Fuzzy Inference for Loss Severity of Operational Risk Quantification

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***Abstract* — To meet the Basel proposal II regulatory requirements for the Advanced Measurement Approaches in operational risk, the statistical methods of estimating operational risk technique have been explored to measure the operational risk losses in financial institutions. A fuzzy inference approach is proposed which the fuzzy parameter of lognormal distribution function is discussed in the distribution of loss severity for operational risk quantification. The distribution of loss severity for operational risk quantification can be described a lognormal distributed by fuzzy parametric statistical inference, in which parameters is characterized as a non-negative fuzzy variable. Prior membership function can be estimated using fuzzy maximum entropy rule, and then a fuzzy simulation method can be designed to estimate posterior mean. The paper shows how fuzzy variables can improve predictive performance. By simulating the operational risk loss of Chinese commercial bank and deriving the regulatory capital allotted for operational risks by China banking industry, the result shows that economical capital for each business line is in accord with the bank's asset.**

Keywords - Operational Risk; Basel II Advanced Measurement Approach; Fuzzy Inference; Loss Distributional Approach; Fuzzy Variable

I. INTRODUCTION

Recently years the operational risk quantification has received increased attention under the new Basel proposal II [1,2].To meet regulatory requirements for the Advanced Measurement Approaches in operational risk, a bank should demonstrate the accuracy of the internal data, relevant external data, and expert opinions.

Some researchers devoted to the statistical methods of estimating operational risk Maryam Pirouz[3]discuss several statistical methods for modelling truncated data, and suggest the best one for modelling truncated loss data, the approach can be useful for increasing accuracy of estimating operational risk capital charge in E-banking. Fengge Yao[4] used Conditional value-at-risk (CVaR) models based on the peak value method of extreme value theory to measure operational risk. Younès, Moutassim[5] used separately a lognormal distribution and a gamma distribution in the mixture models for the zeros losses. Lu[6] explored the large deviation approach for computing the capital charge for operational risk of a bank .in which the negatively-associated structure is utilized to measure the dependence between distinct operational loss cells. the lower and upper bounds of the tail distribution function of total aggregated loss processes are determined. an operational risk assessment model of distribution network equipment based on rough set and D-S evidence theory was built[7]. Ahmed Barakat[8] investigates the direct and joint effects of bank governance, regulation, and supervision on the quality of risk reporting in the banking industry. Pjotrs Dorogovsa[9] discussed new tendencies of management and control of operational risk in financial institutions. Loss Distributional Approach (LDA) is to fit frequency distributions over a predetermined time horizon, typically annual. The financial institutions use a wide variety of frequency and severity distributions for their operational risk data, including exponential, weibull, lognormal, generalized Pareto, and g-and-h distributions [10].

Many researchers devoted to assessing operational risks using Bayesian inference [11, 12], There are potentially many deferent alternatives for the choice of severity and frequency distributions [13-15]. Salahi[16] presented a new approach to compute the capital charge for an E-bank to cover the losses of operational risk, based on Loss Distribution Approach (LDA), which refers to statistical methods for modelling the loss distribution. In this framework, we begin our model with performing the descriptive statistic analysis of internal loss data at bank, and finish it using Value-at-Risk measure, to obtain the capital charge of an E-bank for operational risk. We have tested our model for some operational loss data samples, and have estimated operational risk capital charge. A fuzzy rule-based system can model prior probabilities in Bayesian inference and thereby approximate posterior probabilities. This fuzzy technique allows users to express prior descriptions in words [16].

This fuzzy estimation by expert's judgment may provide valuable information for forecasting and decision making, especially for risk cells lacking internal loss data. Therefore, it is important to apply fuzzy theory [17-20] into the model. In this paper we proposed to use the fuzzy inference method [21] for the quantification of the frequency and severity distributions of operational risks. The method is based on specifying the prior distributions for the parameters of the severity distributions using expert opinions or industry data. Then, the prior distributions are weighted with the actual observations in the bank to estimate the posterior distributions of the model parameters.

