# Design of Controller using Variable Transformations for a Two Tank Conical Interacting Level Systems

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***Abstract*— This paper presents the control of Non Linear Two Tank Conical Interacting Level System (TTCILS) using Globally Linearized Controller (GLC).This approach is based on the hypothesis that a system which is non linear in its original variables becomes linear in some form of transformation of these original variables. The performance of GLC is compared with the performance of conventional decentralized PI controller for different operating points. From the simulation studies, it is observed that GLC outperforms the decentralized PI controller with lower settling time and reduced error indices.**

***Keywords - Non Linear Process; Decentralized PI controller; Globally Linearized controller; Two Tank Conical Interacting Level System; MATLAB***

## I. INTRODUCTION

The control of liquid level is mandatory in all process industries.Most of the practical process control problems will involve non linear systems.The traditional and easiest approach to the controller design problem for non linear systems involves linearizing the modelling equation around a steady state and applying linear control theory results.It is obvious that the controller performance in this case will deteriorate as the process moves further away from the steady state around which the model is linearized.

Apart from the local linearization approach,there are few other special purpose design procedures which can directly be used to design controllers for non linear processes.However,these procedures usually have limited applicability and are often based on accumulated experience with a special type of non linear system[1].

Many researchers have reported on Conical Tank process and Globally linearized controller.The design of controller based on variable transformation for first order non linear process with dead time is presented by Anandanatarajan et al[2].N.Jaya et al [3] designed a GLC and multi region fuzzy logic controller to two capacity interacting level process.Non linear control strategies incorporating input state output models is presented by Konstantinos Dimopoulous [4].V.R.Ravi et al [5] designed a decentralized PID controller for interacting conical tank systems.A GLC is designed for non linear process systems via variable transformations by Babatunde A.Ogunnaike [6].Costas Kravaris et al [7] presented non linear state feedback synthesis by global input/output linearization.A multimodel control of non linear systems using closed loop gap metric is presented by Erdem et al [8].A GLC controller is designed for quadrapule tank process by M.Subba et al [9]. Huajin Tang et.al [10] discussed about Engine control design using globally linearizing control and sliding mode.A case study on Globally linearized control on diabatic continuous stirred tank reactor is presented by Jana AK et.al [11]. Chunyan Du et.al [12] discussed about Control of nonlineat distributed parameter systems based on global approximation. Global stabilization of a class of nonlinear system based on reduced order state feedback control is presented by Chang-Zhong Chen et.al [13]. In this paper a globally linearized controller is designed for a highly non linear process such as Interacting Conical tanks using variable transformation technique.

The paper is organized as follows: Section II deals with Process Description. Conventional decentralized PI controller is designed in Section III. Section IV presents the design of GLC for two tank conical interacting level system. Simulation is carried out using both the controllers for two operating points in MATLAB Environment in Section V. The Simulation studies with respect to servo and regulatory responses are also described in detailed manner. Finally Results and Conclusions are discussed in Section VI.

## II. PROCESS DESCRIPTION

The two tank conical interacting system consists of two identical conical tanks (Tank 1 and Tank 2), two identical pumps that deliver the liquid flows $F_{in1}$ and $F_{in2}$ to tank 1 and tank 2 through the two control valves CV1 and CV2 respectively. These two tanks are interconnected at the bottom through a manually controlled valve, MV12 with a valve coefficient $\beta_{12}$. $F_{out1}$ and $F_{out2}$ are the two output flows from tank 1 and tank 2 through manual control values MV1 and MV2 with valve coefficients $\beta_1$ and $\beta_2$ respectively as shown in Fig.1.

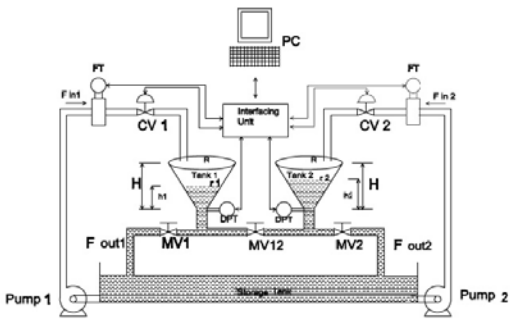


Figure.1 Schematic of TTCILS

The operating parameters of TTCILS is shown in Table.1.

