Research on Reliability Measurement Index of Urban Public Transportation Network Based on the Reliability Measure of Complex Network

Song Minglei, Weng Xiaoxiong*, Liu Yongxin, Guo Juan

School of Civil Engineering and Transportation
South China University of Technology
Guangzhou, Guangdong, 510641, China

Abstract — Objective: To improve the reliability of complex urban public transportation network. Methods: Establishing the reliability evaluation system of urban public transportation network. Process: The paper introduces the development of urban traffic construction, describes the characteristic parameters of complex networks, and establishes the model of the reliability evaluation index system of complex public transportation network. Result & Analysis: This paper studies the complex network theory, analyzes the relevant data of reliability evaluation index system of complex public transportation network, and from the calculation results, it can be seen that the reliability of road network and road service level tends to converge with the increase in the number of traffic flow state. Result: The reliability evaluation system of the complex urban traffic system in this paper is scientific and reasonable.

Keywords - Urban public transportation network; Reliability evaluation; Complex networks

I. INTRODUCTION

With the rapid development of social economy, urban rail transit in many cities in China, has gradually formed a network operation mode. At the same time, urban rail transit has become the priority of urban public transport development, and has gradually become the core of the basic functions of the city, especially the mega cities. With the increasing of urban rail transit in urban public transport, line and site once a failure or attack will have a significant negative impact, therefore, the demand for the reliability of urban rail transit network is also improved. How to use the existing theory and resources to evaluate the reliability of urban rail transit line network, the layout optimization, the optimization of the line network, so as to improve the reliability of urban rail transit, this has become one of the urgent need to study and solve the problem. Railway transportation network reliability research can realize the comprehensive analysis of the strain performance of the rail transit network, it is one of the theoretical basis of constructing urban intelligent transportation system, at the same time, is also an important part of the overall planning and design of the rail transit network, the design of the emergency treatment plan, and the traffic flow organization and management [1]. Fig. (1) (a - b) shows some of the complex traffic networks.

Urban rail transit network is a complex system which is composed of a large number of interacting units. It has the characteristics of organization under certain rules. It
presents dynamic changes and evolution process, and has the openness to the outside world. Urban rail transit network has shown both uncertainty, and there are some characteristics of the internal self-organization principle, can not simply put the urban rail transportation network's question research on the problem of random network or rule network. Urban traffic network system is a complex system. Probability theory is the most important theoretical basis of reliability, the law of large numbers is determined by the reliability test or data analysis, must have sufficient sample size. For urban rail transit system, the research of network reliability has just started, the foundation of the research is very weak, especially in our country, the rail transportation is still in the concentrated period, and the network has not yet formed a certain scale, the actual sample data is scarce, sample data less problem is very prominent. Therefore, how to determine the reliability parameters of the system is an urgent need to solve the problem [2].

II. MATERIALS AND METHODS

A. Computational Complexity

Due to less reliability data, especially in the early stages of the project demonstration and system design, because of the analysis and evaluation of the failure data sample is small, probability models and statistical methods based on large sample data are difficult to apply. At present, it is often used to describe the reliability of the system in the form of expert experience, the description itself is ambiguous, it is difficult to deal with the reliability analysis and evaluation method based on probability theory. The conventional reliability theory is based on the assumption of two state and probability hypothesis. The two state assumption is that the system has only two extreme states: completely normal or completely ineffective. Probability hypothesis is required to meet the definition of the event, a large number of samples, the sample has a probability of repetition, not by human factors affecting 4 conditions [3]. As a system of urban rail transit network, its working state is ambiguous, it is difficult to meet the two state assumption, it is difficult to deal with the reliability analysis and evaluation method based on probability theory. Some systems work in normal or abnormal conditions, and it is difficult to define a clear boundary on the extension, with the concept of ambiguity, such as the part of the line in the network has failed, but the whole network is not completely failed, the system can be degraded to run, etc., with two valued logic and statistical methods, the two state assumption cannot be described accurately, have fuzziness [4].

B. Characteristic Parameters of the Network

Based on the research of complex network topology structure, characteristic parameters of complex networks. The statistical parameters of the commonly used topologies include: Degree and degree distribution, average path length, clustering coefficient and number of referrals, etc., the number of referrals and the number of points and edges. The specific meaning of each characteristic parameter is as follows: Degree and degree distribution: \( k_i \) (Degree) is also known as the degree of connectivity, the number of edges that are directly connected to the node VI [5]. In a directed network, the degree of the node is divided into: degree and degree, the number of edges in the node to other nodes on the grounds.

