

## Dynamic Modulus Master Curve Construction Using the Modified MEPDG Model

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**Abstract**—Dynamic modulus,  $E^*$  is one of the fundamental properties defining the response of hot mix asphalt (HMA) mixtures in flexible pavement systems. Correspondingly, HMA materials are characterized by  $E^*$  master curve incorporating time and temperature effects. One of the widely used models, modified by Bonaquist (2009) is the MEPDG master curve model. The aim of this study is to obtain dynamic modulus data using the modified MEPDG master curve model. The sample has been tested at unconfined pressure where the unconfined master curve typically used in mechanistic-empirical (M-E) pavement analysis methods. From the Goodness-of-fit statistics, most of the sample gave an excellent correlation when  $R^2 \geq 0.90$ . It also has been supported by the standard error ratio when the  $S_e/S_y \leq 0.35$ . Nonetheless, the correlation coefficient,  $R^2$  is not always a reliable coefficient to measure the Goodness-of-fit for nonlinear regression analysis. Furthermore, there may be overall and/or local biases in the predictions that can cause significant reductions in accuracy under certain conditions.

**Keywords:** - Dynamic modulus; Goodness-of-fit; master-curve; MEPDG Model; mechanistic-empirical

### I. INTRODUCTION

#### A. Background

All flexible pavements are sensitive to temperature and the rate of loading. This is because asphalt is a viscoelastic-plastic material. In extreme cases such as at high temperatures and long loading rates (slow speed of passing vehicles), the modulus of a mix may approach to an unbound granular material. But when at cold temperatures and very short load rates, the material will behave in a pure elasticity.

As a matter of fact, dynamic modulus,  $E^*$  is one of the fundamental properties defining the response of hot mix asphalt (HMA) mixtures in flexible pavement systems. It is also the primary HMA material property input at all three

hierarchical levels in the mechanistic-empirical pavement design guide (MEPDG 2004) developed under the National Cooperative Highway Research Program (NCHRP) 1-37A. Moreover, it is a leading candidate for the simple performance test (SPT) recommended by the NCHRP 9-19 [1, 2]. It also has been recommended as a potential quality control or quality assurance parameter [1].

Correspondingly, HMA materials are characterized by  $E^*$  master curve incorporating time and temperature effects. The use of master curves is to describe and represent the characteristic of asphalt mixtures over a wide range of temperatures and frequencies [3]. Various  $E^*$  master curve models have been developed over the last several decades in order to estimate  $E^*$ . One of the widely used models, modified by Bonaquist (2009) is the MEPDG master curve model. In the model, it requires direct measurement of  $E^*$  in the laboratory before further analysis.

#### B. Purpose and Scope of Study

The objective of this study is to obtain dynamic modulus data using the modified MEPDG master curve model. The purpose of this paper is to answer a fundamental question: "How accurate is the modified MEPDG master curve model?" This query will be answered by examining the model equations and perform the Goodness-of-fit statistics.

Since the  $E^*$  test is nondestructive, only six groups needed to complete the portion of this study. There are three replicate in each group. The sample has been tested at unconfined pressure where the unconfined master curve typically used in mechanistic-empirical (M-E) pavement analysis methods [4]. The selected samples were long term aged only. It included field samples, cored from the airport pavement runway in Malaysia. The rest is long term aged sample mixed in the laboratory. For clearer view, the tested sample group is shown in TABLE I.

TABLE I. GROUP OF SAMPLE

Sample ID	Mix Origin	Average Air Void (%)
LGK Old	Airport field cores	3.0
DCP1 New	Airport field cores	8.8
DCP2 Run	Airport field cores	3.1
CLV9	Lab blend	6.9
FLV10	Lab blend	7.2
CHV11	Lab blend	7.0

## II. SIMPLE PERFORMANCE TESTER SPECIFICATIONS

The equipment used in this laboratory testing is Simple Performance Tester (SPT) developed by IPC Global, Australia. The SPT consists of a cabinet that includes the hydraulic pump and actuator, the test chamber, the heating and refrigeration unit, and power and control electronics. The SPT can be used to test HMA mixtures in any region to determine how the mixture will perform in the field [5]. Summary of the SPT specification used in this laboratory testing is indicated in Table II.

TABLE II. SPT SPECIFICATION

Specifications	Criteria
Load Frame	<ul style="list-style-type: none"> <li>• Static load capacity is 25kN</li> <li>• Fatigue load capacity is 20kN</li> <li>• Ram displacement is 30 mm</li> <li>• Max ram speed is 1200 mm/min</li> <li>• Load Cell is 15kN with +/- 0.1% accuracy</li> </ul>
Servo Actuator	<ul style="list-style-type: none"> <li>• Static load capacity is 15 kN</li> <li>• Dynamic load capacity is 13.5 kN</li> </ul>
Confining Cell and Environmental Control Unit	<ul style="list-style-type: none"> <li>• Confining pressure between 0 to 250 kPa</li> <li>• The temperature ranges between 4° to 60°C</li> <li>• Pressure relief valve opens at 275 kPa</li> </ul>

Source: IPC Global SPT Manual (2007)

The testing protocol adapted in this laboratory testing is NCHRP 9-29 PP 02 in “Developing Dynamic Modulus Master Curves for Hot-Mix Asphalt Concrete Using the Simple Performance Test System”.

