Improved Control System Design of Quadrotor Helicopter Based on Integral Neural Sliding Mode control

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Abstract — Regarding the characteristics of the current quadrotor helicopter control system i.e. strong coupling, complex nonlinear, multi-input multi-output and susceptibility to outside disturbance and modeling error, a double closed-loop control structure is proposed, involving attitude integral sliding mode robust control. The proposed controller consists of two parts: an angular velocity controller as the inner-loop and a position controller as the outer-loop. In which the switch function is realized by using integral sliding mode. The dynamic model of the quadrotor helicopter is established based on the Newton-Euler equation. The RBF neural network is used to adjust the gain of the sliding mode switching term and to adaptively learn the upper bound value of uncertain factors such as external disturbance and modeling error of system in order to weaken the chattering phenomenon caused by the conventional sliding mode control. The stability and exponential convergence of the closed-loop system have been proved according to Lyapunov theory, and the feasibility and effectiveness of the proposed approach is verified by numerical simulations.

Keywords - Quadrotor helicopter, RBF neural network, attitude control, sliding mode control

I. INTRODUCTION

Quadrotor helicopter is an under-actuated system with 6 degrees of freedom and 4 control inputs, which demands on real-time control to ensure the stability of the vehicle. Compared with traditional helicopter, quadrotor helicopter has a great advantage in attitude control. Because it has a symmetrical rotor distribution and its four rotors can cancel the reverse torsion moment between each other, it does not require a special reverse torsion moment device, and it can change the attitude or height of the aircraft by adjusting the rotor speed[1] .Therefore, it has been widely used in unmanned reconnaissance, forest fire prevention, disaster monitoring, city patrolling and other fields. With more and more attention is paid to quadrotor helicopter, its quality, safety, and reliability are also becoming significant. In recently years, the modeling and control of the quadrotor helicopter have been sufficiently studied. Many control methods have been proposed, among which the hierarchical control method is frequently used. From inside to outside the control structure can be divided into: motor speed control, attitude control, and position control. At present, the commonly used control algorithm include backstepping control[2], linear quadratic optimal control, PID control[3], H∞ control[4] etc. for the linear part; neural network control, sliding mode control, adaptive control etc. for the nonlinear part. PID control has the advantages of simplicity and robustness. However the control result is poor 9-11 for aircraft system with strong disturbances. The sliding mode control algorithm is simple and insensitive to parameter variations and has a strong anti-interference ability. It has been widely used in quadrotor control, but the sliding mode control is easy to bring the chattering phenomenon to the system. By introducing a disturbance observer, the uncertainty and the external disturbance of the controlled object can be estimated accurately, which can reduce the gain of the sliding mode control and the chattering [5]. In reference[6-7], the problem of under-actuated sliding mode control is studied, and the sliding mode control has good robustness, but the control result is not ideal when the interference is large, because it does not estimate the system disturbance in real-time. The problem of backstepping control is studied in reference[8],but it has a weak anti-interference ability and needs to build a precise math model.

Based on the extended state observer, an active disturbance rejection control attitude decoupling algorithm is proposed in reference [9], which is simulated by a real-time compensation of the disturbance. This method is mainly used to control the attitude of the quadrotor helicopter. In reference [10], an adaptive sliding mode control method is proposed ,it can self-repair the control fault or disturbance of quadrotor helicopte system. However, this method is unable, to analyze the closed-loop system and prove the stability of the system in terms of the overall situation. In reference [11], an intelligent controller with an online correction feedback control law is designed based on
neural network. This controller, however, needs a large number of training data for neural network. In reference [12], a feedback tracking controller for flight attitude controlling is designed combined with the sliding mode technology. In reference [13], Euler angle is selected as the system state in the process of system modeling. When the system works in the vicinity of the equilibrium point, the first-order derivative of the Euler angle can be considered to equal to the angular velocity of the system. This method can simplify the controller design process to a certain extent. Because it still involves strong couplings, it does not reduce the complexity of controller design. In addition, the stability of this kind of system is difficult to analyze. In reference [14], the authors constructed a sliding mode observer, using adaptive adjustment to restrain chattering, and obtained satisfying results.

