Simulation and Multifractal Analysis by Stochastic Ising Dynamic Systems for Commodity Returns

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Abstract — A logarithmic returns process based on Ising model and mean field approximation is considered in this article. By adjusting parameters to get the simulated returns of the dynamic systems with Matlab and Monte Carlo simulation, the commodity market has the similar statistical properties, such as fat-tail behavior. By comparing the MultiFractal De-trended Fluctuation Analysis (MFDFA) results for original series with those for shuffled series, a conclusion can be made that the multi-fractality of the returns is both due to a broad probability density function and long-range correlations.

Keywords - Ising dynamic systems; Monte Carlo simulation; commodity returns; multi-fractal behavior

I. INTRODUCTION

Ising model is an important model in statistical physics, is a kind of equilibrium statistical mechanics [1-3]. It can depict phase interaction between particles and field environment on the influence between particles in paper of M. Bar-tolozzia and a. w. Thomas [4], considering the interaction between the agent business form the financial model cluster, according to the special environment of the commodity market, the Ising model is applied and the characteristics of the operator in the commodity market has the practical significance of the Ising model, makes the formation mechanism of commodity fluctuations from interpretation. The constructed Ising model can also depict the yield sequence distribution of the body exists the interpretation. The constructed Ising model can also depict the yield sequence distribution of the body exists the interpretation. The constructed Ising model can also depict the yield sequence distribution of the body exists the interpretation. The constructed Ising model can also depict the yield sequence distribution of the body exists the interpretation.

II. CONSTRUCT CONFORMS TO THE ISING MODEL OF COMMODITY MARKET RETURNS

The multitude of theories developed to understand.

The $Z^2$ is a square lattice, the elements $i$ in the lattice is a pair of integers $(x_i, y_i)$. Subset of any limited symmetry $A = Z^2$, make $A$ said to $\Omega_A = \{-1, 1\}^A$ on the spin configuration, and the element of $\Omega_A$ can be expressed as $\sigma_i = \{\sigma_i, i \in A\}$, in the case of don't be confused, it can be omitted in some subscript $A$. Has the following the Ising model of Hamiltonian system, for any $\sigma \in \Omega_A$:

$$H_{A, \beta}(\sigma) = -\frac{1}{2|A|} \sum_{i \in A} J_{ij} \sigma_i \sigma_j - h \sum_{i \in A} \sigma_i$$

In the formula, $h$ is a real number, means the environment on the magnetic field, $J_{ij}$ means the strength of the lattice see internal interaction. Under the inverse temperature $\beta$, $\mu_{A, \beta}(\sigma)$ is the probability measure on $\Omega_A$:

$$\mu_{A, \beta}(\sigma) = \frac{1}{Z_{A, \beta}} \exp[-\beta H_{A, \beta}(\sigma)]$$

In the formula, $Z_{A, \beta} = \sum_{\sigma \in \Omega_A} \exp[-\beta H_{A, \beta}(\sigma)]$.

The Ising model is considered [4-5], assume that the market investors stood a point in the lattice. At each time point, investors decided to buy $\sigma_i = +1$ or sell $\sigma_i = -1$ their commodity, every investor decision depends on the local environment in time $t_i(t)$. The formula is:

$$I_i(tans) = \frac{1}{|A|} \sum_{\sigma \in \Omega_A} \exp[-2H_{A, \beta}(\sigma)]$$

Investors change their decision according to the changes in the environment. In every moment $t$, investors can change their investment at a certain probability, and the probability of buying probability is $p$, selling is $1 - p$.

$$p = \frac{1}{1 + \exp[-2I_i(t)\beta]}$$

The first of the $I_i(t)$, $\frac{1}{|A|} \sum_{\sigma \in \Omega_A} \exp[-2H_{A, \beta}(\sigma)]$ represents the attitude of other investors in investment for investors at time $t$ carry out impact investment decisions. Due to the interaction between investors, $I_i(t)\beta$ is changing with time, has the following form $I_i(t) = a\zeta(t) + b\eta(t)$, $\zeta(t)$ reflects the average effect of the market for investors $i$ influence, and $b\eta(t)$ reflects the influence of all traders of different investors $i$ decisions.

The second of the $I_i(t)$, $h_i(t) = c_{i=1} \sum_{\sigma \in \Omega_A} \alpha_i + d_i B(t)$, said
that the investment environment, $q \sum_{j=1}^{N} \alpha x_j$ represents a period of time on the investment's total return to investors $i$, $B(t)$ is a standard Brownian motion, showing random access to external market information, $q \sum_{j=1}^{N} \alpha x_j$ reflects the impact of random information for investment decisions investor $i$'s different.