TABLE.1. OPERATING PARAMETERS OF TTCILS

| Para meter | Description | Nominal values |
|---|---|---|
| R | Top radius of conical tank | 19.25cm |
| H | Maximum height of Tank1&Tank2 | 73cm |
| $F_{in1}$& $F_{in2}$ | Maximum inflow toTank1&Tank2 | 400 &100cm$^3$/sec |
| $\beta_1$ | Valve coefficient of $MV_1$ | 35 cm$^2$/sec |
| $\beta_{12}$ | Valve coefficient of $MV_{12}$ | 78.28 cm$^2$/sec |
| $\beta_2$ | Valve coefficient of $MV_2$ | 19.69 cm$^2$/secs |

In this work, TTCILS is considered as two inputs two output process in which level $h_1$ in tank 1 and level $h_2$ in tank 2 are considered as output variables and $F_{in1}$ and $F_{in2}$ are considered as manipulated variables. The mathematical model of two tank conical interacting system is given as shown in equations (1) and (2).

$$\frac{dh_1}{dt}=\left[\frac{F_{in1}-h_1\frac{dA(h_1)}{dt}-\beta_1\sqrt{h_1}-sign(h_1-h_2)\beta_{12}\sqrt{|h_1-h_2|}}{\frac{1}{3}\pi R^2\frac{h_1^2}{H^2}}\right] \quad (1)$$

$$\frac{dh_2}{dt}=\left[\frac{F_{in2}-\beta_2\sqrt{h_2}+sign(h_1-h_2)\beta_{12}\sqrt{|h_1-h_2|}-h_2d\frac{A(h_2)}{dt}}{\frac{1}{3}\pi R^2\frac{h_2^2}{H^2}}\right] \quad (2)$$

where

$A(h_1)$ - Area of Tank 1 at $h_1$ (cm$^2$)
$A(h_2)$ - Area of Tank 2 at $h_2$ (cm$^2$)
$h_1$ - Liquid level in Tank 1 (cm)
$h_2$ - Liquid level in Tank 2 (cm)

The open loop responses of $h_1$ and $h_2$ are shown in Fig.2.

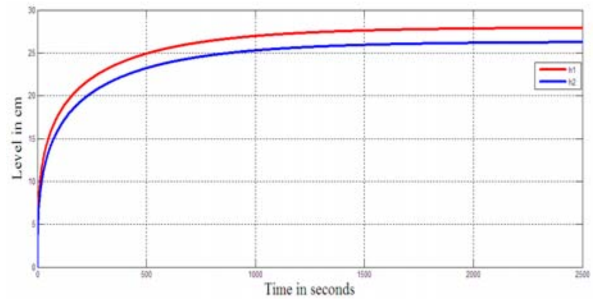


Figure.2. Open loop response of $h_1$ and $h_2$

## III. DECENTRALIZED PI CONTROLLER

The objective in decoupling is to compensate for the effect of interactions brought about by cross coupling of the process variables. There are many different decoupling methods. In this paper, linear decoupling is used.

The basic closed loop block diagram of 2x2 systems is shown in Fig.3 and its closed loop system equation can be written in matrix form as given by equation(3).

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}\begin{bmatrix} G_{c1} & 0 \\ 0 & G_{c2} \end{bmatrix}\begin{bmatrix} r_1 & y_1 \\ r_2 & y_2 \end{bmatrix} \quad (3)$$

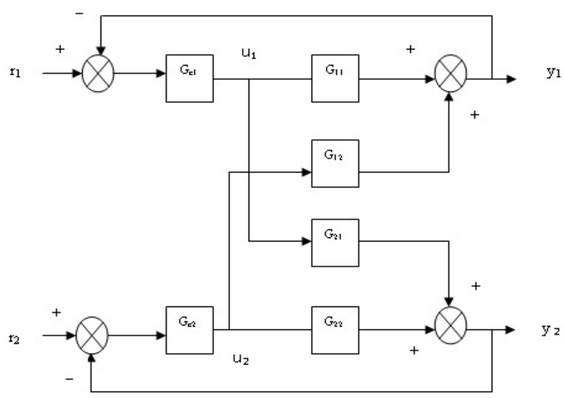


Figure.3. Basic block diagram of 2x2 system

The elements of a 2x2 system for decoupling matrix by to eliminate interactions from all loops are determined as shown by equations (4) to (7).

$$d_{11} = 1 \quad (4)$$

$$d_{12}(s) = \frac{-G_{12}(s)}{G_{11}(s)} \quad (5)$$

$$d_{21}(s) = \frac{-G_{21}(s)}{G_{22}(s)} \quad (6)$$

$$d_{22} = 1 \quad (7)$$

The design of a decentralized control system with a decoupling matrix can be done by combining a diagonal controller $K_d(s)$ with a block compensator D(s) as shown in Fig.4.