Regarding the characteristics of the control system of quadrotor helicopter, i.e. complex nonlinearity, strong coupling and susceptibility to outside disturbance and modeling error, we propose a double closed-loop control structure of sliding mode control. In the scheme, the outer-loop is a position loop, and the inner-loop is a velocity loop. This controller possesses the properties of strong robustness and anti-disturbance. Furthermore, it can eliminate the steady-state error of the system by introducing an integral term, thus attaining a high control precision. The RBF neural network is used to adaptively learn the upper bound of the uncertain part and adaptively adjust switching gain to reduce the chattering of general sliding mode control.

II. DYNAMIC MODEL OF QUADROTOR HELICOPTER

The structure of quadrotor helicopter is shown in Figure 1. It consists of a combination of 4 rotors to achieve translation, pitching, and rolling motion in the space. These movements are achieved relying on the speed difference between rotor 1 and 3 and between rotor 2 and 4, respectively; and the yawing motion is achieved through coordinated rotations of the four rotors. However, because of the cross coupling among the four rotating forces and torques, there will be some effects among the pitching, yawing, and rolling motion. Moreover, because the quadrotor helicopter has 6 degrees of freedom and only 4 motors act as drivers, it is an under-actuated system.

As we described earlier, the quadrotor helicopter is a strongly coupled and nonlinear system. In order to build its mathematical model, it is necessary to make several assumptions as follows.

1. All structures are rigid;
2. Quadrotor helicopter is strictly symmetrical;
3. The gravity center of the aircraft is the structural center;
4. The lifting surface of the motors and the gravity center are located in the same plane;
5. \( \Delta J \) and \( d \) are bounded.

Neglecting the elastic vibration and deformation, the movement of the quadrotor helicopter can be considered as a rigid motion with 6 degrees of freedom, consisting of a 3-axis twirling motion and the line motion of the gravity center along the 3 axes. The body-fixed frame and inertial frame are chosen as the reference benchmark, when establishing dynamic model. As shown in Figure 1, taking \( O_x \) as the origin of the inertial frame, establish \( O_{xyz} \) coordinate system and taking \( O_b \) as the barycenter of quadrotor helicopter, establish the body-fixed frame \( O_{xyz} \), both of which are the right-hand coordinate system. Euler angle is used to denote twirl around the body-fixed frame. Taking pitching angle \( \phi \), roll angle \( \theta \), and heading angle \( \psi \) as the attitude angle of the quadrotor helicopter. Pitch angle \( \phi \) is defined as the angle between the \( x \) axis of the body and the \( X \) axis of the inertial frame, roll angle \( \theta \) is defined as the angle between the \( y \) axis of the body and the \( Y \) axis of the inertial frame, and heading angle \( \psi \) is defined as the angle between the \( z \) axis of the body and the \( Z \) axis of the inertial frame.

According to Newton-Euler equations, we can get rotation equation of quadrotor helicopter rolling body frame as:

\[
(J_0 + \Delta J)\dot{\omega} = -\Omega(J_0 + \Delta J)\omega + M + d
\]

where, \( \omega = [\omega_x, \omega_y, \omega_z]^T \) is the angular velocity of quadrotor helicopter; \( J \in \mathbb{R}^{3\times3} \) is the rotational inertia matrix relative to the body frame; \( \Delta J \) is uncertainties caused by system structure change and so on; \( M = [M_x, M_y, M_z]^T \) is the control torque effect on quadrotor helicopter; \( d \) is external disturbance.

