (18), the optimal $F_1^*$ for the AF SIMO distributed relay system under the relay power constraints can be written as
\[
F_1^* = \frac{\mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H}{\sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}} \sigma^2 \alpha \mathbf{H}_s \mathbf{H}_s^H.
\]

Using (15) and (18), the optimal $\alpha^*$ can be written as
\[
\alpha^* = \frac{\sqrt{p_i}}{\sigma^2 \sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}}.
\]

Substituting (21) into (18), the optimal $F_1^*$ for the AF SIMO distributed relay system under the relay power constraints can be written as
\[
F_1^* = \frac{\mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H}{\sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}} \sigma^2 \alpha \mathbf{H}_s \mathbf{H}_s^H.
\]

Using (15) and (18), the optimal $\alpha^*$ can be written as
\[
\alpha^* = \frac{\sqrt{p_i}}{\sigma^2 \sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}}.
\]

Substituting (21) into (18), the optimal $F_1^*$ for the AF SIMO distributed relay system under the relay power constraints can be written as
\[
F_1^* = \frac{\mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H}{\sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}} \sigma^2 \alpha \mathbf{H}_s \mathbf{H}_s^H.
\]

Using (15) and (18), the optimal $\alpha^*$ can be written as
\[
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\]

Substituting (21) into (18), the optimal $F_1^*$ for the AF SIMO distributed relay system under the relay power constraints can be written as
\[
F_1^* = \frac{\mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H}{\sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}} \sigma^2 \alpha \mathbf{H}_s \mathbf{H}_s^H.
\]

Using (15) and (18), the optimal $\alpha^*$ can be written as
\[
\alpha^* = \frac{\sqrt{p_i}}{\sigma^2 \sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}}.
\]

Substituting (21) into (18), the optimal $F_1^*$ for the AF SIMO distributed relay system under the relay power constraints can be written as
\[
F_1^* = \frac{\mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H}{\sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}} \sigma^2 \alpha \mathbf{H}_s \mathbf{H}_s^H.
\]

Using (15) and (18), the optimal $\alpha^*$ can be written as
\[
\alpha^* = \frac{\sqrt{p_i}}{\sigma^2 \sqrt{\text{tr}(\mathbf{H}_s^H \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_s^H)}}.
\]

KKT conditions [14], the optimal \( f_1^* \) can be determined as

\[
f_1^* = \arg \min_{f_1} J(F_1) \quad \text{s.t.} \quad E[|x_k|^2] = p_2
\]

where \( J(F_1) = \sum_{n=1}^M \left[ y_2 - \alpha s_n^* \right]^2 \)

\[
= \sum_{n=1}^M \sigma_2^2 \| h_n F_2 h_{s,n} \|^2 + \sigma_2^2 \| F_2 F_1 \|^2
- \alpha \sum_{n=1}^M \sigma_2^2 \| h_n F_2 h_{s,n} \|^2
+ \alpha \sum_{n=1}^M \sigma_2^2 \| h_n F_2 h_{s,n} \|^2
\]

\[
p_2 = tr(F_1 H_1 H_1^*)
\]

(42)

Similarly, taking the partial derivative of \( J(F_1) \) with respect to the complex conjugate of \( F_2 \), i.e., \( F_2^* \), and then equating the derivative to zero gives

\[
\frac{\partial J(F_1)}{\partial F_2} = \sigma_2^2 \mu h_n^T h_n F_2 + \sigma_2^2 \mu h_n^T h_n F_2
- \beta^2 \alpha M \sum_{n=1}^M h_n^T h_{s,n} = 0
\]

(43)

where \( \beta = \beta \cdots \beta \). Using (42) and (44), the optimal \( \alpha^\star \) can be written as

\[
\alpha^\star = \sqrt{\frac{p_2}{\sigma_2^2}}
\]

(45)

where

\[
H_j = h_j^H h_j \quad \text{and} \quad H_s = \sum_{n=0}^M h_{s,n}^H h_{s,n}.
\]

