A Deferred Payment Strategy for Risk-Averse Supply Chain based on CVaR

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Abstract — The offer of a delayed period for payment from suppliers may help retailers to order more but also improve the total supply chain performance. In this paper, we adopt Conditional Value-at-Risk, CVaR, as the performance criterion to examine how supply chain performance is affected through the deferred payment contract. Analytical results are yielded for the newsvendor retailer’s optimal order quantity and supplier’s optimal wholesale price, based on which sensitivity analysis is conducted. We show that by use of this deferred payment strategy, both the supplier and retailer can be better off.

Keywords-risk aversion; deferred payment; conditional value-at-risk; supply chain

I. INTRODUCTION

In traditional inventory models, it is implicitly assumed that the retailer must pay for the purchased items as soon as these items are delivered and the accounts receivable proportion out of credit sales is marginal [17][18]. However, for some firms such as start-ups, due to limited available capitals, a speedy payment may be impossible. Therefore, the delayed payment is a very important financing mode in the development of small and medium-sized enterprises. In many practical situations, the supplier is willing to provide the customer with a permissible delay period when the customer obtains enough capital. Delayed payment has grown up to be a common business practice. For example, the Boeing 787 Dreamliner program offers a so-called “risk-sharing” contract, akin to the delayed payment. Under this contract, Boeing’s strategic suppliers will not get paid until Boeing delivers the airplanes to its customers. A major hotel operator in Latin America that operates several chains has a standard Days Delayed Payables Outstanding (DDPO) policy. They allow 90 days delay to their suppliers to pay receivables. This will enable their suppliers to obtain cash after delivery in a predictable and automated manner, thereby strengthening relationships with their suppliers. According to CGI white paper [16], lack of information for an accurate prediction of when shipments will arrive makes it difficult for sellers to obtain timely payments. Delayed payment contract currently represents 85 percent of global trade transactions.

In recent years, delayed payment has been studied widely from using the single-item inventory model under permissible delay in payments to the trade credit model. Scholars have utilized established delay payment models to analyze the supply chain decisions and performance. Existing literature often assume that retailers or the newsvendors are risk neutral. In contrast, in this paper we will examine how the delay payment impacts the risk-averse newsvendor’s decision by introducing Conditional Value-at-Risk (CVaR) to assess the risk-averse newsvendor’s performance under delayed payment. Several questions are interesting to us: how the retailer and supplier determine the optimal ordering and pricing strategies to maximize their expected profit? How the deferred payment contract reduces the newsvendor’s risk and encourages them to order more? What are the implications of such contract for supply chain efficiency and the profitability of each party in the chain?

To answer these questions, in this paper, we consider a newsvendor model in which a risk-averse retailer sells a seasonal product with random demand and places an ordering quantity and a risk-neutral supplier who decides on the wholesale price with the objective of maximizing Conditional Value-at-Risk (CVaR). CVaR is a special mean-risk criterion representing a trade-off between the expected profit and a certain risk measure. Compared with other risk metrics such as value at risk, it is easier to be quantified, because the only subjective parameter for CVaR is the confidence level \( \theta \).

The paper is organized as follows. In section 2, we review the related literature. In section 3, we introduce our model in detail. Section 4 presents the CVaR model. Section 5 presents the retailer’s optimal decisions and supplier’s wholesale price. Section 6 is the parameter analyses. In section 7, we conclude.

II. LITERATURE REVIEW

We briefly survey the most relevant literature in this section. Most existing literature about delay payment often analyzes the retailer’s optimal order quantity under economic order quantity (EOQ). Goyal [1] was the first to establish an EOQ model with a constant demand rate under the condition of permissible delay in payments. Khouja and Mehrrez [2] discuss the trade credit contracts on the EOQ model and the credit terms are related to the order quantity. Many other researches release the assumption of the Goyal’s model in order to derive a realistic ordering strategy.