The remainder of this paper is organized as follows. Section 2 presents a measuring model based on fuzzy estimation for quantitative operational risk, in which this framework is used to quantify distributions for severity of operational losses. Section 3 gave fuzzy estimation of loss severity parameter with operational loss data. Section 4 presents fuzzy simulation method is designed to estimate the posterior mean.

II. FUZZY REFERENCE MODELS FOR THE SEVERITY OF OPERATIONAL LOSSES

For operational loss severity be an log-normal distributed $LN(\tilde{\upsilon},\tilde{\sigma}^2)$ with $(\tilde{\upsilon},\tilde{\sigma}^2)$ characterized as a non-negative fuzzy variable, where $\tilde{\upsilon}$ is shape parameter, $\tilde{\sigma}^2$ scalar parameter and have the following form of probability density function,

$$\pi(x) = \frac{1}{x\sqrt{2\pi\times\tilde{\sigma}^2}}\exp\left\{-\frac{1}{2\times\tilde{\sigma}^2}(lnx-\tilde{\upsilon})^2\right\}, 0\le x\le h \tag{1}$$

$\tilde{\upsilon}$ and $\tilde{\sigma}^2$ is the non-negative fuzzy variable on the credibility space $(\Theta, \mathrm{P}(\Theta), Cr)$ with membership function $\mu_{\tilde{\upsilon}}$ and $\mu_{\tilde{\sigma}^2}$, respectively. Assumed that $\tilde{\upsilon}$ and $\tilde{\sigma}^2$ be independent fuzzy variable, called *prior membership functions*, the parameters of prior membership functions which denoted by $\varphi_1,\varphi_2\ldots\varphi_k$ are refereed as hyper-parameters. The parameters of the prior membership function $\varphi_1,\varphi_2\ldots\varphi_k$ are estimated by fuzzy maximum entropy rule and expert opinions. Then S can be considered as random fuzzy variable on the space $(\Theta, \mathrm{P}(\Theta), Cr)\times(\Omega, \mathrm{A}, \Pr)$. Let x denote the sample $x_1, x_2\ldots x_k$, which can be observed and take crisp values, then the posterior membership function of $\tilde{\upsilon}$ and $\tilde{\sigma}^2$ can be deduced as

$$\mu\left(\tilde{\sigma}^2=\sigma^2\,\middle|\,x\right) = (2\sup_{\sigma^2} Ch\left(\tilde{\upsilon}=\upsilon,\tilde{\sigma}=\sigma^2\,\middle|\,x\right))\wedge 1$$

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Where

$$\sup_{\sigma^2} Ch\left(\tilde{\upsilon}=\upsilon,\tilde{\sigma}=\sigma^2\,\middle|\,x\right) = \begin{cases} \left(\dfrac{\prod_{i=1}^{n}\frac{1}{x_i\sqrt{2\pi\sigma^2}}\exp\{-\frac{1}{2\sigma^2}(lnx_i-\upsilon)^2\}}{\sup_{\upsilon,\sigma^2}\prod_{i=1}^{n}\frac{1}{x_i\sqrt{2\pi\sigma^2}}\exp\{-\frac{1}{2\sigma^2}(lnx_i-\upsilon)^2\}}\right)\wedge 1 \\ if\ \dfrac{\prod_{i=1}^{n}\frac{1}{x_i\sqrt{2\pi\sigma^2}}\exp\{-\frac{1}{2\sigma^2}(lnx_i-\upsilon)^2\}}{\sup_{\upsilon,\sigma^2}\prod_{i=1}^{n}\frac{1}{x_i\sqrt{2\pi\sigma^2}}\exp\{-\frac{1}{2\sigma^2}(lnx_i-\upsilon)^2\}} < 0.5 \\ 0 \quad otherwise \end{cases} \tag{2}$$