Consequently, the optimal nondiagonal \( F_1^* \) for the AF MISO distributed relay system under the relay power constraint can be written as

\[
F_1^* = H_1^{-1} H_s^{-1} \sqrt{\frac{p_2}{\sigma^2}} \quad \text{tr}(H_1^* H_1 H_s^{-1} H_s^* H_s^{-1})
\]

(46)

Additionally, using the KKT conditions, i.e., \( L(F_1 \lambda_2, \gamma_2) \)

\[
J_{KKT}(F_1) + \lambda_2 \left( tr(F_1 H_1 H_1^*) - p_2 \right), \quad \text{the optimal nondiagonal} \quad F_1^* \quad \text{can be written as}
\]

\[
F_1^* = H_1^{-1} H_s^{-1} \sqrt{\frac{p_2}{\sigma^2}} \quad \text{tr}(H_1^* H_1 H_s^{-1} H_s^* H_s^{-1})
\]

(47)

Additionally, using the KKT conditions, i.e., \( L(F_1 \lambda_2, \gamma_2) \)

\[
J_{KKT}(F_1) + \lambda_2 \left( tr(F_1 H_1 H_1^*) - p_2 \right), \quad \text{the optimal nondiagonal} \quad F_1^* \quad \text{can be written as}
\]

\[
F_1^* = H_1^{-1} H_s^{-1} \sqrt{\frac{p_2}{\sigma^2}} \quad \text{tr}(H_1^* H_1 H_s^{-1} H_s^* H_s^{-1})
\]

(48)

where \( h_k = \sum_{n=0}^M h_{k,n}^H \) and \( L(F_1 \lambda_2, \gamma_2) \) can be written as

\[
L(F_1 \lambda_2, \gamma_2) = \gamma^2 - \lambda_2 \left( tr(F_1 H_1 H_1^*) - p_2 \right)
\]

(49)

Here, \( \lambda_2 \) and \( \gamma_2 \) in (49) are the Lagrangian multiplier and the equalizing gain factor at the destinations, respectively. Note that, using (47) and (48), the same BER performance will be produced because \( H_1^{-1} h_k^H \) in (47) is equivalent to \( h_k^H / h_k \) in (48).

2) Diagonal Relay Amplifying Matrix Design: Similar to the case of the nondiagonal relay amplifying matrix design, the desired optimization with the relay power constraint can be written, using the vector form \( f_1 \), as

\[
f_1^* = \arg \min_{f_1} J(f_2) \quad \text{s.t.} \quad E[|x_k|^2] = p_2
\]

(50)

where

\[
J(f_2) = \sum_{n=1}^M \left[ y_2 - \alpha s_n \right]^2
\]

\[
= \sum_{n=1}^M \mu \| h_n F_2 h_{s,n} \|^2 + \mu \| F_2 F_1 \|^2
- \alpha \sum_{n=1}^M \mu \| h_n F_2 h_{s,n} \|^2
+ \alpha \sum_{n=1}^M \mu \| h_n F_2 h_{s,n} \|^2
\]

\[
p_2 = tr(F_2 H_1 H_1^*)
\]

(51)

Similarly, taking the partial derivative of \( J(f_2) \) with respect to the complex conjugate of \( F_2 \), i.e., \( F_2^* \), and then equating the derivative to zero gives

\[
\frac{\partial J(f_2)}{\partial F_2} = \mu \mu h_n^T h_n F_2 + \mu \mu h_n^T h_n F_2
- \beta^2 \alpha \mu \mu \sum_{n=1}^M h_n^T h_{s,n} = 0
\]

(52)

where \( \beta \cdots \beta \). Using (42) and (44), the optimal \( \alpha^\star \) can be written as

\[
\alpha^\star = \sqrt{\frac{p_2}{\mu \mu}}
\]

(53)

Following the previous procedures of the SIMO case, the optimal \( f_1^* \) and \( \alpha^\star \) can be written, respectively, as

\[
H_j = h_j^H h_j \quad \text{and} \quad H_s = \sum_{n=0}^M h_{s,n}^H h_{s,n}.
\]

(54)
distributed relays, and \(2\) sources, \(2\), \(4\)
distributed relay network under the transmit power constraints at the relays.