To examine the impact of delay payment contract on the integrated inventory models, Abad and Jaggi [19] provided a seller–buyer integrated inventory model; Jaber and Osman [20] proposed a supplier–retailer supply chain model. Yang and Wee [21] developed a vendor–buyer integrated inventory model. The difference of these three work lies in trade credit policies and lot-for-lot shipment strategies. In the supplier–retailer model, the permissible delay in payments is considered as a decision variable. The vendor–buyer model is for deteriorating items with permissible delay in payment.
In addition, Ho et al. [22] investigated the production and ordering policy under a two-part trade credit in an integrated supplier–buyer inventory model.

In addition to focusing on determining optimal policy for the retailer or the supplier only, some scholars consider how to relieve the conflict between the buyer and the vendor and attempt to create a win–win strategy under delayed payment. Goyal [1] developed a single-vendor single-buyer integrated inventory model and illustrated that the inventory cost can be reduced significantly if the vendor’s economic production quantity is integer times of the buyer’s purchase quantity.

In most business transactions, the vendor usually offers a delayed credit period to the buyer, which then passes on this credit period to the end customers. Huang [11] modified this assumption to assume that the retailer will adopt the trade credit policy to stimulate his/her customers’ demand to develop the retailer’s replenishment model. It is a two-level trade credit model. Mahata [24] investigated the EOQ-based inventory model under two levels of trade credit to reflect the supply chain management situation in a fuzzy condition. Factors such as inventory cost and ordering cost are ignored.

All above work assume that the supply chain system is risk-neutral. In practical situation, retailer’s risk preferences are usually reflected in delayed-payment contracts. In this paper, we consider both the delay payment and risk preferences, and assume that the retailer’s initial capital is insufficient.

We now review the literature on newsvendor models using CVaR that is also relevant to our work. CVaR is a financial risk measure and widely applied in the newsvendor problem recently. Rockafellar and Uryasev described the properties and application of CVaR in detail. For connection and difference between CVaR and VaR, we refer to Kibzun and Kuznetsova [28].

The literature of CVaR applied to the risk-averse newsvendor problem is as follows. Wu and Zhu [30] discussed quantity competition and price competition under CVaR criterion. In a risk-averse newsvendor model they considered two demand segmentation rules and discussed the trade-off between the retailer’s CVaR measure and his expected profit.

Ahmed [25] analyzed an extended model of the classical multi-period, single-item, linear cost inventory problem where the objective function is a coherent risk measure. Choi and Ruszczynski [7] derived an equivalent representation of a risk-averse newsvendor problem as a mean-risk model and proved that the higher the weight of the risk, the smaller the order quantity. In this paper, the risk-averse retailer we discuss takes CVaR as his performance measure. Given a delay payment contract by the supplier, the retailer chooses an order quantity to maximize his own performance measure.

III. Model Description

In this section, we first introduce the $\eta$–CVaR (conditional value-at-risk) criterion and then present the model. The notations are summarized in Table 1.

<table>
<thead>
<tr>
<th>$p$ : Retail price</th>
<th>$q$ : The retailer’s order quantity</th>
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<tr>
<td>$W$ : The supplier’s wholesale price</td>
<td>$c$ : The supplier’s unit cost</td>
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<tr>
<td>$\xi$ : Market demand</td>
<td>$\eta$ : The degree of risk aversion</td>
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<tr>
<td>$B$ : The retailer’s initial capital</td>
<td>$\Pi$ : Retailer’s expected profit</td>
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<td>$\Pi_S$ : Supplier’s expected profit</td>
<td>$\Pi_I$ : Integrated supply chain expected profit</td>
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<td>$f(\xi)$ : The probability density function of the demand distribution</td>
<td>$\Phi(\xi)$ : The cumulative distribution function of the demand distribution</td>
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<td>$V$ : The value at risk for a given $\eta$</td>
<td>$\eta$ : Critical fractile of $Q$</td>
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<td>$r_f$ : Risk-free interest rate</td>
<td>$r_s$ : Supplier interest rate</td>
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In this subsection, we describe the settings for the business environment, the downstream demand, the timeline of the game and the decision-making process.

