

Artistic Works Classification Model Based on Sparse Coding Algorithms

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Abstract — This paper performs feature analysis and image understanding on an image principle function and sparse coefficients of different artistic styles, acquired by sparse coding algorithms. Then the obtained features are used to study classification implementation of different artistic works. We adopt absolute value of kurtosis as sparse measuring standard of parameter component ensuring the sparsity and independency of coefficients. Simultaneously, fixed parameter component variance items are introduced to target function, to ensure the balance between minimal image reconstruction error and sparse penalty function. We also adopt Gabor wavelet to initialize Sparse Coding (SC) feature principle function to search optimal basis vectors, which further raises convergence speed of SC network. The experiments verify the sparse coding algorithm maximized by kurtosis which can be used to classify styles of artistic works.

Keywords - sparse coding; principle function; classification; artistic work; kurtosis.

I. INTRODUCTION

Image classification is one of research focused on computer vision field, including image retrieval and human-computer interaction. The main modes of image classification research are extracting image characteristics and it adopts machine learning mode to classify image eigenvector. Its early research mainly adopts overall characteristics to express image and implements classifier to classify eigenvector [1,2]. Global characteristics contain image color and texture and accuracy rate is low to apply it to image classification. In order to improve accuracy rate of image classification, sparse coding theory is introduced in the study of image classification field. Fieldy is the first scholar to propose sparse distributed coding method [3]. This coding method reduces neural cell numbers responding any special input information. Sparse coding of signal exists in kurtosis. Olshausen and Field points that [4] the basis function which obtains natural image after sparse coding approximates to receptive field reaction characteristics of simple cell in U1 area. The basis function which is extracted from sparse encoding models successfully simulates three response characteristics of simple cell receptive field in VI area for the first time. They are locality in space domain, directivity and selectivity in time domain and frequency domain. Recently, foreign researchers also put forward many new sparse coding algorithms [5-7]. For instance, Joshua Tenenbaum and William's bi-linear sparse coding model, Ol proposes wavelet pyramid structure is used to learn students' image code, Hyvarien and Hoyer propose double sparse code model.

Sparse code algorithm is very suitable to describing non-Gaussian distribution signal. However, in practical application, it is very few for signals to follow Gaussian distribution. Thus, sparse code algorithm has broad application prospect. Basis function and sparse code which are obtained through sparse code algorithm have similar

characteristics with humans' visual cortex receptive field cell. It can be further used to perform style classification of visual art works.

Based on fully understanding sparse code algorithm theory, visual art works in different styles are encoded to obtain different characteristics basis and sparse coefficient. Through studying these characteristic basis functions and sparse coefficient, image understanding of visual art works in different styles is realized and its feasibility for style classification is further analyzed. According to visual art works in different styles after sparse code algorithm, extracted characteristic basis function and sparse coefficient display large distance between classes and small inner-class distance. Fourth-order statistics and image data are used for high order statistics, we depend on mathematical statistics to realize style recognition and style classification by specific experiments.

II. PRELIMINARY WORKS

A. Mathematical Description of Sparse Coding

The key of sparse coding is using few basis vector to effectively and succinctly describe the input vectors. Assuming input vector is $x \in R^m$, and sparse vector is $\alpha \in R^n$ ($m < n$), the relation between them is $x \approx D\alpha$. $D \in R^{m \times n}$ is sparse changing matrix and its row vectors is basis vectors $b_i \in R^m$ ($1 \leq i \leq n$). D is over-completed and it contains modes to express x . So α has infinite group of solutions.

It is important for sparse coding to train D , to make α as sparse as possible. Main features of x can be expressed by basis vectors b_i , which eliminates noise in x . During training process of D , we adopt training vector set $X = [x_1, x_2, \dots, x_k]$ (k is the number of training vectors) to optimize price function $f_k(D)$:

$$f_k(D) = \frac{1}{k} \sum_{i=1}^k l(x_i, D) \quad (1)$$

$$\begin{cases} f_k(D) = \frac{1}{k} \sum_{i=1}^k \frac{1}{2} \|x_i - D\alpha\|_2^2 + \lambda \|\alpha\|_1 \\ \text{s.t. } \|b_i\|_2^2 \leq 1, \quad \forall j = 1, 2, \dots, n \end{cases} \quad (2)$$

In this equation, λ is regularization parameter and b_i is element of basis vectors.