III. THE FUZZY SEVERITY MODEL WITH OPERATIONAL LOSS DATA

The sample from historical operational risk loss data is $x=(0.01,...,5789)$, the sample size is 4659. Experts estimate that the loss severity of corporate finance $\tilde{\upsilon}$ and $\tilde{\sigma}^2$ range is $-1\sim 6$ and $2\sim 13$, respectively. Let $\tilde{\upsilon}$ be trapezoidal fuzzy variable $(-1,a_1,b_1,6)$, $\tilde{\sigma}^2$ trapezoidal fuzzy variable $(2,a_2,b_2,13)$. Then the prior membership of $\tilde{\upsilon}$ is

$$\mu_{\tilde{\upsilon}}(\upsilon) = \begin{cases} \dfrac{\upsilon+1}{a_1+1}, & if\ -1\le\upsilon\le a_1 \\ 1, & if\ a_1\le\upsilon\le b_1 \\ \dfrac{\upsilon-6}{b_1-6}, & if\ b_1\le\upsilon\le 6 \\ 0, & otherwise \end{cases} \tag{3}$$

The expectation of $\tilde{\upsilon}$ is

$$E[\tilde{\upsilon}] = \frac{a_1+b_1+15}{4} \tag{4}$$

The entropy of is

$$H[\tilde{\upsilon}] = (ln2-0.5)(b_1-a_1)+3.5$$

The prior membership of

$$\tilde{\sigma}\ \mu_{\tilde{\sigma}}(\sigma^2)=\begin{cases}\dfrac{\sigma^2-2}{a_2-2} & if \quad 2\le\sigma^2\le a_2\\ 1 & if \quad a^2\le\sigma^2\le b_2\\ \dfrac{\sigma^2-13}{b_2-13} & if \quad b_2\le\sigma^2\le 13\\ 0 & otherwise\end{cases} \tag{5}$$

where M is sufficiently larger number.

The expected value of $\tilde{\sigma}$

$$E[\tilde{\sigma}]=\frac{a_2+b_2+11}{4} \tag{6}$$

The entropy of $\tilde{\sigma}$ is

$$H[\tilde{\sigma}]=(ln2-0.5)(b_2-a_2)+5.5. \tag{7}$$

By applying the graphic method, then the value of a_1, b_1, a_2, b_2 can be obtained:

$$a_1^*=-1, b_1^*=2, a_2^*=4, b_2^*=13,$$

It follows from equation (18), the hyper parameters can be estimated by the following models

$$\max_{a_1,b_1}(ln2-0.5)(b_1-a_1)+3.5-M\left|\frac{a_1+b_1+15}{4}-4\right| \tag{8}$$

$$s.t. \quad -1\le a_1\le b_1\le 6,$$

where M is sufficiently larger number.

By applying the graphic method, then the value of a_1, b_1, a_2, b_2 can be obtained:

$$\max_{a_2,b2}(ln2-0.5)(b_2-a_2)+5.5-M\left|\frac{a_2+b_2+11}{4}-7\right| \tag{9}$$

$$s.t. \quad 2\le a_2\le b_2\le 13,$$

By applying the graphic method, then the value of a_1, b_1, a_2, b_2 can be obtained:

$$a_1^*=-1, b_1^*=2, a_2^*=4, b_2^*=13,$$

That is $\tilde{\upsilon}$ is a trapezoidal fuzzy variable $(-1,-1,2,6)$ and $\tilde{\sigma}$ is a trapezoidal fuzzy variable $(2,4,13,13)$, Once the prior distribution parameters $\tilde{\upsilon}$ are $\tilde{\sigma}$ estimated, then, the posterior distribution of the posterior membership of $\tilde{\upsilon}$ and $\tilde{\sigma}$ are

$$\mu\left(\tilde{\upsilon}=\upsilon\left|x\right.\right)$$

$$=(2\sup_{\sigma^2}Ch\left(\tilde{\upsilon}=\upsilon,\tilde{\sigma}=\sigma^2\left|x\right.\right))\wedge 1$$