According to the structural characteristics of the quadrotor helicopter, the inertia matrix is obtained:

\[
J = \begin{pmatrix}
J_x & 0 & 0 \\
0 & J_y & 0 \\
0 & 0 & J_z
\end{pmatrix}
\]

From the conversion relation between the body frame and the inertial frame, the attitude kinematic equations of quadrotor helicopter is obtained:
\[
\left(\phi \ \dot{\theta} \ \dot{\psi}\right)^T = R^{3\times3}\omega
\]

Transformation matrix from the body frame to the inertial frame is as follows:

\[
R^{3\times3} = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & \sin \phi \\
0 & \sin \phi \sec \theta & \cos \phi \sec \theta
\end{bmatrix}
\]

In summary:

\[
\begin{align*}
(J_0 + \Delta J) \cdot \dot{\omega} &= -\Omega \cdot (J_0 + \Delta J) \cdot \omega + M + d \\
\left(\phi \ \dot{\theta} \ \dot{\psi}\right)^T &= R^{3\times3}\omega \\
y &= (\phi \ \theta \ \psi)^T
\end{align*}
\]

By designing the control torque \(T\), the output of system \(y\) can track the attitude angle \(y^*\), that is:

\[
\lim_{t \to \infty} \|y - y^*\| = 0
\]

Quadrotor Helicopter is mainly affected by the following torque:

\[
\begin{align*}
M_x &= M_{\phi} + M_{\theta} + M_{\psi} + M_{\phi\theta} + M_{\phi\psi} + M_{\theta\psi} \\
M_y &= M_{\phi} + M_{\theta} + M_{\psi} + M_{\phi\theta} + M_{\phi\psi} + M_{\theta\psi} \\
M_z &= M_{\phi} + M_{\theta} + M_{\psi} + M_{\phi\theta} + M_{\phi\psi} + M_{\theta\psi}
\end{align*}
\]

Where, the subscript (R, G, D, d) are represented respectively as torque generated by rotor lift, gyroscopic effect, air resistance and external disturbance. The rotor gyroscopic effect is that when the four-rotor pitch and roll moving, high-speed rotation of the rotor generated additional torque to impede pitching and rolling motion. Because the quadrotor helicopter rotor area and rotational inertia is small and its flight speed is slow, normally gyroscopic effect and resistance torque can be neglected[16-17].

There is a close connection between the attitude information of the quadrotor helicopter and the speed of the four motors. The torque generated by rotor lift is:

\[
\begin{align*}
M_{\phi} &= l k_c (\omega_x^2 - \omega_z^2) \\
M_{\theta} &= l k_c (\omega_y^2 - \omega_z^2) \\
M_{\psi} &= k_c (\omega_x^2 + \omega_y^2 - \omega_z^2 + \omega_z^2)
\end{align*}
\]

Where, \(k_c\) is the lift coefficient which is relevant to the rotor area, the radius of rotor, the air density and other factors. \(\omega_i\) is the rotate speed of the rotor, \(l\) is the vertical distance between motor shaft and the barycenter of quadrotor helicopter. \(k_c\) is the reactive torque coefficient. Quadrotor helicopter nonlinear coupling model is decomposed into four separate control channel and the control input of the system is defined as[18]:

\[
\begin{align*}
M_1 &= F_1 + F_2 + F_3 + F_4 = k \sum_{i=1}^{4} \omega_i^2 \\
M_2 &= F_1 - F_2 = k_\theta (\omega_x^2 - \omega_y^2) \\
M_3 &= F_1 - F_3 = k_r (\omega_x^2 - \omega_z^2) \\
M_4 &= -F_1 + F_2 + F_3 = k_\psi (\omega_x^2 - \omega_y^2 + \omega_z^2 - \omega_z^2)
\end{align*}
\]

where, \(M_1\) is the vertical control input, \(M_2\) is roll control input, \(M_3\) is pitch control input, \(M_4\) is yaw control. The model adopts the basic physics equations, and assuming that the aircraft aerodynamic coefficient is fixed, ignoring the body mechanical vibration, sensor delay in attitude perception and sensors noise effect. The external torque of quadrotor helicopter can be expressed as:

\[
\begin{align*}
M_{x} &= l k_c (\omega_x^2 - \omega_z^2) + M_{sd} \\
M_{y} &= l k_c (\omega_y^2 - \omega_z^2) + M_{sd} \\
M_{z} &= k_c (\omega_x^2 + \omega_y^2 - \omega_z^2 + \omega_z^2) + M_{sd}
\end{align*}
\]