According to equation 3, $f_k(D)$ can be expressed by federal optimized problem of D and sparsing vector set $A = [\alpha_1, \alpha_2, \dots, \alpha_k]$.

$$\begin{cases} f_k(D) = \min_{\alpha_i \in R^n} \frac{1}{k} \sum_{i=1}^k \frac{1}{2} \|x_i - D\alpha_i\|_2^2 + \lambda \|\alpha_i\|_1 \\ \text{s.t. } \|b_i\|_2^2 \leq 1, \quad \forall j = 1, 2, \dots, n \end{cases} \quad (3)$$

General solution is optimizing another variable and fixing variable D or A alternately, until $f_k(D)$ equals to given minimum value.

A. Kurtosis

Under normal circumstances, the distribution of the sum of independent random variables tends to be more independent of the Gauss distribution. Two independent random variables have more trends to Gauss distribution than any single component. Thus, the problem of determining whether a component is independent can be transformed into a non-Gauss problem of computing random signals. There are a lot of standards to measure non-Gaussian, the most common methods include kurtosis, negative entropy and mutual information, etc.

Classic non-Gauss measuring method is kurtosis or 4-order accumulation amount. The kurtosis of x is defined as:

$$kurt(x) = E\{x^4\} - 3(E\{x^2\})^2 \quad (4)$$

Actually, since we assume x as unit variance, the equation right can be simplified as $E\{x^4\} - 3$.

For one Gauss distribution x , its 4-order matrix is $3(E\{x^2\})^2$. Therefore, the kurtosis is 0 for one Gauss random variable. But for most non-Gauss random variables their kurtosis are not 0. There exists positive or negative value: down Gauss random variable has negative kurtosis and up Gauss random variable has positive kurtosis. Non-Gauss random variable measuring can adopt absolute value or square of kurtosis. If the value is 0, it is Gauss variable; if the value is more than 0, it is non-Gauss variable. The case is very few that the kurtosis of non-Gauss variable is 0.

Kurtosis or its absolute value, due to simple optimization criterion from theory used to solve the problem of sparse coding and its suppleness in calculation and theory, it can be used in non-Gaussian measurement and related field. Because of the effect on sampling data of fourth-order

moments, by the impact of noise data, kurtosis has poor robustness

III. SPARSE CODING AND IMPLEMENTATION BASED ON KURTOSIS SPARSE MEASUREMENT

A. Target Function Modeling

From sc problem, sparse characteristics coefficients should make effort for statistics independence and the reasonable situation is mutually independent. According to relative literatures [9], it is the most difficult non-Gaussian problem whether it is independent or not between components to be transformed into calculating components. Based on previous description, kurtosis, negative entropy and mutual information are three common non-Gaussian measurement standards. Meanwhile, kurtosis is the most simple and effective method to measure random signal non-Gaussian character. The larger absolute value of kurtosis, the stronger the non-Gaussian. When kurtosis value is positive, signal obeys hyper-Gaussian distribution. Research shows sparseness of random variable equalizes super-Gaussian [10]. Since most natural images have sparse structure and follow super-Gaussian distribution, the maximized random coefficient kurtosis is to maximize its sparseness towards these kinds of natural images and independence between coefficient variables is guaranteed.

A good coding model can not only remove statistics correlation in the largest degree, but the recovered image should also remain original image information in the largest degree, that is, to minimize re-establishing error. Therefore, similar to Olshausen and Field, we also consider the least mean square error principle. Meanwhile, in order to avoid requirement satisfying sparseness for enlarging image reconstruction error, we add a fixed coefficient variance item to target functions.