## III. LABORATORY PROCEDURE

The simple performance test specimens were prepared to a target air void content of 7.0 percent. From these, 100 mm diameter by 150 mm high specimens was cored using the portable core drilling machine. Then it sawed using the single bladed saw to trim the top and bottom of the specimen. All coring and sawing was done using water to cool the cutting tools. The cores were measured for compliance with the NCHRP Project 9-29 specimen tolerances [1].

After all cutting was accomplished, the bulk specific gravity of the finished specimen was determined in accordance with AASHTO T166 by first measuring the immersed mass, then the saturated surface dry mass, and finally the dry stack.

The test specimens were conditioned in a separate environmental chamber namely Universal Testing Machine (UTM) at least two hours before testing. The SPT chamber

was also equilibrated to the target testing temperature. Once the specimens and the test chamber reached the target temperature, the specimens were removed from the UTM environmental chamber and placed in the SPT chamber. The SPT chamber was then closed and allowed to equilibrate to the test temperature before the testing began.

Additionally, the specimen mounted deformation system consists of three spring-loaded LVDTs distanced 120° apart. While the designated holder for the LVDTs can be used for unconfined and confined testing. Each holder has a stiff spring that grips the glued gauge points.

The three unconfined  $E^*$  tests, 4, 21, and 40 °C, were performed on the same test specimen. For each condition,  $E^*$  and phase angles were measured at frequencies of 25, 10, 5, 1, 0.5, 0.1, and 0.01 Hz. Stress levels were varied automatically by the SPT to achieve a target strain level of 125 microstrain.

## IV. MODIFIED MEPDG MASTER CURVE MODEL

A large number of studies about time-temperature effects on flexible pavement have led to the development of predictive master curve models. Among the various predictive models, the MEPDG model is one of the current. The MEPDG model has the logistic sigmoidal shaped and is claimed the most popular [3].

### A. Master Curve Construction

The MEPDG model in this study incorporated the Hirsch model for the limiting modulus. The general equation of the modified MEPDG model is shown in Eq. (1).

$$\log|E^*| = \log(|E^*|_{min}) + \frac{\log(|E^*|_{max}) - \log(|E^*|_{min})}{1 + e^{\beta + a \log \omega_r}} \quad (1)$$

where  $|E^*|$  is dynamic modulus in ksi,  $|E^*|_{max}$  is limiting maximum modulus in ksi,  $|E^*|_{min}$  is limiting minimum modulus in ksi,  $\omega_r$  is reduce frequency in Hz, and  $a, \beta$  are the fitting parameters.

### 1) Maximum and Limiting Binder Modulus

When referring to Eq. (2),  $|E^*|_{max}$  is computed using the Hirsch model and a limiting binder modulus of 145,000 psi. The relevance behind limiting binder modulus is because all binders reach a maximum shear modulus of approximately 145,000 psi [4]. And  $P_c$  is computed as Eq. (3).

$$|E^*|_{max} = P_c \left[ 4,200,000 \left( 1 - \frac{VMA}{100} \right) + 3|G^*|_{binder} \left( \frac{VFA \cdot VMA}{10,000} \right) \right] + (1 - P_c) \left[ \frac{1 - \frac{VMA}{100}}{4,200,000} + \frac{VMA}{VFA \cdot 3|G^*|_{binder}} \right]^{-1} \quad (2)$$

$$P_c = \frac{(20 + \frac{VFA \cdot 3|G^*|_{binder}}{VMA})^{0.58}}{650 + (\frac{VFA \cdot 3|G^*|_{binder}}{VMA})^{0.58}} \quad (3)$$

where  $|E^*|_{max}$  is limiting maximum modulus in Psi,  $Pc$  is aggregate contact factor,  $VMA$  is voids in mineral aggregate (%),  $VFA$  is voids filled with asphalt (%), and  $|G^*|$  is binder modulus in Psi.