The business environment and downstream demand: we consider a risk neutral supplier that sells a product through a single selling season. The supplier has an opportunity to produce the product before the selling season. Each unit production cost is $c$ and the wholesale price is $W$. There are no other costs for the retailer to sell the product. Let $q$ be the retailer’s order quantity, $B$ is the retailer’s initial wealth.

The retail price equals $p$, which is fixed regardless of the terms of trade. We assume $p \geq w \geq c$.

The uncertain demand is denoted by a random variable $\xi$ with support $[0, +\infty)$. The probability density function (PDF) of $\xi$ is $f(\xi)$, cumulative distribution function (CDF) is $F(\xi)$, $\Phi(\xi) = 1 - F(\xi)$.

Let $h(\xi) = \frac{f(\xi)}{F(\xi)}$ be the failure rate, $H(\xi) = \frac{\Phi(\xi)}{F(\xi)}$ be the generalized failure rate which is assumed to be increasing. Many distributions, e.g., the uniform, normal, exponential and gamma distribution are increasing generalized failure rate.

The time line of the model: We formulate the interaction between the retailer and the supplier in the framework as a Stackberg game. In the first stage at time $t = 0$, the supplier offers deferred payment contract to the retailer. The retailer accepts the contract and decides on an order quantity, and then pays the supplier the initial capital $B$ (the retailer is
capital constraint, $B < wQ$. For simplicity, we assume that retailer’s operating cost is zero. At the end-of-the-sales season (to be specific, at time $t = T > 0$), the retailer repays his deferred payment $wQ(1 + r) - B(1 + r)$ to the supplier if he earns conveniently. Conversely we assume a zero salvage value for any excess inventory and zero goodwill costs for lost sales.

IV. THE CVaR MODEL

In the decision-making process for risk-averse policymakers, $\eta$ - CVaR criterion measures the average value of the profit falling below a certain critical value. $\eta$ ( $\eta \in (0,1]$ ) reflects the degree of risk aversion of the decision-maker [4]. The smaller $\eta$ is, the higher the degree of risk aversion is. The definition of $\eta$ - CVaR is as follows.

$$CVaR_\eta(\pi(x,y)) = \mathbb{E}[\pi(x,y)|\pi(x,y) \leq q_\eta(y)] = \frac{1}{\eta} \int_{\pi(x,y) \leq q_\eta(y)} \pi(x,y) g(y) dy$$

where $E$ is the expectation operator throughout this paper, $\pi(w,q)$ is the revenue function. $g(q)$ is the probability density function of $q$. $\eta$ denotes the $\eta$ critical fractile of $q$ expressed as

$$g_q(Q) = \sup \{\alpha | \Pr\{\pi(w,Q) \leq \alpha\} \leq \eta\}$$

where $V$ is the value at risk for a given $\eta$, and denotes the upper limit of the supply chain’s profits. An alternative definition of $\eta$ - CVaR is

$$CVaR_\eta[g(a,b)] = \min_{\alpha} \{\alpha + (1 - \eta)^{-1} \mathbb{E}[g(a,b) - \alpha] \}$$

Under deferred payment contract, the profit of the retailer, the supplier and the whole system are as follows respectively.

$$\Pi_q(q,D) = p \min(q,D) + B(1 + r) - wq(1 + r)$$

$$\Pi_q(w,r) = wq(1 + r) - c$$

$$\Pi_q(q,D) = p \min(q,D) - cq$$

According to the (3) and (4), then the risk-averse and capital-constrained retailer’s CVaR model is given by (7).

$$CVaR_\eta[-\Pi_q(w,r)] = \min_{\alpha} \{\alpha_{\Pi_q}, (1 - \eta)^{-1} \mathbb{E}[\Pi_q(w,r) - \alpha_{\Pi_q}] \}$$

$$= \min_{\alpha} \{\alpha_{\Pi_q}, (1 - \eta)^{-1} \int_{-\infty}^{\Pi_q} [-p(x) + wq(1 + r)] - B(1 + r) - \alpha_{\Pi_q} f(x) dx + (1 - \eta)^{-1} \int_{\Pi_q}^{\infty} [-p(x) + wq(1 + r)] - B(1 + r) - \alpha_{\Pi_q} f(x) dx \}$$