We establish target function to solve the minimized optimization problem. Target function is made up by three items and the constructed target function shows:

$$\begin{aligned} J(A, S) = & \frac{1}{2} \sum_{x,y} [X(x, y) - \sum_i a_i(x, y)s_i]^2 \\ & - \frac{\lambda_1}{4} \sum_i |kurt(s_i)| + \lambda_2 \sum_i \left[\ln \left(\frac{\langle s_i^2 \rangle}{\sigma_i^2} \right) \right]^2 \end{aligned} \quad (5)$$

In above equation, $\langle s_i^2 \rangle$ denotes mathematical expectation of s_i^2 ; (x, y) denotes pixel grey value; $X = (x_1, x_2, \dots, x_n)^T$ denotes nput data of n-dimensional nature image; $D = (s_1, s_2, \dots, s_m)^T$ denotes m-dimensional random parameter matrix and s_i denotes one line vector of S ; $A = (a_1, a_2, \dots, a_m)$ denote $n \times m$ dimensional characteristic base matrix, and a_i denotes one row vector of A ; λ_1 and λ_2 are normal constant; σ_i^2 is scale constant of given random parameter variance and it is determined by variance of image to be processed.

The first item of target function is reconstructed error of image and is also the keeping information item. The smaller the reconstructed error of image is, the stronger the image information expressing ability is. The second item of target function is sparse penalty item and the absolute value of kurtosis is taken as sparse measurement principle. In terms of super-Gaussian signal, if kurtosis is positive, the maximized kurtosis is the maximized non-Gaussian. However, in terms of natural image which obeys super-Gaussian distribution, the maximized kurtosis is to maximize its sparseness.

We use $|kurt(s_i)|$ to denote the absolute value of kurtosis of s_i . Then the sparsity of maximized s_i is maximized $|kurt(s_i)|$. Assuming random parameter s_i is processed as random signals whose mean is 0, the function form of $|kurt(s_i)|$ is:

$$|kurt(s_i)| = \beta kurt(s_i) = \beta(E\{(s_i)^4\} - 3E\{(s_i)^2\}^2) \quad (6)$$

In above equation, β corresponds to symbol of kurtosis. When kurtosis is negative, $\beta = -1$; otherwise, $\beta = 1$. The last item of target function is fixed parameter variance. The degree of variance $\langle s_i^2 \rangle$ deviating preset σ_i^2 , is punished by i_{th} neurons parameter to acquire certain ability of fixed information. During process of adjusting preliminary function and parameter matrix, we should let $X \approx AS$.

When basis function is adjusted to $A \propto \frac{A}{\alpha}$ ($0 < \alpha < 1$), and parameter matrix is adjusted to $S \propto \alpha S$, we have $X \approx AS$. When $\alpha \rightarrow 0$ and $S \rightarrow 0$, though we can acquire the maximum sparse degree. $A \rightarrow \infty$ and the basis function has not a stable border. Therefore, if parameter variance changes too little to satisfy the demand for sparsity, reconstructed image can not obtain better description. Obviously it is the case we want.

Relative research shows that receptive field of neural cell can be expressed by Gabor function [11]. Therefore, from our SC algorithm, in order to accelerate seeking the optimized characteristic basis speed, we use Gabor wavelet basis initialized characteristic function with certain structure instead of randomly selecting initial value of basis function.

B. Principle Function Initialization

Two-dimensional Gabor filter can effectively simulate creature's visual system so it plays an important role in image analysis. Two-dimensional Gabor function has superior time frequency local characteristics so it can be taken as rational edge detecting operator. Two-dimensional Gabor wavelet is a group of complex function which is generated by two dimensional Gabor through scale expansion and rotation. Two-dimensional Gabor wavelet function is shown as following equation:

$$g_{mn}(x, y) = Kg(x_g, y_g) \cos[-2\pi(U_0x + V_0y) - P] \quad (7)$$

K is normalized parameter and m, n denote direction number and scale of each direction. Generally,

$K = 1/(2\pi\delta_x\delta_y)$. (U_0, V_0) is two-dimensional simple harmonic wave parameter and it denotes space frequency of Gabor function. P is modulate parameter.

