Analytical and Numerical Calculations of Synchronous Motors for Industrial Drives

Iossif Grinbaum
ABB Switzerland Ltd., Segelhofstrasse 9P
5405 Baden-Dättwil, Aargau, Switzerland
E-mail: iossif.grinbaum@ch.abb.com

Axel Fuerst
University of Bern (BFH), Pestalozzistrasse 20
3400 Burgdorf, Switzerland
E-mail: axel.fuerst@bfh.ch

Cornelius Jäger
Univ. of Applied Sciences Eastern Switzerland (HSR)
Oberseestrasse 10, 8640 Rapperswil, Switzerland
E-mail: cornelius.jaeger@hsr.ch

Jasmin Smajic
Univ. of Applied Sciences Eastern Switzerland (HSR)
Oberseestrasse 10, 8640 Rapperswil, Switzerland
E-mail: jasmin.smajic@hsr.ch

Abstract—This paper presents the analytical and numerical methods for computing synchronous motors with salient pole rotor without damper winding for industrial drives. The entire industrial drive system and its requirements considering the motor calculation accuracy are described in detail. This result is of paramount importance for the daily motor design and optimization of the entire system. The obtained results of the described fast analytical calculation algorithm are compared against the corresponding results of the in terms of CPU-time more demanding field simulation based numerical algorithm (FEM). This comparison reveals some limitations of the analytical approach compared to the field simulation based numerical methodology.

Keywords—Motor drives; ac machines; numerical simulation; synchronous motors with salient poles; industrial drive; and electromagnetic fields.

I. INTRODUCTION

Synchronous motors (SM) for industrial drives are large due to their high torque requirements. The power of those motors covers the range from 2 to 30MW and the rated speed stays as a rule below 600rpm. These requirements in combination with indirect cooling of motor windings and the need for high efficiency make the magnetic, mechanical, and thermal design of these motors rather demanding.

The operation principle of the industrial drive is rather simple. A SM with a salient pole rotor is supplied by a controlled power converter. The most common power converter types used in industrial drives consist of a rectifier and inverter connected by either a voltage or current DC-link. However, for low speed and high power drives the topology without DC-link shown in Figure 1 is possible.

As mentioned above the SM for industrial drives has a high torque and low speed resulting in the design with many pole pairs due to fabrication-, installation- and service limits. It is worth mentioning that the converter is attached to an adjacent control- and protection system not depicted in Figure 1. This system monitors permanently the state variables of the drive (speed, torque, current, temperature, etc.) in order to ensure reliable operation and long lifetime.

As shown in Figure 1, the 6-pulse converter for the reverse drive consists of two controlled rectifiers connected back to back with one rectifier transformer per motor phase (this connection is valid for the stator’s Δ-winding connection). The low-frequency voltage at the converter output is produced by cutting the sinusoidal segments from the secondary harmonic voltage of the rectifier transformers. The frequency of the output voltage is freely adjustable by selecting the time interval at which the negative rectifier bridge of each motor phase takes over from the positive rectifier bridge.
The high-power system presented in Figure 1 generates significant voltage higher harmonics in the grid due to the switching effects of the converter’s thyristors [1]. This problem is not critical as the current form is smooth and almost without higher harmonics due to the effect of the transformer- and load inductance.

The issue of the power quality is an aspect of the demanding design requirements of this system. Another aspect is a failure of this equipment, which originates a large loss of production with significant economic losses. Therefore, the entire system must be carefully considered and analyzed in the design phase in order to reach the desired level of quality, reliability, and life expectancy especially because any drive and motor is to be optimized for the specific operating conditions of each field. This is possible only if the entire system is mathematically modeled in detail, simulated, and optimized.

The specific desirable and available characteristics of the synchronous motors for industrial drives along with its analytical and numerical (field simulations) considerations are presented in [2].

The motor as the most important component of the system can be designed by using the well-known analytical approach presented, for example, in [2], [3] and [8]. This approach usually consists of the following steps (a) analytical approximations of the motor’s magnetic circuit, calculation of the no-load curve and the excitation current, (b) computation of the motor’s resistances and reactances in order to obtain an equivalent circuit, and (c) computation of the motor torque, excitation current, and losses.