Let

$$g_{\eta}(q,\alpha_{\Pi_q}) = \alpha_{\Pi_q} + (1 - \eta)^{-1} \int_{-\infty}^{\Pi_q} [-p(x) + wq(1 + r)] - B(1 + r) - \alpha_{\Pi_q} f(x) dx + (1 - \eta)^{-1} \int_{\Pi_q}^{\infty} [-p(x) + wq(1 + r)] - B(1 + r) - \alpha_{\Pi_q} f(x) dx$$

That is

$$CVaR_\eta[-\Pi_q(w,r)] = \min_{\alpha} g_{\eta}(q,\alpha_{\Pi_q})$$

If $\alpha_{\Pi_q} = -pq + wq(1 + r) - B(1 + r)$, we have

$$\Pi_{\Pi_q}(w,r) = wq(1 + r) - B(1 + r)$$

$$\Pi_{\Pi_q}(q,D) = B(1 + r) - pq wq(1 + r)$$

We take the first order derivative of the above equation (10) and obtain

$$\frac{dg_{\eta}(q,\alpha_{\Pi_q})}{d\alpha_{\Pi_q}} = 1 - (1 - \eta)^{-3} \leq 0$$

If $\alpha_{\Pi_q} > -pq + wq(1 + r) - B(1 + r)$ or $\alpha_{\Pi_q} \leq wq(1 + r) - B(1 + r)$, then

$$g_{\eta}(q,\alpha_{\Pi_q}) = \frac{wq(1 + r) - B(1 + r) - \alpha_{\Pi_q}}{p}$$

We have

$$g_{\eta}(q,\alpha_{\Pi_q}) = \alpha_{\Pi_q} + (1 - \eta)^{-1} \int_{-\infty}^{\Pi_q} [-p(x) + wq(1 + r)] - B(1 + r) - \alpha_{\Pi_q} f(x) dx$$

We take the first order derivative of (11) and obtain

$$\frac{dg_{\eta}(q,\alpha_{\Pi_q})}{d\alpha_{\Pi_q}} = 1 - (1 - \eta)^{-3} F(q,\Pi_q)$$

If $\alpha_{\Pi_q} > wq(1 + r) - B(1 + r)$, then

$$g_{\eta}(q,\alpha_{\Pi_q}) = \alpha_{\Pi_q} + (1 - \eta)^{-1} \int_{-\infty}^{\Pi_q} [-p(x) + wq(1 + r)] - B(1 + r) - \alpha_{\Pi_q} f(x) dx$$

$$\alpha_{\Pi_q} = wq(1 + r) + B(1 + r) - pF^{-1}(1 - \eta)$$

$$CVaR_{\eta_{\Pi_q}}[-\Pi_{\Pi_q}(w,r)] = \Pi_{\Pi_q}(q) = wq(1 + r) - B(1 + r)$$

$$\Pi_{\Pi_q}(q) = \Pi_{\Pi_q}(w,r) - pF^{-1}(1 - \eta)$$

The same procedure may be easily adapted to obtain CVaR model for supplier. Thus the supplier’s CVaR model is

$$CVaR_{\eta_{\Pi_q}}[-\Pi_{\Pi_q}(w,r)] = \Pi_{\Pi_q}(q) - \Pi_{\Pi_q}(w,r)$$

According to the CVaR subadditivity, we can obtain the supply chain’s CVaR model as follows.

$$CVaR_{\eta_{\Pi_q}}[-\Pi_{\Pi_q}(w,r)] + CVaR_{\eta_{\Pi_q}}[-\Pi_{\Pi_q}(w,r)]$$

$$= pq - B(1 + r) - pF^{-1}(1 - \eta)$$

$$\Pi_{\Pi_q}(w,r)$$

$$\Pi_{\Pi_q}(q)$$

$$\Pi_{\Pi_q}(q) - \Pi_{\Pi_q}(w,r)$$

$$p(1 - \eta)^{-1} \int_{\Pi_q}^{\infty} F(x) dx$$

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V. Optimal Ordering Quantity and Wholesale Price

