Application of Partial Differential Equation Based on PSO Algorithm

Chong Tian1,*, Chunshan Xue1, Yongfeng Cui2

1 School of Mathematics and Statistics
Zhoukou Normal University
Zhoukou, Henan, China
2 School of Computer Science and Technology
Zhoukou Normal University
Zhoukou, Henan, China
tianchong@zknu.edu.cn

Abstract — This paper uses PSO algorithm to analyze the solving of the optimal solution of partial differential equation, and ultimately builds an application model of partial differential equation based on PSO algorithm. First, this paper introduces PSO algorithm initially, points out that PSO algorithm is one of the mathematical models to solve the optimal solution, and proposes that the application of PSO algorithm in the solving of partial differential equation is reasonable. Secondly, taking the application of partial differential equation in population issues as a specific example, this paper gives the expression of using partial differential equation to solve population distribution, and introduces PSO algorithm to calculate the optimal solution, and proves that the application model of partial differential equation based on PSO algorithm is credible through reliability analysis.

Keywords - PSO algorithm; optimal solution of partial differential equation; reliability analysis

I. INTRODUCTION

Partial differential equation is widely applied in real life, and the solving of nearly all complex problems may apply partial differential equation, such as the solving in physics. In addition, with the rapid development of modern technology, partial differential equation has also been applied to a variety of high-tech technologies. And image processing technology is a relatively common one [1].

PSO algorithm is a commonly used and relatively effective mathematical method in solving the optimal solution. And there has been a lot of research on its application. Hu Wang points out that compared with other genetic algorithms, PSO algorithm is an efficient optimization algorithm that has accurate calculation and is relatively easy to implement. Hu Jianxiu studies second-order PSO algorithm, which lays foundation for its application in practical problems. In the PID parameter selection of water turbine’s speed governing system, Wang Bo introduces PSO algorithm to select the parameters that are most favorable for system operation, and also makes some improvements, which lays mathematical foundation for the determination of the optimal parameters and makes the searching process of parameters more accurate; in the study of XML population, [2, 3] in order to achieve the purpose of intelligent search, and after comparing advantages and disadvantages of PSO algorithm and ant colony algorithm, Liu Bo integrates the two algorithms and achieves the optimization and selection of intelligent information search by using the combination, which is a breakthrough in the field; in the study of network route, Feng Jun proposes that due to the complexity of the Internet, in order to determine the optimal path from the complex relative network under the condition in conformity with various constraints, and improved PSO algorithm simplifies the process, and obtains the exact optimal solution through calculating; in the study on the injury degree of structure and the testing process, Wan Zuyong introduces improved PSO algorithm and selects the optimal path of detecting structural damage with this algorithm [4-5].

Through learning from a number of scholars’ research results, this paper uses PSO algorithm to study the application of partial differential equation, takes population issue as an example, and points out that partial differential equation is applicable to the solving of population distribution, and the unique solution of population distribution model can be obtained through this model [6].

II. RELATED RESEARCH

A. PSO Algorithm

PSO algorithm is short for Particle Swarm Optimization algorithm. It is an optimization algorithm, similar with genetic algorithm, for both of them are algorithms looking for the optimal solution, but they still have some differences. PSO algorithm is simpler, eliminating the steps like crossover and mutation, having high precision, and easy to implement. For this reason, this algorithm has been widely used in the solving of practical problems nowadays [7].

Figure 1. PSO algorithm for solving the optimal solution
The above figure shows the process starting from random solutions to the optimal solution after continuous iteration and fitness evaluation. This process has following advantages [8]:

![The advantages of PSO](image1)

In addition, PSO algorithm also belongs to a parallel algorithm. In solving the optimal solution, the algorithm is favored by researchers because of its above characteristics [9].

**B. Application of Partial Differential Equation**

Partial differential equation is one of ways to solve a variety of complex mathematical problems, and it can describe the relationship between the unknown variable and time derivative, spatial derivative, so as to explain the significance of unknown variable from the aspect of time and space. To this end, the solving of many physical and mechanical problems would apply partial differential equation [10].

![Partial Differential Equations](image2)

Due to the uniqueness of partial differential equation, the solving of practical problems is difficult. In order to obtain a unique solution of the problem, we must use other mathematical algorithms to find the optimal solution from an infinite number of solutions, such as PSO algorithm, genetic algorithm, artificial intelligence, annealing algorithm and so on. PSO algorithm is an easily used and implemented algorithm for solving the optimal solution, so it can be applied in finding the optimal solution from an infinite number of solutions of differential equation.

![Solving the optimal solution](image3)

Partial differential equation is not only applied in solving mathematical problems, but also used in other practical problems, such as population issues, dynamics of infectious diseases, complex force solving in physics and so on.

![Application of Partial Differential Equations](image4)
III. ALGORITHM DESIGN AND IMPLEMENTATION

As ant colony algorithm is inspired by ants crawling, PSO algorithm is inspired by the characteristics of birds foraging, which is a mathematical algorithm derived from birds foraging behavior. The algorithm is based on iteration, and through the initial solution set of the system, finally gets the optimal solution through continuous iteration.

The following figure is the schematic diagram of the PSO algorithm based on birds foraging behavior.

![Figure 6. Foraging of birds](image)

The above figure shows the process of birds foraging. The only food O is located at the origin of the space coordinate, within space x-y-z, birds A, B, C, D and E are searching food O at different speeds and from different directions, and they are unknown of the location of food O.