For the sake of investigating the achievable rate \(R_i\) for the AF SIMO wireless
distributed relay network, we first examine the SNR for the AF SIMO wireless
distributed relay network under the transmit power constraints at the relays.

The Monte-Carlo simulation results are performed to observe the system performances, such as BER
and achievable rate, for AF SIMO/MISO distributed relay systems under the transmit power constraints at the relays.

Additionally, when the optimal \(\alpha^*\) in (57) is applied, the better BER performance will be observed compared to the one applied in (55). This is because there is the third term, i.e., \(\sigma_v^2 p_2^2 (H_w \otimes I_N)\), in \(R_i\). This will be verified in Section V.

IV. ACHIEVABLE RATE BEHAVIOR

A. SIMO Distributed Relay System

For the sake of investigating the achievable rate \(R_i\) for the AF SIMO wireless distributed relay network, we first examine the SNR, at the destinations can be defined as

\[
\text{SNR}_2 = \frac{\sigma_v^2 \|h_M F r_H h_s^t\|^2}{\sigma_v^2 \|h_M F r_H f_s\|^2 + M \sigma_v^2}.
\]

Using (59), the achievable rate \(R_i\) for the AF SIMO wireless distributed relay network under the transmit power constraints at the relays can be written as

\[
R_i = \frac{1}{2} \log_2 \left( 1 + \text{SNR}_2 \right) = \frac{1}{2} \log_2 \left( 1 + \frac{\sigma_v^2 \|h_M F r_H h_s^t\|^2}{\sigma_v^2 \|h_M F r_H f_s\|^2 + M \sigma_v^2} \right).
\]

Here, \(\frac{1}{2}\) is used for the achievable rate per time slot.

Additionally, the case of both nondiagonal and diagonal relay amplifying matrices is the same expression, as shown in (60).

B. MISO Distributed Relay System

Similarly, the SNR, at the destination for the AF MISO wireless distributed relay network can be defined as

\[
\text{SNR}_2 = \frac{\sigma_v^2 \|h_M F r_H h_s^t\|^2}{\sigma_v^2 \|h_M F r_H f_s\|^2 + M \sigma_v^2}
\]

with the assumption that the same signal is transmitted from the sources. Using (61), the achievable rate \(R_i\) for the AF MISO wireless distributed relay network under the transmit power constraints at the relays can be written as

\[
R_i = \frac{1}{2} \log_2 \left( 1 + \frac{\sigma_v^2 \|h_M F r_H h_s^t\|^2}{\sigma_v^2 \|h_M F r_H f_s\|^2 + M \sigma_v^2} \right).
\]

IV. SIMULATION RESULTS

The Monte-Carlo simulation results are performed to observe the system performances, such as BER
and achievable rate, for AF SIMO/MISO distributed relay systems under the transmit power constraints at the relays.

All simulations of the AF SIMO system are performed for one source, \(N = 2, 4\) distributed relays, and \(M = 2\) destinations. Additionally, the AF MISO system is performed for \(M = 2\) sources, \(N = 2, 4\) distributed relays, and one destination. Hence, to have a fair comparison between SIMO and MISO systems, the same signal in a MISO system was transmitted from two sources, as stated earlier. It is assumed that \(p_1\) and \(p_2\) are the same value, i.e., \(p_1 = p_2 = 1\). The perfectly known channel coefficient row vectors \((h_i)\) and matrices \((H_w, H_v)\) are generated from i.i.d. Gaussian random variables with zero
mean and unity variance. All nodes with only one antenna have the same thermal noise power, i.e., \(\sigma_v^2 = \sigma_v^2\). The originally transmitted signal from the source is assumed to be modulated by quadrature phase shift keying with unit power.