For image $I(x, y)$, we choose certain scale of two-dimensional Gabor wave, which can extract upward border at every direction of each point, by rotation. The two-dimensional wavelet transform of $I(x, y)$ is:

$$A_{mn}(x, y) = \int I(x_0, y_0) * g_{mn}(x - x_0, y - y_0) dx_0 dy_0 \quad (8)$$

Each of above Gabor filters corresponds to signal process of an initial visual cortex simple cell space receptive field. For image processing, it is very easy to understand Gabor filter function which will generate strong response on its vertical direction of fluctuation direction but edge is essential to recognize three-dimensional object. In other words, Gabor filter can be seen as a directional telescope which is sensitive to direction and scale. It can detect local characteristics of frequency information which has corresponding directions in image so as to form local characteristic spectrum of brightness image. These local characteristics form one robustness and compacted characteristics expression of originally input image.

However, Gabor convolution process actually generates complex response of two components by real part and imaginary part. Near edge, real part and imaginary part of Gabor transformation will generate fluctuation rather than a smooth peak response. Therefore, it is not beneficial to match image recognition. Because of this, usual solution remains to delete some redundant steps. To abandon linear characteristics of Gabor transformation but only keep the output amplitude of Gabor filter. Amplitude information actually reflects local energy spectrum of image and can also be understood as specific direction edge strength. Meanwhile, there is perfect smoothness near real edge and it is helpful for image recognition. It only keeps amplitude information but abandons phase information which is not stable for recognition. Thus, after determining various parameters of two-dimensional Gabor wavelet filter, based on the obtained amplitude spectrum, this paper takes initial value of characteristics basis function in sparse coding algorithm, to calculate mean of various spectrum filter energy:

$$\phi_1^{mn} = \mu_{mn} = \iint |A_{mn}(x, y)| dx dy \quad (9)$$

C. Learning Rules

We adopt standard gradient descent optimal algorithm to update feature basis function A and sparse parameter matrix S . Before the learning process, we assume input data is centralized and whited. Taking into account input vector graph of single neuron, we acquire standard gradient of i_{th} line vector of feature basis function A as:

$$\frac{\partial J(a_i, s_i)}{\partial a_i} = -[X - \sum_i a_i s_i] s_i^T \quad (10)$$

Then learning rules of row vector a_i is:

$$a_i(k+1) = a_i(k) + [X - \sum_i a_i(k)s_i(k)](s_i(k))^T \quad (11)$$

Similarly, the standard gradient of i_{th} line vector s_i is:

$$\frac{\partial J(a_i, s_i)}{\partial s_i} = -a_i^T [X - \sum_i a_i s_i] - \lambda_1 f_1 + \lambda_3 \frac{\ln(\langle s_i^2 \rangle / \sigma_i^2)}{\langle s_i^2 \rangle} \langle s_i \rangle \quad (12)$$

$\lambda_3 = 4\lambda_2$ is constant; $f_1(s_i)$ is derivative to line vector s_i of the absolute value of kurtosis $|kurt(s_i)|$. The following equation is deduced from above:

$$f_1(s_i) = \frac{\partial |kurt(s_i)|}{\partial |s_i|} = \beta[\langle s_i^3 \rangle - 3\langle s_i \rangle \langle s_i^2 \rangle] \quad (13)$$

Substitute equation 13 to 12 to get the updating rules of vector s_i :

$$s_i(k+1) = s_i(k) + (s_i(k))^T [X - \sum_i a_i(k)s_i(k)] + \lambda_4 \beta [\langle s_i(k)^3 \rangle - 3\langle s_i(k) \rangle \langle s_i(k)^2 \rangle] - \lambda_4 \langle s_i(k) \rangle \quad (14)$$

In above equation,

$$\lambda_4 = 4\lambda_2 [\ln \langle s_i^2 \rangle / \sigma_i^2] / \langle s_i^2 \rangle \quad (15)$$

According to updating equation to update basis vector and parameter vector, the feature basis matrix and parameter matrix learning process can be performed. When updating parameter vector, we believe current basis vector keeps unchanged,

When updating basis vector, we believe current parameter vector keeps unchanged. For convenience, the scalar scale of basis vector and parameter vector can be adjusted. So our algorithm can be summarized as the following procedures:

Step 1: Data pretreatment. Centralize and white input image data X . Set image constructing error accuracy $e \leq 0.01$.