$$=\begin{cases}\left(\dfrac{2\sup\limits_{\sigma^2}\prod\limits_{i=1}^{4659}\dfrac{1}{x_i\sqrt{2\pi\sigma^2}}\exp\{-\dfrac{1}{2\sigma^2}(lnx_i-\upsilon)^2\}}{\sup\limits_{u,\sigma^2}\prod\limits_{i=1}^{4659}\dfrac{1}{x_i\sqrt{2\pi\sigma^2}}\exp\{-\dfrac{1}{2\sigma^2}(lnx_i-\upsilon)^2\}}\right)\wedge 1 & if\ -1\le\upsilon\le 6\\ 0 & otherwise\end{cases}$$

$$=\begin{cases}\left(\dfrac{2\prod\limits_{i=1}^{4659}\exp\{-\dfrac{1}{2\sigma^{*2}}(lnx_i-\upsilon)^2\}}{\prod\limits_{i=1}^{4659}\exp\{-\dfrac{1}{2\sigma^{*2}}(lnx_i-\upsilon^*)^2\}}\right)\wedge 1 & if\ -1\le\upsilon\le 6\\ 0 & otherwise\end{cases} \tag{10}$$

$$\mu(\tilde{\sigma}=\sigma^2|x)$$

$$=(2\sup_{\sigma^2}Ch\left(\tilde{\upsilon}=\upsilon,\tilde{\sigma}=\sigma^2\left|x\right.\right))\wedge 1)$$

$$=\begin{cases}\left(\dfrac{2\sup\limits_{\upsilon}\prod\limits_{i=1}^{4659}\dfrac{1}{x_i\sqrt{2\pi\sigma^2}}\exp\{-\dfrac{1}{2\sigma^2}(lnx_i-\upsilon)^2\}}{\sup\limits_{\upsilon,\sigma^2}\prod\limits_{i=1}^{4659}\dfrac{1}{x_i\sqrt{2\pi\sigma^2}}\exp\{-\dfrac{1}{2\sigma^2}(lnx_i-\upsilon)^2\}}\right)\wedge 1 & if\ 2\le\sigma^2\le 13\\ 0 & otherwise\end{cases}$$

$$=\begin{cases}\left(\dfrac{2\prod\limits_{i=1}^{4659}\exp\{-\dfrac{1}{2\sigma^2}(lnx_i-\upsilon^*)^2\}}{\prod\limits_{i=1}^{4659}\exp\{-\dfrac{1}{2\sigma^{*2}}(lnx_i-\upsilon^*)^2\}}\right)\wedge 1 & if\ 2\le\sigma^2\le 13\\ 0 & otherwise\end{cases} \tag{11}$$

$$\upsilon^*=4.26 \quad \sigma^{*2}=4.37$$

The posterior mean $E[\upsilon]$ of the υ is exploited as fuzzy simulations technique.

IV. FUZZY SIMULATION FOR POSTERIOR MEAN $E[\tilde{\sigma}]$

For simplicity, a fuzzy simulation will be designed to estimate $E[\tilde{\sigma}]$ by the following procedure.

1) Set $e=0$.

2) Randomly generate $\tilde{\sigma}(\theta_k)$ from the ε-level set of u and write $\nu_k=\mu_{\tilde{\sigma}}(\sigma|x)$ for $k=1,2,\cdots,N$, where μ the membership of function of $\tilde{\sigma}$.

3) Set $a = \tilde{\sigma}(\theta_1) \wedge \cdots \wedge \tilde{\sigma}(\theta_N)$, $b = \tilde{\sigma}(\theta_1) \vee \cdots \vee \tilde{\sigma}(\theta_N)$

4) Uniformly generate r from [a,b]. Set $e = e + Cr\{\tilde{\sigma} \geq r\}$,

where $\mathrm{Cr}\{\tilde{\sigma} \in \beta\} = \frac{1}{2}(1 + \sup_{x\in\beta} \mu_{\tilde{\sigma}}(\sigma|x) - \sup_{x\in\beta^c} \mu_{\tilde{\sigma}}(\sigma|x))$.