![Fig. 3. BER performance versus input SNR for AF MISO/SIMO wireless](image-url)
distributed relay networks using $\mathbf{F}_1$ in (22), (25), and $\mathbf{F}_2$ in (47), (48) with two sources and one destination for MISO, and with one source and two destinations for SIMO, and $N = 2$, 4 relays, respectively.

Figure 3 shows the BER performance versus input SNR for AF MISO/SIMO wireless relay networks using the optimal nondiagonal $\mathbf{F}_1$ in (22), (25), and the optimal nondiagonal $\mathbf{F}_2$ in (47), (48) with two sources and one destination for MISO, and with one source and two destinations for SIMO, and $N = 2$, 4 relays, respectively. As analyzed, the BER performance with (25) is better than the one with (22) in Fig. 3. However, the identical BER performance is observed using (47) and (48). Additionally, when $M = N$, almost the equivalent BER performance is produced using (47) and (48) because the number of sources and relays is the same. However, when $N > M$, using (47) and (48), the MISO BER performance is better than the SIMO one. Finally, as $N$ increases, the BER performance improves.

Figure 4 presents the BER performance versus input SNR for AF MISO/SIMO wireless relay networks using the optimal diagonal $\mathbf{F}_1$ in (36), (37), and the optimal diagonal $\mathbf{F}_2$ in (55), (57) with two sources and one destination for MISO, and with one source and two destinations for SIMO, and $N = 4$ relays, respectively. As analyzed, for the AF SIMO wireless distributed relay system, the BER performance with (37) is better than the one with (36). In addition, for the AF MISO wireless distributed relay system, the BER performance with (57) is better than the one with (55). Unlike the case of the optimal nondiagonal amplifying matrices, the SIMO BER performance is better than the MISO one. Finally, it is observed that the BER performance enhances as $N$ increases.

Figure 5 provides the achievable rates ($R_1$ and $R_2$) versus input SNR for AF MISO/SIMO wireless relay networks using the optimal nondiagonal $\mathbf{F}_1$ in (22), (25), and the optimal nondiagonal $\mathbf{F}_2$ in (47), (48) with two sources and one destination for MISO, and with one source and two destinations for SIMO, and $N = 4$ relays, respectively. All results of the $R_1$ and $R_2$ are the same results as the BER performance in Fig. 3. For example, the same achievable rate is yielded using (47) and (48) and the MISO achievable rate is better than the SIMO one.
Figure 6 shows the achievable rates ($R_1$ and $R_2$) versus input SNR for AF MISO/SIMO wireless relay networks using the optimal nondiagonal $F_1$ in (36), (37), and the optimal nondiagonal $F_2$ in (55), (57) with two sources and one destination for MISO, and with one source and two destinations for SIMO, and $N = 4$ relays, respectively. All results of the $R_1$ and $R_2$ are the same results as the BER performance in Fig. 4. For instance, for the AF SIMO wireless distributed relay system, the achievable rate with (37) is better than the one with (36). Additionally, it is observed that the SIMO achievable rate is better than the MISO one.

V. CONCLUSIONS

This paper presented MMSE AF relay schemes in wireless distributed relay networks for both SIMO and MISO systems. Based on the MMSE criterion, optimal nondiagonal and diagonal relay matrices were derived under the transmit power constraints at the relays. It was observed that the BER performance and achievable rate of the AF MISO wireless distributed relay system are better than that of the AF SIMO one when the optimal nondiagonal relay amplifying matrices were applied. In contrast to this, the AF SIMO wireless distributed relay system showed better BER performance and achievable rate than the AF MISO one when the optimal diagonal relay amplifying matrices are used. However, it was found that almost the identical BER performance is yielded when $M = N$ when the optimal nondiagonal relay amplifying matrix derived by the Lagrangian optimization problem is applied.

REFERENCES