In summary, the equation of attitude motion of quadrotor helicopter can be expressed as:

\[
\begin{align*}
\dot{\phi} &= l k_c (\omega_x^2 - \omega_z^2) + M_{sd} / J_x + (J_z - J_y) \omega \sin \phi / J_x \\
\dot{\theta} &= l k_c (\omega_y^2 - \omega_z^2) + M_{sd} / J_y + (J_x - J_z) \omega \sin \phi / J_y \\
\dot{\psi} &= k_c (\omega_x^2 + \omega_y^2 - \omega_z^2 + \omega_z^2) + M_{sd} / J_z + (J_z - J_y) \omega \sin \phi / J_z \\
\dot{x} &= (\sin \phi \sin \psi + \cos \psi \sin \theta \cos \phi) M_{x} / m \\
\dot{y} &= (\cos \theta \sin \phi) M_{y} / m \\
\dot{z} &= (\cos \phi \cos \theta) M_{z} / m - g
\end{align*}
\]
A. Outer Loop Sliding Mode Control

Outer loop sliding mode control is mainly used to design the attitude angular velocity instructions \( \omega_e \), and \( \omega_e \) is the input of inner loop control system. Outer loop is to achieve attitude angle tracking. The outer loop sliding mode surface is designed as follow[19]:

\[
S_w = \theta_e + K_1 \int_0^t \dot{\theta}_e dt, \quad S_w \in \mathbb{R}^3
\]  

(10)

Where, \( K_1 = \text{diag} \{ k_{11}, k_{12}, k_{13} \} \) is the gain matrix, and choosing appropriate gain matrix can make the system tracking instruction deviation in a ideal sliding mode surface sliding to stable. Take attitude angular velocity instructions as attitude angular velocity tracking virtual control items. 

In (3) , \( (\phi \dot{\psi})^T = R^{+3} \cdot \omega \), where, the error between the \( \omega_e \) and \( \omega \) is eliminated through inner loop control. Attitude angular velocity instruction is designed as:

\[
\omega_e = R^{-1}(\theta)(\dot{\theta} + K_2 \dot{\theta}_e) + R^{-1}(\theta)\rho_1 \text{SGN}(S_w)
\]  

(11)

where, \( \rho_1 > 0 \)

\[
\text{SGN}(S_w) = [\text{sgn}(s_1) \text{sgn}(s_2) \text{sgn}(s_3)]^T.
\]

According to (10) , \( S_w \) can be written as:

\[
S_w = \hat{\theta}_e + K_2 \dot{\theta}_e = \dot{\theta}_e - R(\theta)\omega_e + K_2 \dot{\theta}_e
\]

Define a Lyapunov-like function as:

\[
V = \frac{1}{2} S_w^T S_w
\]

Then

\[
\dot{V} = S_w^T \dot{S}_w = S_w^T \left( \dot{\theta}_e - R(\theta)\omega_e + K_2 \dot{\theta}_e \right)
\]  

\[
= S_w^T \left( \dot{\theta}_e - R(\theta) \left( \dot{\theta} + K_2 \dot{\theta}_e \right) + R^{-1}(\theta) \rho_1 \text{SGN}(S_w) \right) + K_2 \rho_1 \dot{\theta}_e
\]

\[
= -\rho_1 S_w^T \text{SGN}(S_w) = -\rho_1 \sum_{i=1}^3 |S_{wi}| < 0
\]

B. Inner Loop Sliding mode control

In order to achieve \( \omega_e \rightarrow \omega \), inner loop sliding mode control law is designed to make \( \omega_e - \omega \rightarrow 0 \). Using integral sliding mode surface to design inner loop sliding mode function[20]. That is:

\[
S_n = \omega_e + K_2 \int_0^t \omega_e dt, \quad S_n \in \mathbb{R}^3
\]  