In the supply chain consisting of a risk-neutral supplier and a risk-averse retailer with \( \eta - \text{CVaR} \) criterion, for any given confidence level \( \eta \in (0, 1] \), the retailer’s optimal order quantity \( q^* \) satisfies

\[
q^* = F^{-1}(\frac{\eta}{p - (1 + r)B + (1 - \eta)(p - w)})
\]  

(16)

Under the same deferred payment contract, for any two retailers \( i \) and \( j \), if retailer \( i \) is more risk averse than retailer \( j \), i.e., \( \eta_i > \eta_j \), then the optimal order quantity for retailer \( i \) is less than that for retailer \( j \). In the special case \( \eta_i = 1 \), retailer \( j \) is risk neutral, and hence orders more quantity than his risk-averse counterpart \( i \) under the CVaR criterion [4]. The following corollary holds.

We take the first order derivative of (15) and obtain the supply chain’s optimal order quantity.

\[
q^* = F^{-1}(\frac{p - c}{(1 - \eta)(p - w)})
\]  

(17)

In order to achieve the perfect coordination of supply chain, we let the retailer's optimal order quantity equal to the optimal quantity of the supply chain's. That is \( q^* = q^* \). The optimal wholesale price is obtained.

\[
w = \frac{(p - c)(1 - \eta) + B(1 + r) - (1 - \eta)(p - w)}{q(1 + r)}
\]  

(18)

Therefore, the supplier can select arbitrary \( w \) and \( r \), which are satisfied (18) to achieve the perfect coordination of the supply chain.

Next we use an example to prove the superiority of deferred payment contract. The market demand follows a Normal distribution with mean \( \mu = 10 \) and standard deviation \( \sigma = 5 \). We set the market price \( p = 300 \), the unit cost \( c = 60 \), and the initial capital \( B = 500 \).

Note: Curve a is retailer’s CVaR with deferred payment contract; Curve b is retailer’s profit with deferred payment contract; Curve c is retailer’s CVaR without deferred payment contract; Curve d is retailer’s profit without deferred payment contract.

Figure 1 depicts the retailer’s CVaR performance with deferred payment contract or without deferred payment contract. From Figure 1 we can see that when the retailer is risk-averse with \( \eta - \text{CVaR} \) criterion, if the deferred payment contract is accepted, then the retailer’s optimal order quantity \( q^* \) is greater than the retailer’s ordering without the deferred payment contract \( \hat{q} \). Furthermore, both retailer’s profit and CVaR performance with deferred payment contract are higher than the one without it. This suggests that deferred payment contract indeed improves retailer’s performance.

VI. Parameter Analyses

The degree of risk aversion \( \eta (\eta \in (0,1]) \) reflects the degree of risk aversion of the decision-maker [4]. The smaller \( \eta \) is, the higher the degree of risk aversion is. In our model we have the following Corollary.

Corollary 1. The retailer’s optimal order quantity is increasing in the risk aversion factor \( \eta \). The wholesale price is decreasing in the risk aversion factor \( \eta \).

Now we consider a simple example, where a market demand follows a Normal distribution with mean \( \mu = 10 \) and standard deviation \( \sigma = 5 \). The retail price \( p = 300 \) and the unit cost \( c = 60 \). The retailer’s initial capital \( B = 0 \). The optimal order quantity is determined by Proposition 1, we depict the ordering curves and wholesale price curves in Figure 2 and 3.
Figure 2 shows the calculation result of the optimal order quantity for different levels of risk aversion $\eta_i$. This example verifies Corollary 1. For the case of $\eta_i \to 0$, the retailer is extremely risk-averse so that he does not order at all. Figure 3 shows the relationship between the wholesale price and risk aversion parameter $\eta_i$. When the value of $\eta_i$ increases, the retailer becomes more risk-neutral and orders more. Then the wholesale price decreases in $\eta_i$.