In the case of discrete time price process $S(t)$ is [6]

$$\ln S(t+1) = \ln S(t) + R(t)$$ (4)

And commodity price gains on the difference between supply and demand is proportional to commodity buyers and sellers, therefore

$$c \sum_{j=1}^{N} \alpha x_j = c \left[ \ln S(t) - \ln S(0) \right]$$ (5)

In the formula, $S(t)$ is the commodity price at time $t$.

Set the initial price for the $S(0)$, and $N \times N$ investors in the market, according to equation (3) and the above discussion, $i$ investor's investment strategy function $I(t)$ can be written as follows:

$$I_i(t) = a \xi(t) x(t-1) + \frac{1}{N^2} \sum_{j=1}^{N^2} b \eta_j(t) \sigma_j(t-1) + c \ln \left[ \frac{S(t)}{S(0)} \right] + dB(t)$$ (6)

among them, $\xi(t)=\xi(t-1)$ allows investors to generate random amplification reaction in the price trend. In addition, when $N$ is large, since the $\eta_j(t)$ uniformly distributed on the $(-1,1)$ interval. Therefore, according to [7],

$$\frac{1}{N^2} \sum_{j=1}^{N^2} b \eta_j(t) \sigma_j(t-1) \rightarrow 0$$ , and when $N$ is large enough, in

$$c_1 = c, c_2 = d, i=1, ..., N^2$$ of the case, use Ising model mean-field approximation theory, magnetization can be introduced $x(0)$ satisfies the mean-field equation [7]

$$x(t) = \tanh \left[ a \xi(t) x(t-1) + c \ln \left[ \frac{S(t)}{S(0)} \right] + dB(t) \right]$$ (7)

So the available commodity established by the Ising model random logarithmic model commodity returns for the income process is

$$R(t) = \ln S(t+1) - \ln S(t)$$

$$= a \xi(t) x(t-1) + c \ln \left[ \frac{S(t)}{S(0)} \right] + dB(t)$$ (8)

III. STATISTICAL ANALYSIS OF THE MODEL

A. Statistical Analysis

Using the Ising model to study the commodity price yields is a new attempt to study commodity price fluctuations [2-5]. Based on the CCPI from the first week of 2010 to the last week of 2014 to yield the number of mean and variance, the formula (8) is adjusted to the parameters, order $a=1.5$, $c=0.002$, $d=0.01$, such that the mean and variance of the number on the card yields approaching. With the Monte Carlo simulation method analog (8) logarithmic returns, a group of yield is got as shown in figure 1(a). By figure 1, we can see that the the logarithmic rate of return (equation (7)) has obvious fluctuation of cluster and the nature of sustainability, that volatility often persistent phenomenon of high or low in a certain period of time. In general, if current market volatility is small, the volatility of the next stage will be small, after the commodity market a big fluctuation, vibration often lasts for a very long time. Figure 1(b) is logarithmic rate of return normality test chart. It can be seen by examining, in the middle of the most places, it is subject to fluctuations in the distribution of normality. But at the end, the apparent deviation from the normal distribution, the distribution tails are thicker than normal.

(a) return series

(b) normal inspection figure

Figure 1. Time series of logarithmic price returns and normal probability plot of $R(t)$ with $a=1.5$, $c=0.0002$, $d=0.01$

B. Multifractal Analysis

Detrended fluctuation analysis (DFA) is a test whether the time series with long memory method, multifractal detrended fluctuation analysis (MFDFA) not only long memory test sequence, It can also test hidden in the time series of multi-fractal characteristics. In this section, the Ising model [7] (1) is derived in this paper (8) defined 2.1 using Monte Carlo simulation method yields simulated sequence $R(t)$, $t=1,2, ..., N$. The steps of multifractal analysis method [8-10].

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Consider yields sequence \( \{ R(t), t = 1, 2, ..., N \} \), where \( N \) denotes the length of the sequence.

\[
y_0(t) = \sum_{i=1}^{N} [R(t) - \langle R \rangle], \quad t = 1, 2, ..., N,
\]

Among them, \( \langle R \rangle \) represents the average of the whole time series. The \( y_0(t) \) is divided into non-overlapping \( N_s = \text{int}(N/s) \) segments, each of length \( s \). For each section of \( m = 1, 2, ..., N_s \) data using least squares fitting function \( y_m(t) \), then to give the function of \( y_m(t) \)

\[
y_m(t) = y_1[(m-1)s+i], \quad m = 1, 2, ..., N_s
\]

\[
y_m(t) = y_N^{-1}N-(m-N_s)s+i] - y_1(i), \quad m = N_s+1, N_s+2, ..., 2N_s
\]