### 2) Shift Factor Law

Master curve is constructed using a time-temperature superposition. The shift factor is calculated using the Arrhenius shift factor law as shown in Eq. (4). The activation energy ( $\Delta E_a$ ) treated as a fitting parameter. The  $\Delta E_a$  describes the minimum energy needed before any intermolecular movement can occur.

$$\log[a(T)] = \frac{\Delta E_a}{19.14714} \left( \frac{1}{T} - \frac{1}{T_r} \right) \quad (4)$$

where  $a(T)$  is a shift factor at temperature  $T$ ,  $T$  is temperature in °K, and  $T_r$  is a reference temperature in °K.

$$\log \omega_r = \log \omega + \frac{\Delta E_a}{19.14714} \left( \frac{1}{T} - \frac{1}{T_r} \right) \quad (5)$$

Finally, Eq. (5) is the reduced frequency,  $\omega_r$ . Where  $\omega$  is the loading frequency at the reference temperature of 21 °C.

### B. Dynamic Modulus, Phase Angle and, MEPDG Input

This section describes the results from the construction master curve using the modified MEPDG model. The representative sample of the results is LGK Old.

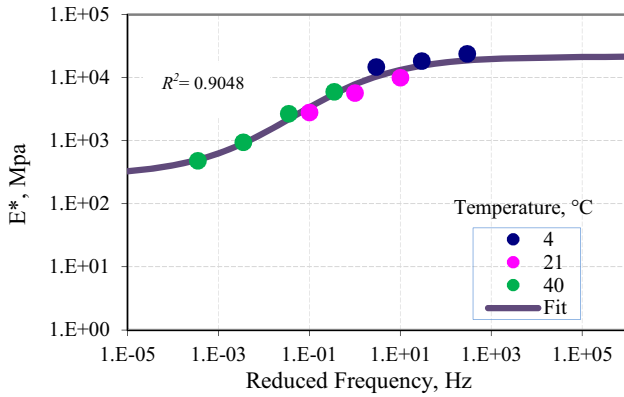


Figure 1. Dynamic Modulus at Reduced Frequency

Figure 1 is the dynamic modulus curve for Langkawi runway sample (LGK Old). This is the logistic sigmoidal shaped of dynamic modulus versus the reduced frequency. As can be seen from the graph, the highest  $E^*$  value is 23618 Mpa and the lowest is 477 Mpa. While the continuous line represents the fitted  $E^*$  value calculated from the MEPDG model. The accuracy of the fitted data is further explained in Section V.

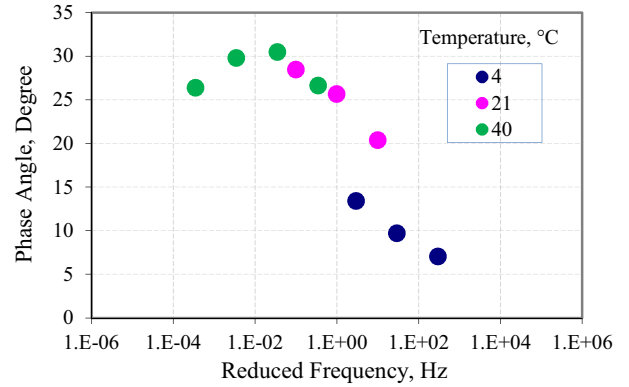


Figure 2. Phase Angle at Reduced Frequency

Figure 2 shows the phase angle over the reduced frequency. Theoretically, the phase angle is the angle in degrees between a sinusoidal applied (peak to peak) stress and the resulting (peak to peak) strain in a controlled-stress test [6]. The reduced frequency is incorporated loading frequency and temperature followed the Arrhenius shift factor law. From the scattered plots, the peak angle is 31°. The peak angle is a sign of the magnitude of the loss modulus value. According to Wahab [7], the degree of peak angle can be an indicator to evaluate pavement distress. But the detailed analysis of phase angle its characteristics will be discussed in another research paper.

TABLE III. M-E DESIGN INPUT

Temp. (°C)	Freq. (Hz)	Shift Factor	Reduced Frequency	E* (Mpa)
-10.0	10	2.8276	6723.78	20691.2
-10.0	5	2.8276	3361.89	20415.9
-10.0	1	2.8276	672.37	19452.3
-10.0	0.5	2.8276	336.18	18850.8
-10.0	0.1	2.8276	67.23	16867.7
4.4	10	1.4315	270.10	18632.2
4.4	5	1.4315	135.05	17837.8
4.4	1	1.4315	27.01	15320.6
4.4	0.5	1.4315	13.50	13936.2
4.4	0.1	1.4315	2.70	10204.8
21.1	10	-0.00906	9.79346	13239.3
21.1	5	-0.00906	4.89673	11641.8
21.1	1	-0.00906	0.97934	7759.5
21.1	0.5	-0.00906	0.48967	6207.1
21.1	0.1	-0.00906	0.09793	3365.9
37.8	10	-1.29523	0.50671	6280.0
37.8	5	-1.29523	0.25335	4897.2
37.8	1	-1.29523	0.05067	2561.2
37.8	0.5	-1.29523	0.02533	1919.2
37.8	0.1	-1.29523	0.00506	1030.2
54.4	10	-2.45056	0.03543	2205.8
54.4	5	-2.45056	0.01771	1658.2
54.4	1	-2.45056	0.00354	911.9
54.4	0.5	-2.45056	0.00177	735.5
54.4	0.1	-2.45056	0.00035	499.8