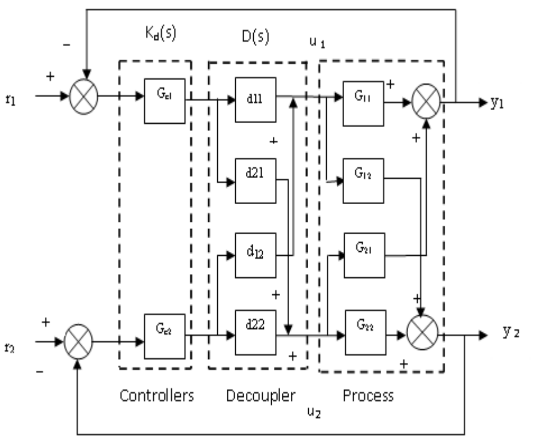


Figure. 4. General 2x2 system with decouplers and single loop controllers

## IV. DESIGN OF GLC

The general single- input, single-output non linear control system will be represented by equation (8).

$$\frac{dx}{dt} = F(x, u) \tag{8}$$

where F(.) is an arbitrary non linear function of x, the system state variable and u the control variable. It is always possible to break F(x,u) up as shown in equation (9).

$$F(x,u)=C_1f_1(x)+C_2f_2(x,u) \tag{9}$$

where $f_1(x)$ is a function of x and x alone and $f_2(x,u)$ is a function of both x and u.Both $f_1$ and $f_2$ are taken to be non linear and no restrictions are placed on their functional forms.$C_1$ and $C_2$ are constants.Thus the process model can be written as given in equation (10).

$$\frac{dx}{dt} = C_1f_1(x)+C_2f_2(x,u) \tag{10}$$

Now consider the transformation given by equation(11).

$$z = g(x) \tag{11}$$

where g(.) is a function of x and x alone, which is to be determined such that the process model equations (8) and (9) ,which is non linear in the original variables x and u , is mapped to model that are linear in z and v as indicated in equation(12).

$$\frac{dz}{dt} = a + bv \tag{12}$$

Differentiating equation (11) with respect to t gives equation (13).

$$\frac{dz}{dt} = \frac{dg}{dx}.\frac{dx}{dt} \tag{13}$$

By introducing equation (10) into equation (11) gives equation (14).

$$\frac{dz}{dt} = C_1f_1(x)\frac{dg}{dx} + C_2f_2(x,u)\frac{dg}{dx} \tag{14}$$

Observe now that if the function z = g(x) to be chosen such as given by equation(15) and (16).

$$C_1f_1(x)\ dg/dx = a \tag{15}$$

$$C_2f_2(x,u)\ dg/dx = bv \tag{16}$$

and the non linear system is mapped as in equation (17).

$$\frac{dz}{dt} = a + bv \tag{17}$$

The conceptual configuration of the transformation controller is shown in Fig.5.

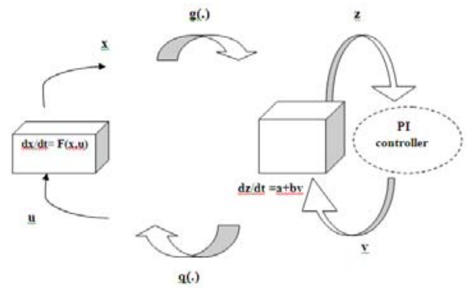


Figure.5. Conceptual configuration of the transformation controller

Also the block diagram configuration of a Globally Linearized Controller is shown in Fig.6.