5) Repeat the fourth step for N times.

6) Compute $E[\tilde{\sigma}] = a \vee 0 + b \wedge 0 + e \cdot (b-a)/N$, then output $E[\tilde{\sigma}]$.

By fuzzy simulation technique, we can get fuzzy point estimation of, then the density function of severity in operational risk losses

$$\pi(x) = \frac{1}{x\sqrt{2\pi\times 4.16}} \exp\left\{-\frac{1}{2\times 4.16}(\ln x - 4.52)^2\right\}, 0 \leq x \leq 6000 \quad (12)$$

V. RESULT ANALYSIS AND DISCUSSION

In this section the results are presented, followed by discussion and analysis of results. Empirical analyses for operational risk accurately grasp the characteristics of a data bank support. First, This study is based on a sample extracted from the commercial banks operational risk loss data, The risk events are collected from a variety of sources including: the commercial banks operational risk internal loss data relevant external data, and scenario analysis. For each product line, the loss of a sample and the parameters of the membership function by using fuzzy inference is given, and then taking posterior mean as a fuzzy point estimate, the intensity loss distribution can be obtained. The values of fuzzy inference for several types of business lines are showed in table I. After obtaining the expected loss severity distribution and using loss data, the annual loss probability distribution of business lines can be depicted by simulations technique. In most cases, there is not enough reliable data to apply the statistical approaches that are commonly used for assessing market risks.

TABLE I THE VALUES OF FUZZY INFERENCE FOR COMMERCIAL BANK SEVERAL TYPES BUSINESS LINES

business lines	υ	σ
Corporate Finance & Commercial Banking	5.15	5.14
Retail Banking & Retail Brokerage	2.12	9.13
Payment and Settlement	2.16	8.12
Agent Service & Asset Management	3.13	5.15
Trading and Sales	5.14	6.11

The annual loss probability distribution of business lines can be described .For the purposes of the regulatory capital calculations of operational risk, the annual loss distribution, in particular its 0.999 quintile (VaR) as a risk measure, should be quantified for each Basel II risk cell in business lines and for the whole bank. As usual, assume independence between business lines. Table II presents loss mean for capital estimates at the 99.9% level for business lines and for the whole bank. This table presents the business lines that were used in this study. The test results show proposed method in assessing risks of commercial banks operating is effective, fuzzy inference risk model Bank provides an efficient operational risk warning tools.

TABLE II SEVERAL TYPES BUSINESS LINES THE LOSS MEAN WITH FUZZY PARAMETERS OF BUSINESS LINE

business lines	loss mean
Corporate Finance & Commercial Banking	415.21
Retail Banking & Retail Brokerage	7.1723
Payment and Settlement	15.163
Agent Service & Asset Management	1.2461
Trading and Sales	1.23457

VI. CONCLUSION

This paper firstly analyzed the method assess operational risk. The main conclusions of this paper are as follows:

(1)This article describes fuzzy inference techniques can be used to assess operational risk loss.The fuzzy inference model accommodated a combination of banks collective losses data and expert opinion better is suited for practitioners in banks and financial institutions.

(2) The proposed measuring method allows using the banks collective losses data and expert opinions to improve the correctness of estimates value. Fuzzy reference models works well within an operational risks management framework by estimating parameter in the distribution of loss severity for operational risk. Financially, we simulate the operational risk loss of Chinese commercial bank and derive the regulatory capital allotted for operational risks by China banking industry.

(3)It offers a promising alternative for measuring operational risks when a few data are available. We hope that the presented method provides an attractive and feasible approach in which to realize these models. Because of China's commercial banks operational risk effectively history data is less, the proposed method in operational risk assessment has broad application prospects in the small sample situation.

(4) Further research based on model is necessary for other risk loss estimation, such as insurance risk estimation, enterprise risk estimates.

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