As an alternative to the mentioned analytical approach, it is possible to design the motor (the steps (b) and (c)) by using numerical field simulations. This method is also well-known and already reported for example in [2]. It is also worth mentioning here that the field simulation approach can be very detailed, but therefore also costly in terms of CPU-time and memory requirements.

The modern design process of the industrial drive motors aims at finding an optimum from the performance, material cost, and operating cost points of view. The process is an iterative loop requiring a high number of motor calculations for slightly different motor parameters defined by the optimization procedures. Therefore, the analytical method for computing the motor’s parameters has usually advantage over the numerical approach based on field simulations due to its significantly shorter CPU-time. The advantage of the numerical field simulations, on the other hand, is their high level of accuracy that cannot be reached by the analytical procedure. This is so due to (a) the complete modeling of physical effects within the numerical simulation scheme and (b) the operating with local differential equations of the numerical field simulation scheme instead of global integral equations within the analytical methods.

The original contribution and purpose of this paper is manifold: (a) to review the basic steps of the analytical calculation adapted to the properties of the industrial drive synchronous motors, (b) to present the equivalent numerical method based on field simulations, and (c) to compare the obtained results to show the accuracy and limits of the both methodologies considering this class of salient poles synchronous motors without damper winding. The damper winding is here not required because the drive has a frequency control.

The remaining part of the paper is organized as follows. Section II contains the description of the analytical methodology for computing the parameters of the synchronous motors with salient rotor for industrial drives. Section III gives the details of the corresponding numerical procedure based on field simulations. Section IV presents the obtained results and their comparison. Section V concludes the paper.

II. ANALYTICAL METHOD FOR COMPUTING SMS

The theory of synchronous motors with salient pole rotor is well-known and its analytical calculation available in the literature, as already reported, for example, in [2] and [4].

The calculation procedure starts with the input data specified by the product requirements: industrial size, output mechanical power, rated and maximal speed, maximal starting torque, altitude of the installation, cooling system and the temperature of the cooling medium, etc.

The input data are at the beginning usually used to determine the motor proportions by using the following well-known equation for the mechanical power of the SM [7]:

$$P_m = \frac{\pi^2}{\sqrt{2}} \beta_B k_w L n^2 B A \cos \phi 1000 (kW)$$

where $\beta_B$ is the flux density factor in the air gap (taking into account the relation of the flux density main wave amplitude to its average value and the pole shoe shape), $k_w$ is the stator winding factor for the fundamental harmonic component, $D$ is the air gap diameter (m), $L$ is the stator core effective length (m), $B$ is the average air gap magnetic flux density (T), $n$ is the rotor speed (rpm), $A$ is the specific current loading of the stator periphery (A/m), and $\cos \phi$ is the power factor.

As a general rule, the power $P_m$ and the speed $n$ are given by the customer, the magnetic flux density in the air gap $B$ is limited by saturation of the magnetic steel and the stator current load $J$ is limited by the insulation class of the stator winding.

As mentioned above, according to the mechanical limitation of pole dimensions, the number of pole pair is mainly defined by rated torque and available air gap circuit, which is as a rule limited by prescribed outer- or/and inner
diameter of the motor frame. This means also that the air gap diameter \( D \) is chosen.

Once the air gap diameter is chosen, it is possible to define the air gap length \( L \) between rotor and stator according to the common recommendations for industrial machines [2].

According to the rated motor voltage \( U \) and the chosen magnetic flux density \( B \), which in combination of pole face area defines the magnetic flux, it is possible to define the stator winding structure (the winding type, turn numbers, etc.).

The required magnetic flux determines the main geometrical dimensions of the magnetic circuit of the motor according to the allowable saturation level of the stator core magnetic steel and the rotor pole steel, i.e. the stator yoke height, the stator tooth width, the rotor pole width, and the rotor pole shoe shape. The corresponding magneto motive forces (MMF) for each part of the motor’s magnetic circuit are to be calculated after the dimensions of the magnetic circuit are obtained and after the level of the magnetic flux density is chosen.

Due to the fact that the air gap in the industrial machines is usually not constant between the stator inner surface and the outer surface of the rotor shoe, the B-field of the motor will depend on the angle between the stator rotating field axis and d-axis (the so called load angle) [2]. The well-known theory of two reactions (d-q theory) solves this problem by transforming the stator MMF to the two axis of the excitation winding [2], [4], and [5].