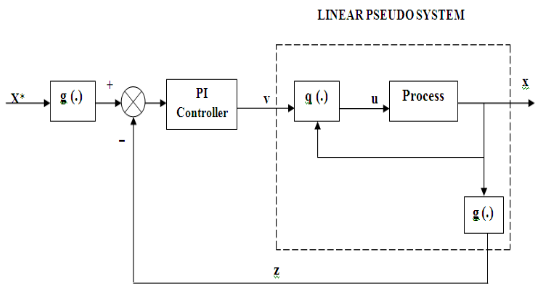


Figure.6. Block diagram configuration of GLC

The entire design procedure is based on the determination of g(.) and to obtain the corresponding q(.) which recovers the actual u to be implemented from the output of the transformed system controller. In the two tank conical interacting system, two GLC's are designed one with respect to tank 1 and the other with respect to tank 2.

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From Equation (1), the process model can be written as in equation(18).

$$A\frac{dx}{dt} = u - 0.1455h_1^2 - h^{1/2}(\beta_1+\beta_{12}) \quad (18)$$

where $x = h_1$, $Fin1 = u$.
From Equation (14), g(.) is required to be chosen as given in equation(19).

$$g(.) = \int (k_1/h_1^2).dh_1 \quad (19)$$

From equation (15), the control implementation law is obtained as given by the equations (20) and (21).

$$f_2(x, u) = k_2\ v\ f_1(x) \quad (20)$$

$$u + h^{1/2} = k_2\ v\ h_1^2 \quad (21)$$

Similarly another GLC is designed for equation (2) and the necessary equations for g(.) and q(.) are obtained. The transformation g(.) maps the non linear system represented by equations (1) and (2) into a linear system as shown in Figure.5. The transformed system is a linear pseudo system, whose mathematical model is given by equation(22).

$$\frac{dz}{dt} = a + bv \quad (22)$$

The values of a and b are assumed as -0.0128 and -1.278 respectively. a and b values are chosen such that it gives satisfactory performance for the desired setpoint. Mathematically there are no restrictions on the values of a and b.

## V. SIMULATION STUDIES

Simulation studies are carried out on TTCILS,using MATLAB (R2010a) and both the controllers are taken up for study.For decentralized PI controller,the controller settings are obtained as the proportional gain, Kc = 101.19 and Integral time Ti = 0.85s.Similarly the tuning parameters for GLC technique are Kc = 1200 and Ti = 0.2 s. The simulation is carried out by considering the nominal values of $h_1$ and $h_2$($h_1$ = 28cm and $h_2$ = 26 cm).

### A. *Performance of Servo response*

The setpoint variations are introduced for assessing the tracking capability of decentralized controller and GLC as shown in Fig. 7-10.