Step 2: Initialize basis function and parameter matrix. Choose any one nature image to set its scale and direction value, to make two-dimensional Gabor wavelet transformation. The mean of all amplitudes of spectra is adopted as initialized value of feature basis function A . Assume W is inverse matrix or pseudo inverse matrix of A , compute $S = WX$ to get the initial value of parameter matrix S . Adjust scalar scale of A and S let $S = S / \|S\|_2$, $A = A / \|A\|_2$.

Step 3: Begin learning iteration process. First, keep a_i unchanged and adjust s_i according to equation 14. Let $s_i = s_i / \|s_i\|_2$. Second, keep current parameter vector s_i unchanged and update basis vector a_i according to equation 11.

Step 4: Determine whether image constructing error $e = \sqrt{\|X - AS\|_2}$ satisfies condition $e \leq 0.01$. If it is, the iteration process is completed; otherwise, repeat step 3.

In actual implementation, dimensional number of Gabor filter is determined by size of randomly collected sub-image. For convenience, Gabor filter is often set as a square matrix

D. Sparse Coding-based Artistic Work Classification

The components of sparse coding are dependent. Besides mean and variance, we also use higher order statistics of kurtosis to describe non-Gauss distribution of sparsity when describing the characteristic. When sparse coding algorithm is used to classify artistic styles, kurtosis maximization based on image space is adopted. On one hand, it can acquire better classification effects; on the other hand, it avoids wavelet method which depends on complicated statistics framework. Kurtosis maximization is basis of sparse coding used to artistic works. The classification flow of sparse coding is depicted as figure 1:

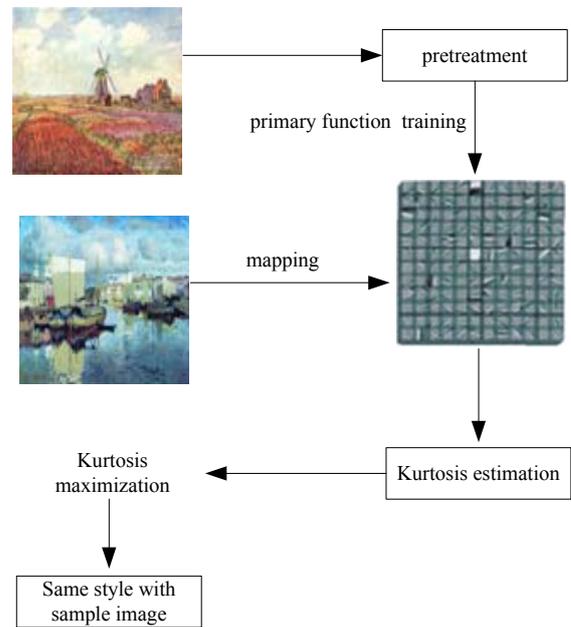


Fig. 1. Flowchart used in style classification based on sparse coding algorithm

Based on flow chart 4, it can be seen that sparse coding algorithm-based style classification steps of art works are:

Step 1: it needs train a group or more groups of basis function of certain style image. Before training basis function, it needs to be clear sample image belongs to which style of visual art works and needs appropriate pretreatment.

Step 2: multiple image blocks are randomly selected from test style images to construct test matrix and the test matrix is mapping basis function.

Step 3: finally, response results after mapping are performed kurtosis estimation. Test figure of large kurtosis value will belong to the same style of sample image of training basis function. On opposite, it does not belong to the same style.

IV. EXPERIMENTAL RESULTS ANALYSIS

A. Recognition and Classification of Artistic Works in Different Styles

First, the experiments for style classification are divided into positive and negative ones. If testing matrix is taken as positive experiment to be mapped with one style of principle function, the other experiments are taken as negative ones. For example of 8×8 scale of principle function, the functions in positive experiment are all selected from 36 pieces of material in traditional Chinese painting. The rest 24 amplitudes heavy colored drawing materials and the randomly selected 24 Monet’s works are taken as test set of this basis function. From back experiment, each group of 8×8-scaled basis function is that 8 works are randomly selected from Monet’s 36 materials to be obtained by training from a group of training set. The rest 24 Monet’s materials and the randomly selected 24 works from heavy color works are taken as test set of this group of basis function.