As a result of this theory two reactances of the stator winding are defined:

\[
\begin{align*}
X_d &= X_{as} + X_{ad} \\
X_q &= X_{as} + X_{aq}
\end{align*}
\]

where \( X_{as} \) is the leakage inductance of the stator winding, \( X_{ad} \) is the d-axis armature reaction reactance, and \( X_{aq} \) is the q-axis armature reaction reactance.

The reactances given by Equations (2) and (3) can be analytically determined by calculating the magnetic permeance of the radically simplified path of magnetic flux [2], [4], [7]. These reactances are essential for defining a detailed equivalent circuit of the SM and for computing the armature reaction of the SM.

After computing the reactances and by assuming no influence of the grid, no losses in the stator laminations and neglecting the stator resistance, the power of the SM with salient pole rotor can be computed as follows [2], [4]:

\[
P(\theta) = \frac{U \cdot U_g}{X_d} \cdot \sin(\theta) + \left(\frac{U^2}{2} \cdot \left(\frac{1}{X_q} - \frac{1}{X_d}\right)\right) \cdot \sin(2\theta)
\]

where \( U \) is the stator terminal voltage, \( U_g \) is the synchronous generated voltage, and \( \theta \) is the angle between the d-axis and the stator MMF (the load angle).

The detailed equivalent circuit of the SM with salient pole rotor without damper winding is presented in Figure 3. It is evident that the armature reaction reactances along with the stray inductances of the stator and field excitation circuit play a decisive role in computing the synchronous generated voltage \( U_g \) and the corresponding field excitation current \( I_{df} \) in each operating point of the SM.

As already emphasized the analytical computation of the reactances appearing in the equivalent circuit presented in Figure 3 involves radically simplification of the motor’s geometry and the distribution of the magnetic field lines. Modern analytical design algorithms, as for example [8], allow for designing electrical machines with practical accuracy of about 10%. This accuracy is not sufficient to estimate the influence of some parameters, such as for example the pole shoes shape, on finding an optimal solution.

Fig. 3. The detailed equivalent circuit of a SM with salient pole rotor without damper winding is presented. \( R_s \) is the stator resistance, \( I_{df}^* \) is the field excitation current recomputed to the stator side.

Fig. 4. An example of the motor power calculation according to Equation (4) is presented. Two cases with the armature reaction reactances computed with and without magnetic saturation effect were compared.
In this simplified picture it is almost impossible to accurately take into account complicated effects such as partial magnetic saturation of some parts of the stator’s and rotor’s magnetic circuits and unequal length of the air gap between the rotor’s pole shoe and stator inner air gap surface.

These effects can accurately take into account only a FEM field simulation that is presented in the next section.

To illustrate the need to accurately take into account the effect of magnetic saturation two analytical computations were performed. In the first case the stray and armature reaction reactances were computed by assuming that no part of the motor’s magnetic circuit is saturated, i.e. by taking the magnetic permeability of the core material being very high (theoretically infinite). This analysis resulted in the solid red motor’s power curve presented in Figure 4. On the other hand, if a certain level of magnetic saturation in some parts of the motor’s magnetic circuit is assumed the dashed red line presented in Figure 4 is obtained.

This analysis of the saturation effects clearly demonstrate that the corresponding power curves differ significantly from each other. In this particular case, this is essential for the motor’s capability to reliably deliver a certain torque or power in operation. If the operating point is close to the maximal deliverable torque known as breakdown torque and if the saturation effects are not well taken into account, it could happen, according to Figure 4, that the motor cannot reach the predefined operating point at the calculated excitation current. To avoid such situation a more accurate approach for computing the reactances based on field simulations is needed.

III. NUMERICAL METHOD FOR COMPUTING SMS

The leakage stator and rotor reactances and the armature reaction reactances of the SM can be very accurately computed by using field simulations. The corresponding 2-D magnetic field simulation boundary value problem (BVP) has the following form [6]:

\[
\frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial x} \right) - \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial y} \right) = J_z \quad \text{in} \quad \Omega \subseteq \mathbb{R}^2 \tag{5}
\]

\[
A_z = 0 \quad \text{on} \quad \partial_D \Omega \tag{6}
\]

\[
A_z(r, \theta_z + p) = -A_z(r, \theta_z) \quad \text{on} \quad \partial_P \Omega \tag{7}
\]

where \( A_z \) is the z-component of the magnetic vector potential perpendicular to the motor’s cross section (xoy-plane), \( \mu \) is the magnetic permeability of the material, \( J_z \) is the electric current density (existing only in the winding regions), \( \Omega \) is the computational domain (the cross section of the motor), \( \partial_D \Omega \) is the Dirichlet’s boundary with the known vector magnetic potential, and \( \partial_P \Omega \) is the periodic boundary.