(12)

Where, \( \omega_e = \omega_e - \omega, K_2 = \text{diag} \{ k_{21}, k_{22}, k_{23} \} \) is the gain matrix. The control law is designed as follows

\[
M = J_0 \omega + J_2 \dot{\theta} + \Omega J_0 \omega + \mu S_n + \rho_2 \text{SGN}(S_n)
\]  

(13)

Where, \( \rho_2 > 0, \mu > 0 \). According to (4) and (13),we can get

\[
s_n = \omega_e + K_2 \omega_e = \omega_e + (J_0 + \Delta J)^{-1} \Omega (J_0 + \Delta J) \omega - (J_0 + \Delta J)^{-1} M - (J_0 + \Delta J)^{-1} d + K_2 \omega_e
\]

Define a Lyapunov-like function as

\[
V = \frac{1}{2} S_n^T (J_0 + \Delta J) S_n
\]

Because of \( (J_0 + \Delta J) \) is positive definite matrix, therefore, \( V > 0 \), then:

\[
\dot{V} = S_n^T \left[ \frac{1}{2} \Delta J \dot{S}_n + (J_0 + \Delta J) \dot{S}_n \right]
\]  

\[
= \frac{1}{2} S_n^T \Delta J S_n + S_n^T (J_0 + \Delta J) \left[ \omega_e + (J_0 + \Delta J)^{-1} \Omega (J_0 + \Delta J) \omega \right] - (J_0 + \Delta J)^{-1} M - (J_0 + \Delta J)^{-1} d + K_2 \omega_e
\]

Substituting the control law in (13) into formula above ,then

\[
\dot{V} = \frac{1}{2} S_n^T \Delta J S_n + S_n^T \left[ J_0 K_2 \omega_e - \Omega J_0 \omega - \mu S_n - \rho_2 \text{SGN}(S_n) \right] - \left( J_0 + \Delta J \right) K_2 \omega_e
\]

\[
\dot{V} = -\rho_2 \sum_{i=1}^3 |s_{ni}| - \mu \| \omega \| + \frac{1}{2} S_n^T \Delta J S_n + S_n^T \left[ \Delta J \omega_e + \Omega \Delta J \omega - d + \Delta K_2 \omega_e \right]
\]
Where, \( \Delta J \dot{\omega} + \Omega \Delta J \omega - d + \Delta J K_2 \omega \leq \rho_i \leq \rho_i \), \( i = 1,2,3 \).

Assuming \( \lambda_{\text{max}} \) is the maximum eigenvalues of the \( \Delta J \), and through designing \( \mu \), we can get \( \mu - \lambda_{\text{max}} > 0 \), then:
\[
V \leq -\left( \mu - \lambda_{\text{max}} \right) \||S_n|| \leq 0 \quad (14)
\]

### C. Improved Sliding Mode Controller Based on RBF Neural Network

When the system trajectory deviation from sliding mode surface because of external disturbance and the system modeling error, the discrete control \( \Delta w \) makes the system trajectory toward the sliding mode surface. But the switching term gain of controller will bring the chattering to sliding mode control. The chattering phenomenon can lead to severe concussion, which will bring difficulty to the actual control, and may even damage the system stability[21-22].

In order to further reduce the chattering of sliding mode control, the neural network is used to online real-time adjust the switch gain values \( \rho_i \). Due to the ordinary BP algorithm easily lead to local minimum and cannot get the overall optimum and has a slow convergence speed, the RBF neural network is used in this paper. RBF neural network has the following advantages: simple structure, fast convergence speed[23], and can approximate any nonlinear function. The network structure is as shown in figure 3.

![Figure 3. Architecture of RBF neural network](image)

The input of the RBF network is \( x = [\dot{\theta} \ \dot{\phi}] \) and the output is \( \rho_i \), where \( i = 1,2 \).