For 24 pieces of test images of traditional Chinese painting we get 1200 response kurtosis value. For principle functions of 24 pieces of impressionism works. We adopt mathematical statistics to compare two kinds of kurtosis value. The probability of traditional Chinese painting’s kurtosis value that is bigger than that of impressionism works is the probability to determine a work as impressionism one. So it is also the recognition rate of principle function on impressionism images. On the contrary, the probability of traditional Chinese painting’s kurtosis value that is smaller than that of impressionism works is miscalculation probability of impressionism works. After comparison of 1000000 kurtosis value, the statistics results are shown as table 1:

From table 1 we can see, for experiments of positive and negative aspects, the test images are the same, and they are all images out of training sample set. Through mathematical statistics method to compare effect and performance based on kurtosis classification, basis function of heavy color drawings can generate larger kurtosis response on the style of heavy color drawings. In contrast, basis function with Monet’s style can generate large kurtosis response on works in Monet’s style. We conclude that this sparse code algorithm-based basis function and style classification method of visual art works with kurtosis obtain perfect experiment results and this method feasibility is proved.

TABLE 1. POSITIVE AND NEGATIVE EXPERIMENTS RESULTS BASED ON KURTOSIS

Positive and negative principle function	Style of test images	12×12 scale of classification rate
Traditional Chinese painting	Heavy color style	91.46%
	Other style	9.48%
Impressionism works	Heavy color style	21.13%
	Other style	8.95%

In order to verify influence of basis function in different scales on experimental results, basis function in 8×8, 12×12, 16×16 are respectively trained for experiment. Correspondingly, images of 8×8 pixel, 12 × 12 pixel and 16× 16 pixel are respectively extracted to establish three kinds of corresponding test matrix and map basis function in the same scale. Meanwhile, the repeated kurtosis times in one drawing increase from 50 to 100. With works in heavy color drawing style and Monet’s style to perform classification experiment, experiment results are shown as table 2. Through increasing kurtosis times in experiment and enhancing basis function with different scales for experiment, we can see that although experiment effect does not improve too much, it proves robustness and stability of classification experiment with sparse coding.

TABLE 2. EXPERIMENTS RESULTS OF THREE SCALES OF PRINCIPLE FUNCTIONS BASED ON KURTOSIS

Times of statistics	Positive and negative function	Style	Recognition of different scale (%)		
			8×8	12×12	16×16
1000000	Traditional Chinese painting	Heavy color style	60.34	90.63	88.67
		Other style	39.62	9.38	11.34
	Impressionism works	Heavy color style	19.43	21.05	23.36
		Other style	80.55	78.95	76.65
5000000	Traditional Chinese painting	Heavy color style	60.31	91.77	91.96
		Other style	39.42	8.29	8.06
	Impressionism works	Heavy color style	19.18	18.78	20.11
		Other style	80.83	81.22	79.84

4.2 Sparse Code-based Style Classification

Test images are taken from outside training set. They contain 36 Monet's impressionism style images, 36 heavy

color style images and 36 cross striation images drawn by MATLAB software. Kurtosis value of some images and experiments are offered in figure 2.

