An Algorithm of Attribute Reduction Based on Granular Computing in Manufacturing Grid System

Ying Dong¹, Dongbo Li¹*, Yifei Tong¹

School of Mechanical Engineering
Nanjing University of Science and Technology
Nanjing, Jiangsu, China

Abstract — There are a large number of dynamic, heterogeneous, and even redundant mercurial conflicting information in the manufacturing grid environment. It is of great importance for grid users to discover manufacturing grid resources. As for resource discovery, the attributes recorded in the manufacturing system are the key. To address this aim, attribute reduction based on granular computing is studied. The nuclear of the reduction information system is calculated. Based on the nuclear, the reduced attributes which are important are obtained. A case is presented to illustrate the application of the proposed method.

Keywords - granular computing; attribute reduction; knowledge granularity; manufacturing grid

I. INTRODUCTION

When facing increasingly fierce global competition, the huge investments in high-tech manufacturing resources become an issue that must be considered for each enterprise [1]. Manufacturing grid system was proposed for coordinated resource sharing and collaboration in dynamic, multi-institutional virtual organizations to solve the problems [2]. There are a large number of dynamic, heterogeneous, and even redundant mercurial conflicting information in the manufacturing grid environment. The root cause of the manufacturing grid system is complicated. As the basis of forecast and decision making, the unascertained information needs knowledge discovery and access. Knowledge reduction is the core of knowledge discovery and access [3, 4], which is proved a NP - Hard problem. In recent years, many scholars have made a lot of research results about attribute reduction [5,6]. However, unascertained information is not fully considered.

Granular computing is a powerful tool, during 1996 and 1997, the concept of which was first proposed by L.A.Zadeh [7,8]. The essence of granular computing is divide and conquer, based on relationships between grain and grain to analyze and process the unascertained information.

This article will achieve attribute reduction based on granular computing. Firstly, calculate the system’s reduction nuclear, and then obtain the reduced attributes.

II. RELATED KNOWLEDGE

The basic concepts and properties of knowledge granularity are introduced briefly as follows [7,8].

Definition 1: \( U \) is a non-null limited universe of discourse, \( R \) is an equivalence relation on \( U \). \( K = \langle U, R \rangle \) is a knowledge base, \( u_R \) which is generated by \( R \) is basic knowledge granularity. \( U/R = \{ u_R | u \in U \} \) is the grain division of \( U \).

Definition2: \( P \in R \), \( P \) is an equivalence relation on \( R \), undistinguishable relation \( Ind(P) = \bigcap_{P \in R} x_p \) which is generated by \( P \) indicate the highest recognition that \( P \) can achieve. \( Ind(R) \) indicate the highest recognition that \( K = \langle U, R \rangle \) can achieve.

Definition3: \( K = \langle U, R \rangle \) is a knowledge base, knowledge \( P \in R \) is an equivalence relation on \( U \). The granularity of \( P \in R \) is denoted as \( GK(P) \).

\[
GK(P) = \frac{|P|}{|U \times U|} = \frac{|P|}{|U|^2}
\]  

(1)

Where \(|P|\) is cardinal number of \( P \subseteq U \times U \).

\( GK(P) \) can denote the resolving capacity of \( P \). \( \forall u, v \in U \), if \( (u, v) \in P \), then objects \( u \) and \( v \) are not indistinguishable under \( P \), they are the same equivalence relation on \( P \). In general, the larger \( GK(P) \) is, the weaker the resolving capacity of \( P \); otherwise the smaller \( GK(P) \) is, the stronger the resolving capacity of \( P \).
Theorem 1: \( P \in R \) is the knowledge in knowledge base \( K = \{U, R\} \), then \( U/R = \{X_1, X_2, \cdots X_n\} \), then
\[
GK(P) = \left( \sum_{i=1}^{n} |X_i|^2 \right) / |U|^2
\]

It can be proved as follows:
\[
\forall(u,v) \in P, \exists u,v \in X_i, P = \sum_{i=1}^{n} |X_i|^2
\]

\[
GK(P) = \frac{|P|}{|U|^2} = \frac{\sum_{i=1}^{n} |X_i|^2}{|U|^2}
\]

Character 1: \( S = (U, A, V, f) \) is a information system, \( P, Q \subseteq A \).

Then (1) if \( P \Rightarrow Q \), then \( GK(P) \leq GK(Q) \);

(2) if \( P \Leftrightarrow Q \), then \( GK(P) = GK(Q) \).

It can be proved as follows:

(1) From \( P \Rightarrow Q \), \( Ind(P) \subseteq Ind(Q) \) can be got then
\[
GK(P) = \frac{|Ind(P)|}{|U|^2} \leq \frac{|Ind(Q)|}{|U|^2} = GK(Q)
\]

(2) From \( P \Leftrightarrow Q \), \( P \Rightarrow Q \) and \( Q \Rightarrow P \), from (1) \( GK(P) \leq GK(Q) \), and \( GK(Q) \leq GK(P) \) so \( GK(P) = GK(Q) \).