The number of connected edges of the node to the other nodes is pointed out by the degree. To a certain extent, the degree of node can show the importance of this node, the greater the degree \( k_i \), the more important the node VI, and vice versa. In the transportation network, the node is the traffic hinge station, and the transportation hub station in the entire transportation network also has the status. \( k_i \) can be calculated by adjacency matrix [6], for the degree \( k_i \) of the undirected network node, the formula is as follows:

\[
k_i = \sum_{j \in V} k_{ij}
\]

Among them, \( k_i \) represents the value of the node \( k_i \), \( k_{ij} \) indicates the value of element \( j \) in the \( i \) column of the adjacency matrix. For the calculation of the degree \( k_i \) of the network node is as follows:

\[
k_i^\text{in} = \sum_j a_{ji} \quad k_i^\text{out} = \sum_j a_{ij}
\]

Among them, \( k_i^\text{in} \) is the node degree, \( k_i^\text{in} = \sum_j a_{ji} \); \( k_i^\text{out} \) of the node, \( k_i^\text{out} = \sum_j a_{ij} \). The average value of all the nodes in the network can reflect the connectivity of the whole network, the formula for calculating the average <\( k_i^\text{in} \)> is:

\[
<k> = \frac{1}{n} \sum_{i=1}^{n} k_i
\]

Another characteristic parameter is the degree of distribution of complex networks, it can reflect the type and nature of the network, and it is the important geometrical property of the network. In general, the distribution of \( P(k) \) is used to describe the distribution of degrees, the distribution function \( P(k) \) indicates that the probability of a node whose degree is \( k \) is random. Previous research found that the node degree distribution of random network approximate Poisson distribution, Poisson distribution functions are as follows:

\[
P(k) \approx e^{-ck} \frac{k^k}{k!}
\]

And in a complex network of empirical research on discovery, the distribution of node degree of the real world in the vast majority of the network obviously do not follow Poisson distribution. The \( r \) is a power law exponent, which obeys the power law distribution \( P(k) \propto k^{-r} \). The distribution of the degree is defined as a network of the power law distribution. On the basis of the distribution of degree and degree, the distribution of the degree of accumulation, a parameter describing the network characteristics [7]. Especially when the network size is relatively small, when the data points are less, the
cumulative distribution is more accurate. The \( F(k) \) value of the cumulative degree is not less than the probability of the node of the \( k \):

\[
F(k) = \sum_{k \geq k} P(k) \quad (5)
\]

If the degree distribution is a power law distribution, that is, \( P(k) \propto k^{-\alpha} \), and \( \alpha \in [2,3] \) is usually, then the distribution function of the cumulative degree is the power law distribution of \( \gamma - 1 \):

\[
F(k) \propto \sum_{k \geq k} K^{-\gamma} \propto k^{-(\gamma-1)} \quad (6)
\]

If the distribution is exponential distribution, that is, \( P(k) \propto e^{-\kappa k} \), the \( \kappa \) is a constant, and \( \kappa > 0 \) is the exponential type, and has the same index:

\[
F(k) \propto \sum_{k \geq k} e^{-\kappa k} \propto e^{-\kappa k} \quad (7)
\]

The power distribution in the holding type corresponding logarithmic coordinates on a straight line, the exponential distribution in the semi logarithmic coordinate system corresponds to a straight line, so the power law distribution and exponential distribution can be identified by using the log and semi log coordinates respectively [8].