The initial capital $B$

Next we discuss the influence of the retailer’s initial capital $B$ to the retailer’s ordering strategy. A risk-averse retailer with less initial capital is highly in need of deferred payment. Thus, the deferred payment contract is especially useful to small enterprises or those with insufficient capital.

Corollary 2. When both a deferred payment contract and $\eta_i$-CVaR criterion are employed, the risk-averse retailer’s optimal ordering quantity $q^*$ increases in retailer’s initial capital $B$, and finally constant for large $B$.

According to Corollary 2, increase in $B$ relaxes the boundary on retailer’s cash position. When the retailer has a small initial wealth, he orders less and more inclined to accept deferred contract. When the retailer has a large initial wealth, he will not go bankruptcy, so $B$ has no effect on the optimal order quantity decision. On the other hand, the implementation of the deferred payment contract suggests that the repayment risk will occur. For a risk-averse retailer, when he has more initial capital, the probability of taking deferred payment contract will reduce. When retailer has enough initial wealth that he cannot go bankruptcy, his initial wealth has no effect on the optimal order quantity, suggesting that $q^*$ keeps constant. We continue to use the mathematical examples above to analyze Corollary 2 further. Figure 4 shows the relationship between the initial capital $B$ and the optimal ordering $q^*$, the retailer’s final profit $\Pi_r$, the wholesale price $W$ and the supplier’s profit. When the retailer has such little initial wealth he must accept deferred payment contract. His optimal ordering quantity increases with small initial capital. With the growth of $q^*$, the supplier sets the wholesale price to make his profit from a smaller order quantity with no bankruptcy risk dominates that from a larger order quantity with bankruptcy risk from the retailer. Thus the wholesale price $W$ decreases in $B$. The retailer and the supplier’s final profits continue to increase in $B$.

Figure 4. (a) the optimal order quantity $q^*$, (b) the retailer’s profit $\Pi_r$, (c) the wholesale price $W$, (d) the supplier’s profit $\Pi_s$. Supplier’s interest rate $r_s$. 
Figure 5 shows retailer’s ordering response according to different $r_s$ compared with integrated supply chain’s optimal ordering quantity. Observe in Figure 5, when $r_s \leq r'_s$, the supplier makes the retailer get $q=q_e$. That means the supplier can fully coordinate the decentralized supply chain by manipulating wholesale price $w$ and delay rate $r'$. From Figure 5 we can see as $r_s$ decreases, the wholesale price and order quantity increase, i.e. $w$, $\Pi$, and $q$ decrease in $r_s$, but $Z$ increases in $r_s$. The risk-neutral retailer is willing to bear more risk due to his limited liability. In order to encourage the retailer to take risks by lowering the delay rates, the supplier can charge a larger wholesale price that would have resulted with the same retailer’s order quantity.

![Figure 5. Retailer’s order quantity with different $r_s$.](image)

### VII. CONCLUSIONS

In this paper, we have investigated the behavior of the risk-averse retailer and the risk-neutral supplier on quality investment and wholesale price decision in a supply chain with uncertain demand. The supplier offers deferred payment contract to the retailer. On the basis of the preference theory, we use $\eta$—CVaR criterion to determine the optimal ordering of a retailer and optimal wholesale price of a supplier. We analyze how the risk preferences impact the retailer’s decision strategy and the relationship between the optimal order quantity and the wholesale price. Finally, we have discussed how the deferred payment contract improve the supply chain efficiency by adjusting $\eta$. We show that the retailer’s ordering strategy is highly correlated to initial wealth $B$ and risk parameter $\eta$. Moreover, the smaller $\eta$ is, the better the deferred payment contract performs. Further work includes investigation of the effect of using comprehensive performance measure such as mean-CVaR on supply chain coordination and performance management.

### REFERENCES

QIANQIAN CHEN: A DEFERRED PAYMENT STRATEGY FOR RISK-AVERSE SUPPLY CHAIN BASED ON CVAR