III. ATTRIBUTE REDUCTION BASED ON KNOWLEDGE GRANULARITY

A. Significance of attributes

\( S = (U, A, V, f) \) is a information system, where \( A \) is the attribute set, \( a \in A \). The significance of \( a \) can be analyzed by knowledge granularity. If there is a great change of the knowledge granularity by removing attribute \( a \) from \( A \), then \( a \) is important for \( A \).

Definition 4: In a information system \( S = (U, A, V, f) \), the significance of attribute \( a \) in the attribute set \( A \) is indicated as \( \text{Sig}_{A \setminus \{a\}}(a) \).

\[
\text{Sig}_{A \setminus \{a\}}(a) = GK(A \setminus \{a\}) - GK(A) \quad (3)
\]

Particularly, when \( A = \{a\} \), \( \text{Sig}_{\{a\}}(a) \) is denoted by \( \text{Sig}(a) \).

\[
\text{Sig}(a) = \text{Sig}_{\{a\}}(a) = GK(\emptyset) - GK(A) = 1 - GK(\{a\})
\]

where \( GK(\emptyset) = 1 \).

Character 2: \( 0 \leq \text{Sig}(a) \leq 1 / |U| \).

Definition 5: \( S = (U, A, V, f) \) is a information system, where \( A \) is the attribute set, \( B \subseteq A \), \( \forall a \in A - B \), the significance of attribute \( a \) in the attribute set \( B \) is indicated as \( \text{Sig}_B(a) \).

\[
\text{Sig}_B(a) = GK(B) - GK(B \cup \{a\}) \quad (4)
\]

From definition 5, the significance of attribute \( a \) in the attribute set \( B \) can be measured by the change of knowledge granularity caused by adding \( a \) to \( B \). The larger the change is, the more important \( a \) is to \( B \).

Definition 6: \( S = (U, A, V, f) \) is a information system, \( a \in A \). If \( GK(A \setminus \{a\}) = GK(A) \), then \( a \) is unnecessary in \( A \); otherwise \( a \) is necessary in \( A \). If each attribute \( a \in A \) is necessary in \( A \), then attribute set \( A \) is independent; otherwise \( A \) is dependent.

Definition 7: \( P \subseteq A \), if \( P \) is dependent and \( GK(P) = GK(A) \), then \( P \) is a reduction of \( A \). All the reductions of \( A \) are denoted as \( \text{red}(A) \). The set composed of all the necessary attributes of \( A \) is denoted as \( \text{Core}(A) \).

\[\text{Core}(A) = \bigcup \{a \in A | \text{Sig}_{A \setminus \{a\}}(a) > 0 \}\]

Character 3: If and only if \( \text{Sig}_{A \setminus \{a\}}(a) > 0 \), attribute \( a \) is necessary in \( A \).

Character 4: \( \text{Core}(A) = \bigcup \{a \in A | \text{Sig}_{A \setminus \{a\}}(a) > 0 \} \)

B. Arithmetic of attribute reduction based on knowledge granularity

Part A provides a method of attribute reduction from the perspective of knowledge granularity. The method is as follows: By calculating \( GK(A \setminus \{a\}) \) and \( GK(A) \), they are equal or not to determine whether the attribute \( a \) is removable or not. If \( GK(A \setminus \{a\}) = GK(A) \), then \( a \) is removable; otherwise \( a \) is irremovable. The set composed of all the irremovable attributes of the attribute set is the \( \text{Core}(A) \). Next \( \text{Sig}_{\text{Core}(A)}(a) \) standing for the significance of the left attributes is calculated. Finally, the \( \text{red}(A) \) is composed of \( \max_{a \in \text{Core}(A)} \text{Sig}_{\text{Core}(A)}(a) \) and \( \text{Core}(A) \).

Form above, the complete algorithm is shown as follows:
Input: information system $S = (U, A, V, f)$;
object set $U = \{x_1, x_2, \ldots, x_n\}$;
attribute set $A = \{a_1, a_2, \ldots, a_m\}$.
Output: $\text{red}(A)$ and $\text{Core}(A)$