### C. Complex Network Theory

Complex network theory is a common method to deal with the problems of complex networks that look like each other. Complex network theory based on the statistical characteristics of the actual network structure, to reveal the relationship between the network characteristics and network behavior, and to construct the network model, to help people understand the practical significance and mechanism of these statistical properties, forecast network behavior, and then consider the countermeasures to improve the network behavior. Since 1960s, random graph theory has always been a basic theory to study the structure of complex networks, but the vast majority of the actual network structure is not entirely random. From the end of twentieth Century, due to the rapid development of computer data processing and computing capabilities, scientists have made a large number of empirical research on a large number of real networks, such as power network, computer Internet, food chain network, actor network, scientists cooperation network. Scientists have found, a large number of real networks are neither regular networks nor random networks, but with the same statistical characteristics of the former two are not the same. Some of these networks are called networks complex, which marks the beginning of a new era in complex networks. The study of complex network theory is no longer confined to the field of mathematics, is penetrating into many different fields, such as mathematics, life and engineering disciplines [9]. Scholars begin to pay attention to the overall characteristics of the large number of nodes, the complex structure of the connection structure, the scientific understanding of the complex network quantitative and qualitative characteristics, has become an extremely important and challenging topic in the research of the network times, and even is called "the new science of the network". The relationship between the geometrical properties of the network and the efficiency and stability of the network is studied, it is the core content of complex network research. Study on the reliability of urban rail transit network from the perspective of complex network, is a more accurate and comprehensive assessment of urban rail transit, not only from the perspective of general network reliability research on urban rail transit network, and the special topology of the network is included in the scope of the study, so as to form a more complete conceptual model. The reliability index of the previous (such as punctuality rate, failure rate) more network characteristics [10].

### D. Research on Reliability Evaluation Index of Road Network

As a complex system of urban transportation system, the uncertainty in the system is often difficult to avoid. Uncertainty is the system, the state of the event, the process is uncertain, and the structure and (or) parameters of these systems (or events) have some uncertain factors, have not determined the interference effect in the system, there are many possibilities for the answer to the question. For urban traffic system, it is usually influenced by the two mechanisms, on the one hand, the road users try to achieve the lowest cost through choosing the best path. On the other hand, the impedance of the user is closely related to the supply capacity of the system, and the traffic on the road is high, the essence of these two kinds of mechanisms is the interaction between traffic demand and traffic supply. The cause and the performance of the uncertainty of traffic system are studied by domestic and foreign scholars. The main source of the uncertainty of traffic system is the random fluctuation of traffic demand and random fluctuation of traffic demand, the random fluctuation of the road capacity is easy to form the traffic jam, and the random fluctuation of traffic demand is easy to lead to the traffic congestion, two will lead to the variability of travel time, and then affect the traveler decision-making behavior. Section capacity is also known as road traffic capacity, is the maximum number of vehicles that can pass through a section of the road. Road capacity as a function of road, is the ability to measure the road to ease the vehicle, the actual capacity of the road section is affected by many factors. There are many factors that cause the change of the actual capacity of the road, intersection and road network, and can be divided into the following categories: Permanent influence factor [11]. Such as lane width, lane number, road longitudinal slope, etc., the impact of these factors on the capacity of the road section is basically determined, if there is a change is often due to the establishment of statistical information based on Lu Duanrong the amount of analysis of the model itself is not accurate, therefore, it can be considered as a deterministic effect. Persistent effects (interference) factors. Such as vehicles, non motor vehicles, pedestrian interference, traffic the control scheme, the
adjacent intersection or sections of the interference, driver level, condition and the influence of road pavement condition etc., although these factors have a strong randomness, but in general, in a certain period of time, these random factors have a more durable and stable effect on the capacity. Short term impact (interference) factors. Such as traffic accidents, vehicle broke down, signal failure, short-term traffic management, road maintenance, bad weather or the occurrence of major disasters, such as the interference is not often occur, but this kind of interference occurs once, the impact of the road capacity is often large and has a cascading effect, there may even affect the operation of the entire network. These episodic events are strongly random [12].

E. Small World Network

In practice, a large number of networks are not completely random, nor is it complete rule, but the network is not only show the randomness also show the rules, has a small world effect, such a network is small world network academic, small world network is also known as a network with small world effects, Strogatz and Watts constructed a kind of classical small world network model, which is called WS, which has smaller average path length and large clustering coefficient. The evolution mechanism of WS small world network model is: The initial network is composed of N nodes, and K is an even number; then with probability p randomly to each edge in the network of reconnection, even if a side end of a fixed, randomly select a node in the network to another endpoint for the edge, and provides for the most connected between two nodes at most one edge, any node can not be connected with the node itself. In the construction of WS small world network model, the adjustment of the probability p, can be the rule of the nearest neighbor of the network into a completely random network [13]. The evolution of the WS small world network model is shown in Fig. (2).