Table III is the Level 1 design input for M-E pavement design. Important to realize, the MEPDG model also can forecast data outside the measured range. These inputs are very important for mixture evaluation and for characterizing the modulus of HMA. From the table, the  $E^*$  value at 54.4 °C and 0.1 Hz is only 499.8 Mpa. This explained that in extreme cases such as at high temperature (54.4 °C) and a very low loading frequency (0.1 Hz), the pavement reflects very poor performance. However, this does not explain poor performance in all conditions. Conversely, the pavement performs extremely well at very low temperature (-10 °C) and low loading frequency (0.1 Hz) when the dynamic modulus value achieved 16867.7 Mpa. To be noted, the dynamic modulus below 1500 Mpa is considered a poor performance [8].

## V. STATISTICAL ANALYSIS

### A. Goodness-of-Fit Test Statistic

Statistical methods have been used to determine the Goodness-of-fit between measured and predicted data. From Eq. (6) to Eq. (8) are the standard error estimation before the final adjusted coefficient of determination,  $R^2$  established in Eq. (9).

Standard Error Ratio

$$\frac{S_e}{S_y} \quad (6)$$

Standard Error of Estimation

$$S_e = \sqrt{\frac{\sum(Y - \hat{Y})^2}{n - k}} \quad (7)$$

Standard Error of Deviation

$$S_y = \sqrt{\frac{\sum(Y - \bar{Y})^2}{n - 1}} \quad (8)$$

Adjusted Coefficient of Determination

$$R^2 = 1 - \frac{(n-1)}{(n-k)} \cdot \left[ \frac{S_e}{S_y} \right]^2 \quad (9)$$

where  $n$  is sample size,  $k$  is the number of independent variables in the model,  $Y$  is a measured  $a_T$ ,  $\hat{Y}$  is predicted  $a_T$ , and  $\bar{Y}$  is the mean value of measured  $a_T$ .

The coefficients are as previously defined. For the perfect fit,  $R^2 = 1$ . The criteria for the  $S_e/S_y \leq 0.35$  is considered excellent and  $R^2 \geq 0.90$  is also considered excellent [3].

### B. Model Parameter

The final parameters were obtained from the modified MEPDG model. These values were optimized from the nonlinear least-squares fitting (NLSF) procedure using the Excel Solver add-in function.

TABLE IV. MODEL PARAMETER AND THE GOODNESS-OF-FIT STATISTICS

Mix ID	Model Parameter	Se/Sy	R <sup>2</sup>	
LGK Old	$\Delta E_a$	135236	0.22	0.9048
	$\beta$	1.1838		
	$\gamma$	-0.8854		
	$E_{max}$	21631.5		
	$E_{min}$	279.9		
DCP1 New	$\Delta E_a$	238884	0.20	0.9161
	$\beta$	-1.4085		
	$\gamma$	-0.4475		
	$E_{max}$	21631.5		
	$E_{min}$	37.6		
DCP2 Run	$\Delta E_a$	114676	0.35	0.7613
	$\beta$	-0.5798		
	$\gamma$	-0.9608		
	$E_{max}$	21631.5		
	$E_{min}$	282.7		
CLV9	$\Delta E_a$	207644	0.08	0.9871
	$\beta$	-1.3771		
	$\gamma$	-0.6136		
	$E_{max}$	21829.2		
	$E_{min}$	48.6		
FLV10	$\Delta E_a$	208392	0.05	0.9944
	$\beta$	-1.2260		
	$\gamma$	-0.6062		
	$E_{max}$	21435.5		
	$E_{min}$	47.5		
CHV11	$\Delta E_a$	185223	0.08	0.9886
	$\beta$	-0.9313		
	$\gamma$	-0.7627		
	$E_{max}$	21664.6		
	$E_{min}$	99.4		

Once again, the final parameters indicated the accuracy between measured and predicted data. From Table IV, most of the sample gave an excellent correlation when the  $R^2 \geq 0.90$ . Standard error ratio also shows an excellent correlation when the  $S_e/S_y \leq 0.35$ . However ‘DCP2Run’ sample only gives about 76% ( $R^2 = 0.7613$ ) of accuracy, but it can be judged as ‘good’ correlation. Even so, according to Md. Yusoff, et al. [3] the correlation coefficient ( $R^2$ ) is not always a reliable coefficient to measure the Goodness-of-fit for nonlinear regression analysis. There may be overall and/or local biases in the predictions that can cause significant reductions in accuracy under certain conditions [9].

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