![Figure 7. Particle Swarm Optimization](image)

PSO algorithm finally gets the optimal solution of a problem based on this idea. In this algorithm, each initial random solution can be treated as a "particle", as birds foraging, continues to narrow the scope of the solution in the known solution space and finally finds the optimal solution.

In the process of "particle" finding the optimal solution, its speed, direction and position coordinate are changing, and the formula applied is shown as follows:

\[
\begin{align*}
    v_i(t+1) &= \omega \cdot v_i(t) \\
    &+ c_1 \cdot \text{rand} \cdot (p_{best} - x_i(t)) \\
    &+ c_2 \cdot \text{rand} \cdot (g_{best} - x_i(t)) \\
    x_i(t+1) &= x_i(t) + v_i(t+1)
\end{align*}
\]

where \( t \) is the times of particle updating and iterating.

For standard PSO algorithm, its calculation process is listed as follows.

1. Initialize the random velocity and position of a group of particles \( m \);
2. Use fitness to evaluate each particle;
3. For each particle, compare its adaptive value with its best position \( p_{best} \) in the searching process, and it is considered as the best position \( p_{best} \) for the moment if the result is good;
4. For each particle, compare its adaptive value with the best position \( p_{best} \) of all particles in the searching process, and reset \( g_{best} \) index number if the result is good;
5. Change the speed and position according to equation (1);
6. Based on the results of a-e, if the termination condition has not been met, return to b, until the termination condition is met and the preset maximum algebra \( G_{max} \) is found.

Through above preliminary understanding and discovery, it finds that the algorithm can be used to solve the optimal solution.

IV. SIMULATION AND TEST

In order to more clearly explain the application of PSO algorithm in the solving of partial differential equation, some specific examples are applied for explanation. The most common population issues are used for calculation.

In population issues, the partial differential equation describing population distribution is shown as follows:

\[
\begin{align*}
    \frac{\partial p(t,x)}{\partial t} + \frac{\partial p(t,x)}{\partial x} &= -d(x)p(t,x) \quad (t \geq 0, 0 \leq x \leq A) \\
    t &= 0: p = p_0(x) \quad (0 \leq x \leq A) \\
    t &= 0: p = p(t,0) = \int_{-\infty}^{0} b(\xi)p(t,\xi)d\xi \quad (t \geq 0)
\end{align*}
\]

In order to get this unique global solution, PSO algorithm is used, and its specific solving process is shown as follows.
Figure 8. PSO algorithm process

Through above calculation process, the unique global solution \( p(t, x) = 0 \) of the above partial differential equation (2) can be obtained by substituting specific values, and \( p(t, A) = 0 \).

Reliability analysis is used to further evaluate the reliability of the above model. As we all know, reliability analysis is to test the reliability of things. In reliability analysis, Cronbach \( \alpha \) reliability coefficient is the most commonly used. Its expression is:

\[
\alpha = \frac{k}{k-1} \times (1 - \frac{\sum S_i^2}{S_T^2})
\]  

(3)

In addition, the evaluation principles of reliability coefficient are listed as follows:

<table>
<thead>
<tr>
<th>Reliability</th>
<th>Cronbach ( \alpha ) Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Credible</td>
<td>Cronbach ( \alpha &lt; 0.3 )</td>
</tr>
<tr>
<td>Almost Credible</td>
<td>( 0.3 \leq \text{Cronbach} \leq 0.4 )</td>
</tr>
<tr>
<td>Credible</td>
<td>( 0.4 \leq \text{Cronbach} \leq 0.5 )</td>
</tr>
<tr>
<td>Very Credible (most common)</td>
<td>( 0.5 \leq \text{Cronbach} \leq 0.7 )</td>
</tr>
<tr>
<td>Very Credible (second most common)</td>
<td>( 0.7 \leq \text{Cronbach} \leq 0.9 )</td>
</tr>
<tr>
<td>Extremely Credible</td>
<td>( 0.9 \leq \text{Cronbach} \leq 1 )</td>
</tr>
</tbody>
</table>

By comparing the size of Cronbach \( \alpha \) coefficient, we can observe the reliability of the system. Conduct evaluation on the above model, and by calculating, obtain: Cronbach \( \alpha = 0.672 \).

Based on reliability analysis criteria, credibility of \( 0.5 \leq \text{Cronbach} \leq 0.7 \) is very credible (most common), so that the model is credible. Therefore, using PSO algorithm to find the optimal solution of partial differential equation is reasonable.

IV. CONCLUSION

Partial differential equation is one of the key means to solve practical problems in physics and some other problems, and is significant for problem solving. This paper takes the application of partial differential equation in population issues as a specific example, applies PSO algorithm to conduct analysis and constructs the application model of partial differential equation based on PSO algorithm.

First, this paper introduces PSO algorithm initially, points out that PSO algorithm is one of the mathematical models to solve the optimal solution, and is widely applied in this kind of problems. In addition, it briefly introduces the application background of partial differential equation and points out the extensive application of partial differential equation.

Secondly, taking the application of partial differential equation in population issues as a specific example, this paper gives the expression of using partial differential equation to solve population distribution, introduces PSO algorithm to calculate the optimal solution and constructs the application model of partial differential equation based on PSO algorithm. In addition, the model is proved credible through reliability analysis.

ACKNOWLEDGMENT

This work was supported by Foundation and cutting-edge technology project of Henan province (No.162102310619, No. 162102310614, No. 142300410464).

REFERENCES