In this section, three function which is fitting \( y_m(t) \) expressed as MFDFA-3. \( y_m(t) \) the variance is represented as \( F_s^q(m,s) = \frac{1}{s} \sum_{i=1}^{s} (F^q(m,s))^2 \), to get the wave function of order \( q \)

\[
F_q(s) = \left[ \frac{1}{2N_s} \sum_{m=1}^{N_s} \left( F_s^q(m,s) \right)^{q/2} \right]^{1/2}
\]

Among them \( q \) can take any real number. For a fixed order \( q \), \( s \) depending on the value calculated \( F_q(s) \) of the value, and draw out \( F_q(s) \) with \( s \) changes to the base 10 of the double-log plot, as 2(a). Analysis of fluctuation scale which simulated by the formula (8) to a number yields. If the time series \( R(t) \) is related to the long, \( F_q(s) \) increases with increasing \( s \), and has a power-law characteristics.

By the method of comparison of the original time sequence and shuffled time series, based on the yield of long memory sequence is determined by the correlation between sequences, or sequences exist between certain function caused. Thus the analog time series of random numbers yield shuffled 10 times, 10 times that of the average of seeking to get shuffled sequence \( \{ R_{shuf}(t), \quad t = 1, 2, ..., N \} \), The same with Matlab to draw \( s \) base 10 double log plot in Figure 2(b). If the correlation is caused by the probability density, With the changes of \( s \) the \( F_q(s) \) and have the \( F_{shuf}(s) \) same trend, it can expressed by \( F_{corr}(s) \) (Figure 2(c)) ratio between them

\[
F_{corr}(s) = F_{shuf}(s) \times \frac{1}{F_{corr}^{1/2}} = F_{corr}^{1/2} = g_{corr}(s)
\]

If only have the distribution of multi-fractal, \( h(q) = h_{shuf}(q) \) and \( h_{corr}(q) = 0. \) If only have the correlation multi-fractal, the \( h_{shuf}(q) = 0.5 \) and \( h(q) = 0.5 + h_{corr}(q) \). If \( h(q) \) and \( h_{corr}(q) \) are dependent on the \( q \), The presence of density and relevance multi-fractal. The results are shown in Table I.

Figure 2. Title Multi-fractal behavior of simulated time series on equation (8) with \( a=1.5 \), \( c=0.0002 \), \( d=0.01 \)
According to the analysis with Figure 2 and Table 1, it can be learned, With the construction of Ising logarithm returns model Simulated yields Both multi-fractal density, but also relevant multi-fractal, The multi-fractal nature of the process is the introduction of local-scale features diversity, in order to describe price volatility over time inhomogeneity [7]. The larger of $q$, Highlighting the role of earnings volatility, The smaller of $q$, Highlighting the role of small fluctuations in earnings. The Figure 2(a) instructions for different $q$ have different scaling exponent, Fluctuation in different ranges have different scaling relations, Explanation. Ising model than the ARCH or GARCH model is more suitable in describing the have the income of scale change. It can be seen from Figure 2(d), when $q$ take a smaller positive number, $h(g) > 0.5$. Effect of earnings minor fluctuations are amplified. At this time showing Return series in a state of continuous side, Highlighting the role of internal factors in the market, when $q$ is less than 0.5 start, At this earnings volatility plays a major role, showing Return series in a state of anti-continuous side, The impact of External regulation is more prominent, Therefore, the sustainability of large fluctuations and sustainability of small fluctuations are not the same. The multi-fractal spectrum estimation formula $f(g) = qL - h(g) + 1$, among them $L = h(g) + qL(q)$, the analog of logarithm rate of return multi-fractal spectrum is estimated as shown in Figure 2(e). From the multi-fractal spectrum, Local Hölder index vary over a wide range, and the multi-fractal spectrum was maximum when $L = 0.25$. So the analog of a logarithmic rate of return is a multi-fractal process.

IV. Conclusions

Using the Ising model structure commodity returns process, and by the method of simulation Monte Carlo simulation yields the logarithm of time series. Analysis of the Simulate the yield time series, the time sequence about rate of return and actual commodity market showed the same conclusion. Flaring, volatility persistence, etc. By analyzing the multi-fractal, a model obtained yields have the characteristics of multi-fractal which can describe the sequence yields the existence of different scaling relation, in a word, the yield process model which is built in this article is reasonable which has some innovative research on commodity price volatility.

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