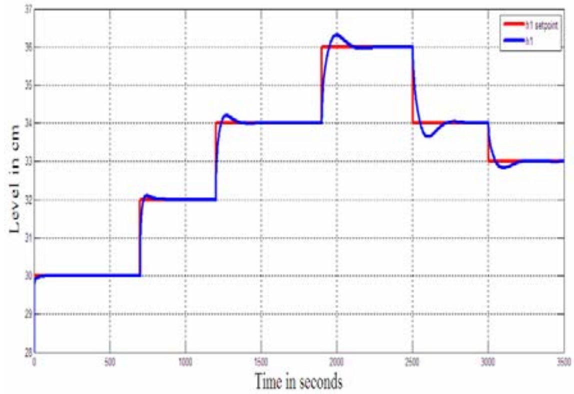


Figure.7. Servo Response of $h_1$ in TTCIS using Decentralized PI controller

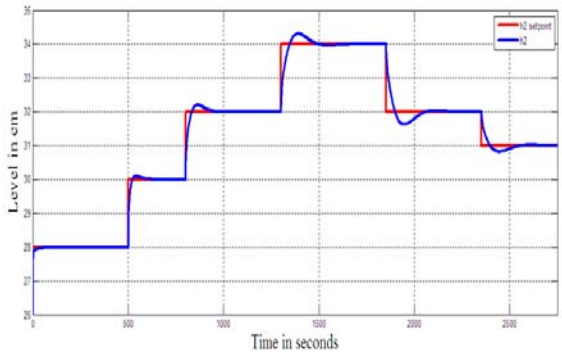


Figure.8. Servo Response of $h_2$ in TTCIS using Decentralized PI controller

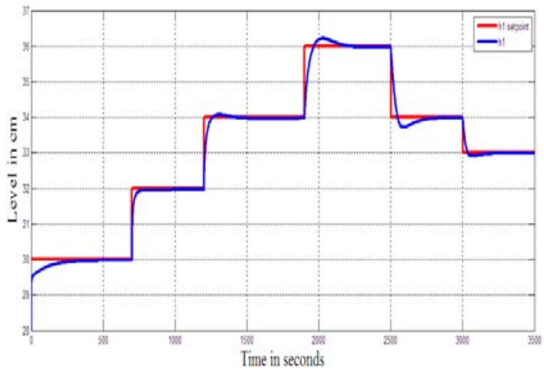


Figure.9. Servo Response of $h_1$ in TTCIS using GLC

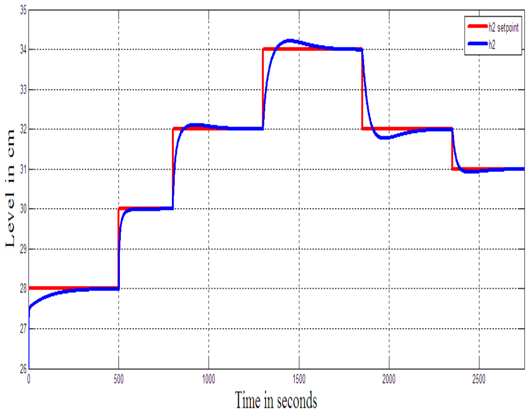


Figure.10. Servo Response of $h_2$ in TTCIS using GLC

From the responses, it is inferred that GLC is able to maintain the tank levels $h_1$ and $h_2$ at the respective setpoints with better settling time and integral square error compared to the decentralized controller . The performance indices for decentralized controller and GLC are reported in Table 2 and Table 3.

TABLE.2. COMPARISON OF TRACKING CAPABILITY PERFORMANCES OF $h_1$

| Operating points of $h_1$ in cm | Conventional decentralized controller | | GLC controller | |
|---|---|---|---|---|
| | Settling time (secs) | ISE | Settling time (secs) | ISE |
| 30-32 | 100 | 0.00836 | 200 | 0.00210 |
| 32-34 | 200 | 0.02770 | 100 | 0.00330 |
| 34-36 | 500 | 0.06115 | 300 | 0.00428 |
| 36-34 | 400 | 0.10590 | 300 | 0.00424 |
| 34-33 | 400 | 0.11600 | 200 | 0.00466 |

TABLE.3. COMPARISON OF TRACKING CAPABILITY PERFORMANCES OF $h_2$

| Operating points of $h_2$ in cm | Conventional decentralized controller | | GLC controller | |
|---|---|---|---|---|
| | Settling time (secs) | ISE | Settling time (secs) | ISE |
| 28-30 | 250 | 0.00770 | 50 | 0.00132 |
| 30-32 | 200 | 0.02420 | 200 | 0.00173 |
| 32-34 | 210 | 0.05660 | 200 | 0.00204 |
| 34-32 | 250 | 0.10101 | 210 | 0.00194 |
| 32-31 | 210 | 0.11101 | 180 | 0.00191 |

3. *Performance of Regulatory response*

Simulation studies are also carried out to explain the disturbance rejection capability of GLC and conventional controller. In TTCILS, the changes in input flow rates are considered as disturbance.The step change in input flow rates $F_{in1}$ and $F_{in2}$ which corresponds to 25% change in output level in tank1 and tank2 are introduced as disturbances. These disturbances are introduced at output levels of $h_1$ = 32cm and $h_2$ = 30cm.The regulatory response of TTCILS using conventional controller is shown in Fig. 11 and 12 for $h_1$ and $h_2$. Fig. 13 and 14 shows the regulatory response of TTCILS using GLC.