Step 1: Calculate $\text{GK}(A)$;
Step 2: For each $a \in A$, calculate $\text{Sig}_{d-[a]}(a)$;
Step 3: Calculate $\text{Core}(A) = \{a \in A | \text{Sig}_{d-[a]}(a) > 0\}$;
Step 4: If $\text{GK}(\text{Core}(A)) = \text{GK}(A)$, then return $\text{red}(A) = \text{Core}(A)$.
If $\text{GK}(\text{Core}(A)) > \text{GK}(A)$, then to next;
Step 5: $B = A - \text{Core}(A)$, calculate $\text{Sig}_{\text{Core}(A)}(b)$;
$max_{b \in B} \text{Sig}_{\text{Core}(A)}(b)$;
Step 6: If $\text{GK}(\text{Core}(A) \cup b) = \text{GK}(A)$, then return $\text{red}(A) = \text{Core}(A) \cup b$ and $\text{Core}(A)$.
If $\text{GK}(\text{Core}(A) \cup b) > \text{GK}(A)$, then to Step 5;
Step 7: $C = A - \text{Core}(A) \cup b$, calculate $\text{Sig}_{\text{Core}(A),b}(c)$;
$max_{c \in C} \text{Sig}_{\text{Core}(A),b}(c)$; Repeat step 6 until $\text{GK}(\text{Core}(A) \cup b \cup c \cup \cdots) = \text{GK}(A)$ or each attribute is calculated.
Step 8: Return $\text{red}(A) = \text{Core}(A) \cup b \cup c \cup \cdots$ and $\text{Core}(A)$.

C. Improved arithmetic of attribute reduction and application in manufacturing grid system

1) Improved algorithm

The improved algorithm is shown as follows:
Input: information system $S = (U, A, V, f)$;
object set $U = \{x_1, x_2, \ldots, x_n\}$;
attribute set $A = \{a_1, a_2, \ldots, a_m\}$.
Output: $\text{red}(A)$ and $\text{Core}(A)$

Step 1: Calculate $\text{GK}(A)$;
Step 2: For each $a \in A$, calculate $\text{Sig}_{d-[a]}(a)$;
Step 3: For $1 \leq i \neq j \leq m$
Calculate $\text{Sig}_{d-[a_i,a_j]}(a_i,a_j)$;
Step 4: Calculate $\text{Sig}_{d-[a_i,a_j]}(a_i,a_j)$;
Then return $\text{red}(A) = \{\text{Sig}_{d-[a_i,a_j]}(a_i,a_j)\}$.
Step 5: Calculate $\text{Core}(A) = \{a \in A | \text{Sig}_{d-[a]}(a) > 0\}$;
Step 6: If $\text{GK}(\text{Core}(A)) = \text{GK}(A)$, then return $\text{red}(A) = \text{Core}(A)$.
If $\text{GK}(\text{Core}(A)) > \text{GK}(A)$, then to next;
Step 7: $B = A - \text{Core}(A)$, calculate $\text{Sig}_{\text{Core}(A)}(b)$;
$max_{b \in B} \text{Sig}_{\text{Core}(A)}(b)$;
Step 8: If $\text{GK}(\text{Core}(A) \cup b) = \text{GK}(A)$, then return $\text{red}(A) = \text{Core}(A) \cup b$ and $\text{Core}(A)$.
If $\text{GK}(\text{Core}(A) \cup b) > \text{GK}(A)$, then to Step 5;
Step 9: $C = A - \text{Core}(A) \cup b$, calculate $\text{Sig}_{\text{Core}(A),b}(c)$;
$max_{c \in C} \text{Sig}_{\text{Core}(A),b}(c)$; Repeat step 8 until $\text{GK}(\text{Core}(A) \cup b \cup c \cup \cdots) = \text{GK}(A)$ or each attribute is calculated.
Step 10: Return $\text{red}(A) = \text{Core}(A) \cup b \cup c \cup \cdots$ and $\text{Core}(A)$.

2) Application

A given information system $S = (U, A, V, f)$ is shown in Table 1.

The question is to get $\text{red}(A)$ and $\text{Core}(A)$.

<table>
<thead>
<tr>
<th>$U$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
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Answer:
1. Calculate \( GK(A) \)
\[
U/A = \{ x_1, x_2, x_3, x_4, x_5 \}
\]
\[\begin{align*}
GK(A) &= \left( \sum_{i=1}^{5} |X_i| \right) / |U| = 5/25 = 1/5 ; \\
\end{align*}\]
2. For each \( a \in A \), calculate \( \text{Sig}_{A-a}(a) \)
\[
U/A - \{ a \} = \{ x_1, x_2, x_3, x_4, x_5 \}
\]
\[\begin{align*}
GK(A - \{ a \}) &= \left( \sum_{i=1}^{5} |X_i| \right) / |U| = 5/25 = 1/5 , \\
\text{Sig}_{A-a}(a) &= GK(A - \{ a \}) - GK(A) = 0 ; \\
U/A - \{ a \} &= \{ x_1, x_2, x_3, x_4, x_5 \}
\end{align*}\]
3. \( \text{Sig}_{A-\{a,b\}}(a,b) \)

IV. CONCLUSION

In this paper, a new reduction algorithm is presented. The algorithm is able to handle attributes reduction without reduction of kernel. Granular Computing provides a new technical support for solving the problem of index reduction, whose application value still needs to be further studied and developed. For those cases not about nuclear situation, the above algorithm is powerless. In total, attributes reduction of information system is not only the results. Some may have more than a reduction.

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