Where, \( \rho_i = \left| w^T h(x) \right| \quad (15) \)

In \( (15) \), \( w^T \) is the weight of the neural network, \( h(x) \) is the Gauss function.
\[
h_i(x) = \exp(-\frac{||x - m_i||^2}{\sigma_i^2}), \quad i = 1,2,3
\]

Where, \( m_i \) is the center of the No. \( i \) neurons, \( \sigma_i \) is the width of the No. \( i \) neurons. The weights of neural network are adjusted:
\[
E = \frac{1}{2} e^2 \quad (16)
\]

From the equation (16), the weights learning algorithm of neural network is:
\[
\Delta w = -\eta_1 \frac{\partial E}{\partial w} = -\eta_1 \frac{\partial e}{\partial w} = -\eta_1 \frac{\partial y}{\partial w} = -\eta_1 \frac{\partial u}{\partial w} \quad (17)
\]

\[
\sigma = -\eta_1 \text{sgn} \left( \frac{\partial y}{\partial u} \right) \frac{\partial u}{\partial \rho_i} = -\sigma^{-1} \text{sgn} \left( \frac{s}{h(x)} \right) \text{sgn} \left( \frac{h(x)}{h(x)} \right) \quad (18)
\]

The weights of network learning algorithm is:
\[
\Delta w(t) = -\eta_1 \left( -\sigma^{-1} \text{sgn} \left( \frac{s}{h(x)} \right) \text{sgn} \left( \frac{h(x)}{h(x)} \right) \right)
\]

The weights of network learning algorithm is:
\[
\eta_i \in (0,1), \alpha \text{ is inertia coefficient, } \alpha \in (0,1).
\]

Similarly, the external disturbance and the upper bound of the uncertain factors such as modeling error values are also learned by RBF network adaptively. The inputs of network are \( x = [\theta \ \dot{\theta}] \) and \( x = [\omega \ \dot{\omega}] \), the outputs are the estimate value of the upper bounds of the uncertain parameters \( \Delta J \) and \( \dot{\theta} \).

### IV. Numerical Simulation

In order to verify the effectiveness of the proposed method, the simulation model of quadrotor helicopter attitude control system is established by MATLAB/SIMULINK.

Numerical simulation parameter are as follows:
- \( m = 0.8kg \), \( l = 0.8m \), \( J_{xx} = 0.033Kg \cdot m^2 \), \( J_{yy} = 0.033Kg \cdot m^2 \), \( J_{zz} = 0.061Kg \cdot m^2 \),
- \( d = [\sin t \ \cos t \ \sin(2t)] \), the initial
angle $\theta = [\varphi \ \theta \ \psi]^T = [0 \ 0 \ 0]^T$, the initial angular velocity $\omega = [\omega_x \ \omega_y \ \omega_z]^T = [0 \ 0 \ 0]^T$. Angle tracking order is cosine signal $\Theta_c = [\cos t \ \cos t \ \cos t]^T$. In the outer loop control law, saturation function is used to replace switching function. $K_1 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$. The parameters of inner loop sliding mode control law are $\mu = 8$, $K_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. The simulation results are showed in figure 4–figure 6. From the figure 4, we can know that angle tracking of yawing and angle tracking of pitching can achieve stability in 4–5 s, while angle tracking of rolling can achieve stability in 6 s and the steady-state error is approximately equal to 0. The result shows that attitude angle tracking is fast and stable and the proposed method is effective.

V. CONCLUSIONS

Aiming at the characteristics of the quadrotor helicopter control system i.e., complex nonlinear, strong coupling and susceptibility to outside disturbance and modeling error, a double loop control structure of sliding mode control is proposed. In the scheme, the integral sliding mode switching function is used and the RBF neural network is used to adjust the gain of the sliding mode switching term and to adaptively learn the upper bound value of uncertain factors such as external disturbance and modeling error of system. The result shows that the proposed approach is feasible and effective.

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