The BVP (5)-(7) is usually solved by using the well-known Finite Element Method (FEM) [6]. The model setup of the chosen SM with salient pole rotor for industrial drives is shown in Figure 5b. In this representation is visible that 10 poles of the motor are modeled and the corresponding periodic boundary conditions are used. This is so because the stator winding is of the fractional type and has different symmetry planes compared to the rotor. In other words, as presented in [5], one basic motor having 10 poles is considered to reduce the calculation time.

The Figure 5a shows the geometry of the rotor and stator magnetic circuit, the winding structure and the air gap. Such a complicated geometry is very difficult to analytically compute by using the methodology presented in Section II.

The BVP (5)-(7) is discretized by using the standard FEM approach [6]. This includes the meshing of the computational domain (Figure 5b), the discretization of the energy functional of the field described by Equation (5)-(7) on the obtained mesh, the approximation of the unknown function \( A_z \) over each element of the mesh with respect to the unknown approximation coefficients, the computing of the unknown coefficients by finding the minimum of the energy functional.
The obtained solution of the vector potential $A_z$ is then used to compute the magnetic field and its energy [6]:

$$\vec{B} = \nabla \times \vec{A}$$

$$W_m = \frac{1}{2} \iint (\vec{H} \cdot \vec{B}) \, dS = \frac{1}{2} \iint (\vec{J} \cdot \vec{A}) \, dS \quad (J / m)$$

Once the magnetic energy is obtained it is possible to compute the corresponding inductance of the winding that produces this magnetic field [6]:

$$L = \frac{2 \cdot W_m}{I^2} \quad (H / m)$$

It is also worth mentioning that the energy (9) and the inductance (10) are obtained per unit length of the motor, because all the involved integrations were performed over the surfaces of the motor’s components. Therefore the units for energy and inductance are $(J/m)$ and $(H/m)$, respectively.

The situation in one SM with salient pole rotor is more complicated than it is given by Equations (9) and (10) because several different inductances should be computed after one field simulation. The rotor pole position also plays an important role in order to obtain the inductances $L_{ad}$ and $L_{aq}$.

The magnetic flux density distribution over the motor’s cross section at the moment of overlapping of the stator rotating field axis with the rotor’s d-axis is shown in Figure 6a. Similarly, Figure 6b shows the field distribution when the stator field axis overlaps with the rotor’s q-axis.

Although the magnetic field distributions presented in Figure 6 are rather complicated, it is possible to obtain separately the reactance components given by Equations (2) and (3). The total inductance for the both rotor’s positions is obtained according to the magnetic energy as given by Equations (9) and (10). Additionally, the magnetic flux that through the air gap reaches the pole shoe can be obtained as follows:

$$\Phi_{ad} = \int \vec{B}_{ad} \cdot \vec{n} \, dl \quad (Wb / m)$$

$$\Phi_{aq} = \int \vec{B}_{aq} \cdot \vec{n} \, dl \quad (Wb / m)$$

Equation (11) and (12) present the magnetic flux through the air gap for the rotor position showed in Figure 6a and 6b, respectively. The corresponding inductances can be computed according to the following equations:

$$L_{ad} = \frac{N \Phi_{ad}}{I} \quad (H / m)$$

IV. NUMERICAL RESULTS

At the beginning of the analysis the inductances of the ABB synchronous motor with salient poles presented in Figure 5 for the conditions of unsaturated magnetic circuit were computed. This ABB motor has the following parameters:

$$P_n = 15.6(MW), U_b = 3900(V), f = 5.6(Hz)$$

The unsaturated magnetic circuit of the motor is also presented in Figure 6, since the magnetic flux density does not go above 0.3T. The obtained results for this unsaturated case (the linear case) are shown in Table I.

According to the inductances presented in Table I and their corresponding reactances, the motor characteristics
with respect to the load angle were computed and visualized in Figure 7.