When \( p = 0 \), the small world network is the nearest neighbor called the rule, at this time, the average shortest path length and clustering coefficient are larger; When \( p = 1 \), the small world network is a random network, at this time, the average shortest path length and the smaller clustering coefficient are smaller. From the process of the evolution of the small world network model can be seen, the average shortest path length and clustering coefficient of the small world network model are between the average shortest path length and clustering coefficient of the rule network and random network. When the re connection probability \( p \) is gradually increasing from 0 to a very small increment, the clustering coefficient changes slowly, and the length of the characteristic path is rapidly changed. Based on this rule, a small world network model with a smaller characteristic path length and clustering coefficient, the small world network model can be used to describe the characteristics of these real networks [14].

F. Public Transportation Network Connectivity Reliability Evaluation Index System

To evaluate the reliability of its reliability, we need to establish the reliability quantitative index. In the establishment of quantitative indicators, firstly, the factors affecting the reliability of complex networks are clear, including internal factors, external factors, topology and network synchronization, draw lessons from the reality of the evaluation index system has been shown in Fig. (3), Including: global efficiency, average shortest path, the average clustering coefficient, the Dalian through sub graph, from network survivability, effectiveness and survivability and synchronization of four aspects to a comprehensive measure of network reliability [15].
The survivability described in attacking strategy failure under the action of complex network reliability; survival presents the complex network in random failure under the action of the reliability, reflecting the influence of random failure and network topology on the reliability of complex networks; effectiveness is based on the reliability index of the business performance. It is pointed out that the complexity of the complex network in the condition of the failure of the network components to meet the requirements of the business performance, which is an important measure of the efficiency of network utilization, validity refers to the availability of the network to human beings; Synchronization is a complex network with synchronization phenomenon, the research finds that there are beneficial and harmful synchronization in the network, the purpose of the study is to find some nodes or edges in a network, network synchronization and its effect on the reliability of the network [16].

III. RESULTS AND ANALYSIS

Service level reliability calculation example and analysis, the network is shown in Fig. (4), from two OD (Q24, Q14), 4 nodes and 5 sections. The time function of each section of the road is used as the BPR function:

\[ t_a(x_a) = t_a^0 \left[ 1 + 0.15 \left( \frac{x_a}{c_a} \right)^4 \right] \]  

Where, \( t_a^0 \), \( x_a \), \( c_a \), respectively, the free flow time, the traffic volume and the actual capacity of the road section \( a \). The free flow time of each section, the maximum capacity of the section and the length of the section are as table 1. Assuming the average demand of OD to (1 - > 4) is 120pcu/h, the average demand of OD for (2 - > 4) is 60pcu/h, the changes in traffic demand and the corresponding probability of each OD are shown in Table 2; The parameters of SUE assignment model for the \( \theta = 5.0 \).

![Figure 3. Reliability Evaluation Index of Complex Network](image)

![Figure 4. Example Network](image)

<table>
<thead>
<tr>
<th>Section number</th>
<th>Section of the free flow time :min</th>
<th>Section maximum capacity: pcuh(^1)</th>
<th>Section length :km</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>80</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>120</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>120</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OD with(1-&gt;4)</th>
<th>OD with(1-&gt;4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Requirement: pcuh(^1)</td>
<td>Probability</td>
</tr>
<tr>
<td>135</td>
<td>20%</td>
</tr>
<tr>
<td>108</td>
<td>60%</td>
</tr>
<tr>
<td>81</td>
<td>20%</td>
</tr>
</tbody>
</table>

Because each section of the road corresponds to 2 different capacity state, each pair of OD corresponds to 3 different needs state, so the traffic flow state space, a total of 288 state vector, and epsilon =2.0, then the most likely state space there are 83 state vector [17]. Method for reliability analysis of service level established in this section, under the condition of two levels of service level, from the
calculation results we can see that with the increase of the traffic flow state, the reliability of road network and road service level tends to converge.

IV. RESULTS

Along with the development of modern economy, the road of city traffic is getting more and more complicated, therefore, it is necessary to establish the reliability evaluation system of the complex urban public transportation network. On the basis of studying the characteristic parameters of complex network, this paper establishes the evaluation index system of public transportation network connectivity, and through the analysis of the system model and the simulation data, the reliability evaluation system of the complex public transportation network is more scientific and effective.

ACKNOWLEDGMENT

This work is supported by Research on Passenger Trip Distribution of Urban Public Transportation Under a Multi-source Data Environment 2015 Science and technology project of Guangdong Provincial Department of transportation Item number: science and technology - 2015-02-076.

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