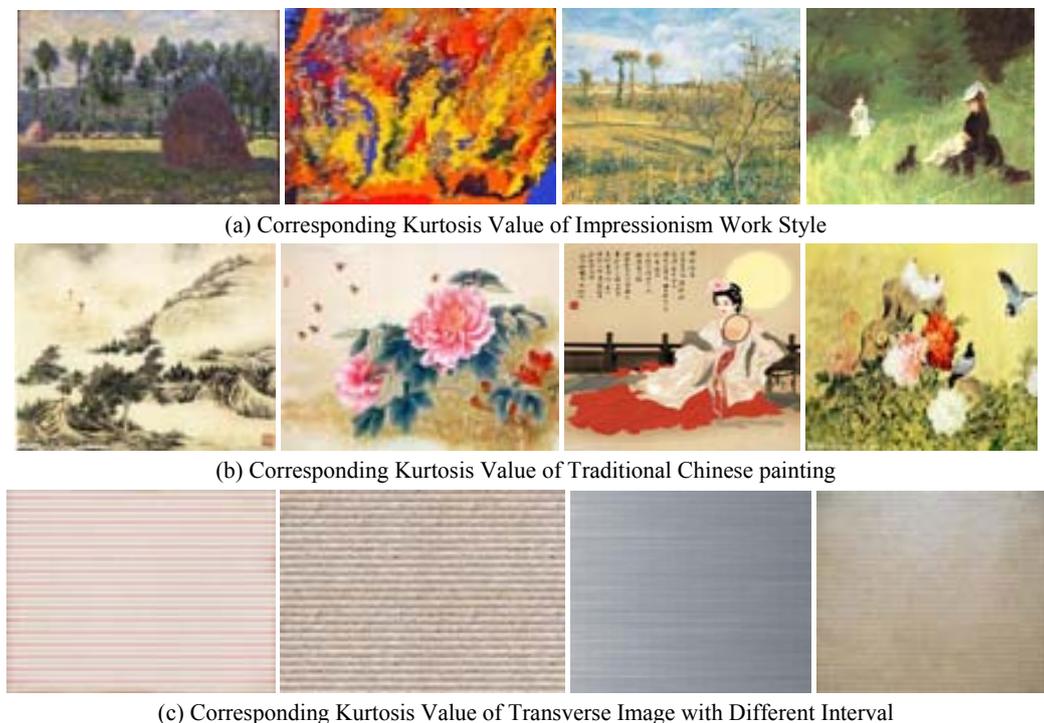


Fig.2. Part of test Images and kurtosis value

In this experiment, test figures in different styles and kurtosis distribution of coefficient S which is obtained from Impressionism image basis function response are shown as figure 2(a). From figure 2(a), for the function in impressionism works, kurtosis value of test image in Impressionism style is the largest. This indicates that the works in the same style and basis function response coefficient in the same style is most sparse. Towards test images in other styles, kurtosis value is lower than the value of Monet test. This indicates that Impressionism works and those test images do not belong to the same style. With this method, the works in different styles can be distinguished. Meanwhile, through further analysis of kurtosis value in figure 2(b) and 2(c), it is not hard to find that kurtosis value distribution in these three styles of works are respectively in three different hierarchies. Impressionism work locates in the top while heavy color drawing is in the middle and cross striation locates at the bottom. The reason is when non-Impressionism style works are responding impressionism training basis, the sparse degree of obtained coefficient is also different.

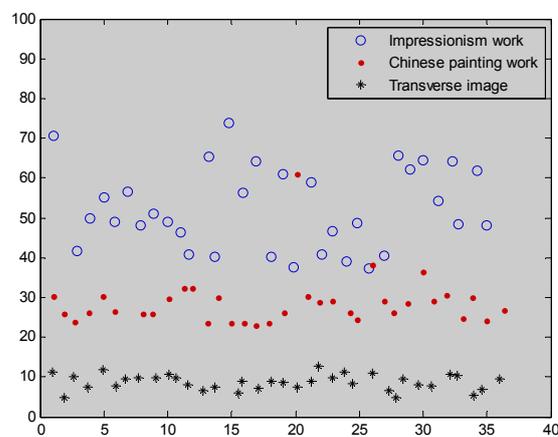


Fig.3. Kurtosis distribution graph

Compared to cross striation image, the image with heavy color drawing style more approximates to Impressionism style. Therefore, its kurtosis value locates at upper hierarchy of cross striation image. However, cross striation image is very regular and impressionism does not have this regularity. Furthermore, this regularity is totally different from impressionism style, so the obtained coefficient is the least sparse during responding Impressionism style basis function

and its kurtosis value is the least. Since kurtosis distribution lies in different layers, we can not only distinguish between non-impressionism style image and impressionism style image, but distinguish between drawing works approximating to Impressionism style and drawing works which are the least approximating to other styles.

CONCLUSION

This paper makes study from simulation of brain process processing mode based on sparse coding theory. According to artistic work with different styles, we can acquire sparse parameter feature with bigger extern-object distance and smaller inter-object distance, after sparse coding. They are applied to image understanding and style classification of different artistic work. The experiments adopt training set of positive and negative objects to test the mapping of test set. The result shows that it can differentiate the works well, which verifies the effectiveness of sparse coding algorithm implemented to work style classification.

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