It is evident from Figure 7b that the total magnetizing current of the motor surpasses the nominal stator current over the entire theoretical operating range (0°-71.1°). Additionally, at the peak of the power the total magnetizing current reaches 150% of the nominal stator current.

This shows that the effect of magnetic saturation must be taken into account very carefully during the reactance computing procedure. To further clarify this statement the inductances of the SM for various magnetizing currents were computed. These results are presented in Figure 8. It is evident that the inductance $L_{ad}$ significantly reduces its value if the magnetizing current goes above 0.8 (p.u.). On the other hand the magnetizing current showed in Figure 7 has values between 1.2 (p.u.) and 1.5 (p.u.) within the theoretical operating range (the load angle between 0° and 71.1°). The remaining two inductances $L_{aq}$ and $L_{ad}$ also change their values with the saturation level but much less significant compared to the inductance $L_{ad}$.

Fig. 7. The power (a) and the currents (b) of the SM showed in Figure 5 with respect to the load angle are visualized. The operating point is defined by the load angle of 40.7°. $I_s$ is the stator current, $I_u$ is the magnetizing current, and $I_d$ and $I_q$ are the projections of the stator current to the rotor’s d- and q-axis, respectively.

Fig. 8. The inductances of the SM presented in Figure 5 with respect to the magnetizing current are depicted. It is evident that the inductance $L_{ad}$ is significantly reduced when the magnetizing current is above 1 (p.u.) and $L_{aq}$ and $L_{ad}$ are only slightly affected by the magnetic saturation.

Fig. 9. The power of the SM presented in Figure 5 with respect to the load angle (a) and the corresponding magnetic flux density distribution at the operating point of the SM ($\theta=40.7^\circ$) (b) is presented. The curves (a) are a result of the calculation based on the inductances presented in Figure 8 that are calculated by using FEM field simulations.

The last result shown in Figure 8 means that the inductance differences due to the saturation level should be taken into account in every operating point, i.e. for each value of the load angle, for computing the power curve. This analysis yielded the blue power curve depicted in Figure 9 by fully taking into account the nonlinear effects and the
corresponding inductance differences. Evidently, the nonlinear curve has a significant offset from the corresponding linear power curve for higher values of the load angle.

The comparison between the breakdown torque calculated using the analytical method (Fig. 4) and using the FEM method (Fig. 9) reveals a good agreement. The numerical difference between the two methods computed for the linear and nonlinear case of the breakdown torque amounts to 2.4% and 6.4%, respectively.

This comparison result should not be considered as an argument to exclusively use the analytical methodology, as those results are design dependent. It is recommended to validate every new design by using the corresponding FEM method and/or accurate measurements.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>THE INDUCTANCES OF THE ABB SYNCHRONOUS MOTOR WITH SALIENT POLES SHOWN IN FIGURE 5 FOR THE UNSATURATED CONDITIONS.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical method</td>
</tr>
<tr>
<td>$L_{m1}(mH)$</td>
<td>3.2</td>
</tr>
<tr>
<td>$L_{m2}(mH)$</td>
<td>22.1</td>
</tr>
<tr>
<td>$L_{m3}(mH)$</td>
<td>13.6</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

This paper presents and compares the analytical method and the field simulation based numerical method for computing large synchronous motors for industrial drives with salient pole rotor without damper winding. The influence of the magnetic saturation effects on the numerical results of the both methodologies are analyzed in detail.

A careful comparison of the results from Figure 9 with the results from Figure 4 yielded the following conclusions:

(a) the presented analytical method can predict well the character of the saturation influence on the power curve, (b) the agreement between the analytical method and the numerical simulations can be very high, if the correction factors in the analytical computation valid for the considered design are known and used, and (c) due to the fact that the analytical nonlinear method uses the approximated inductance to take into account the magnetic saturation independent from the load angle, the accuracy of the analytical results for each motor and operating regime can be guaranteed only by using the step-up safety factor of at least 1.1. In general only the presented field simulation method yields always reliable results and can thus reduce the corresponding inaccuracy risk factor for optimized design of synchronous motors.

ACKNOWLEDGEMENTS

This work was supported in part by the Swiss Commission for Technology and Innovation (CTI) under Grant KTI 15736.1 PFEN-IW.

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