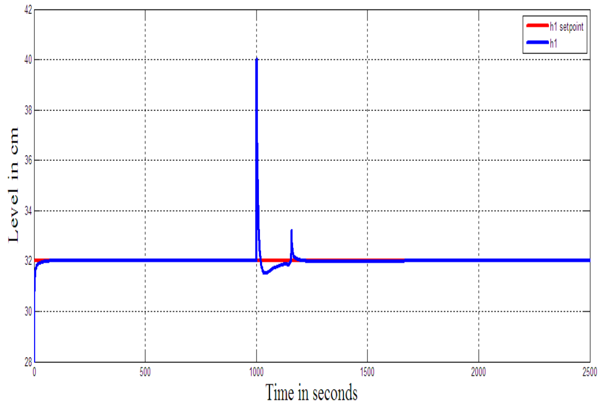


Figure.11. Regulatory response of $h_1$ in TTCIS using decentralized controller

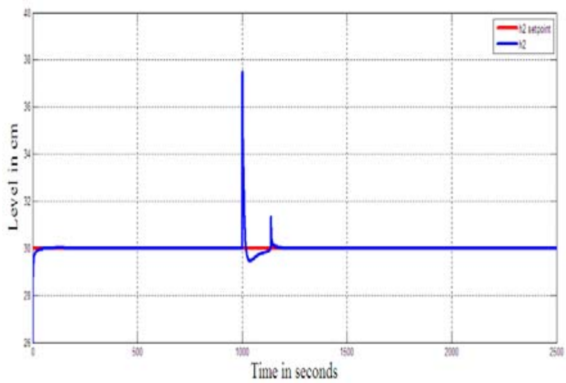


Figure.12. Regulatory response of $h_1$ in TTCIS using decentralized controller

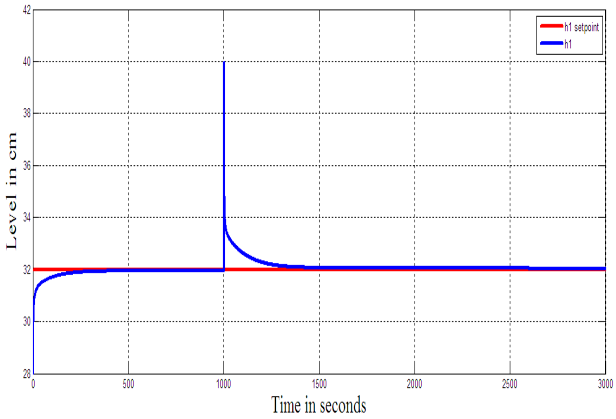


Figure.13. Regulatory response of $h_1$ in TTCIS using GLC

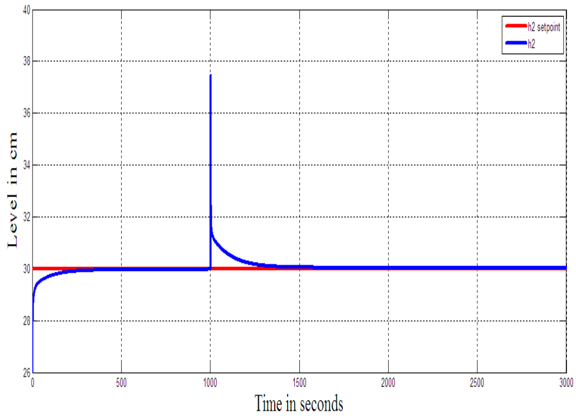


Figure.14. Regulatory response of $h_2$ in TTCIS using GLC

On comparing both the controllers,GLC controller exhibits better disturbance rejection capabilities and the performance due to disturbance rejection capabilities of both the control schemes are listed in Table.4.

TABLE.4. COMPARISON OF DISTURBANCE REJECTION CAPABILITY PERFORMANCE

| Controllers | 25% Disturbance in $h_1$ | | 25% Disturbance in $h_2$ | |
|---|---|---|---|---|
| | Settling time(secs) | ISE | Settling time(secs) | ISE |
| Conventional decentralized PI controller | 650 | 0.1955 | 450 | 0.1919 |
| GLC controller | 450 | 0.03889 | 400 | 0.04086 |

## VI. CONCLUSION

In this paper a Globally Linearized Controller (GLC) and decentralized PI controller are designed and incorporated on the two tank conical interacting non linear process. The servo and regulatory responses are obtained for different operating points with various set point and load changes through simulation. In all simulation analysis, both servoand regulatory responses settles quickly when GLC is employed. Also GLC offers minimum integral square error than decentralized controller. It can be concluded that GLC has good setpoint tracking, disturbance rejection capabilities and hence outperforms decentralized controller in all aspects for the chosen process.

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