Financial Time Series Analysis Model for Stock Index Forecasting

Zhang Jian, Li Song

Institute of Economics
Sichuan Agricultural University
Chengdu, Sichuan, China 611130
bjxcdsm@163.com

***Abstract* — There are many defects when current data mining methods are implemented to forecast and analyze a stock index. To adapt the characteristics of financial transaction data, this paper proposes an analysis model for the main indicators of stocks transaction based on an improvement of original mining technologies. It is financial time series analysis model based on Elman wavelet neural network and ARMA. We introduce wavelet analysis and ARMA model to time series modeling and forecasting. Wavelet decomposition is adopted to extract and separate all the hidden periods and nonstationary factors. The feature of wavelet decomposition series and rules of data decomposition are fully applied on Elman dynamic neural network and ARMA model. Then ARMA model and neural network are respectively adopted for scale transformation series and wavelet transformation series. The final forecasting is acquired by the integration of forecasting results of all the scale regions using wavelet reconstruction. The case study shows our model approaches expected standard for high precision, and the forecasting results are prior to artificial neural network**

Keywords -- financial time series; stock forecasting; wavelet neural network; ARMA; WEA model

I. INTRODUCTION

As the main body of financial market, stock market has always been playing an essential role in national economic development. Stock price index reflects overall level and changing situation of various stock market prices in stock market, and provides forecasting analysis. From macroscopic aspect, stock price index provides the basis for national macroscopic decision; From microscopic aspect, it can affect investors' investment strategy and means [1,2]. Thus, forecasting research of stock index is always focused by domestic and foreign scholars. It has been proved to be an effective way to study stock index in time series. At present, the methods on time series of stock index have two patterns: traditional time series model and data mining. The development of wavelet analysis theory [3] offers effective tool for the integration of the advantages of previous two methods, to perform multi-resolution analysis and forecasting for stock index time series.

Literature [4] establishes a hybrid model used for predicting price fluctuation in stock market. It uses genetic algorithm for the optimal decomposing selection of wavelet packet and parameter selection in neural network. By empirical study of the Shanghai composite index, it shows that the performance of this hybrid model is superior to similar neural network model and wavelet decomposition-based neural network model. Reference [5] establishes the forecasting model of stock price which combines wavelet and neural network according to local scale and multi-scale reciprocity in the multi-fractal process. Its predicting accuracy is obviously higher than similarly neural network model. Reference [6] integrates genetic algorithm, wavelet analysis, artificial neural network and simulated annealing to propose a genetic wavelet network. That is, it uses genetic algorithm to learn weight, scale parameter and location parameter between wavelet neural network layers, and applies it to forecast ground pressure and obtain better effect than traditional neural network. From current literatures of wavelet analysis-based stock index forecasting model, most wavelet applications concentrate on de-noising and multi-resolution analysis about stock index research. However, most of the studies still apply single method after wavelet decomposition on series such as data mining model or traditionally random time series model. Furthermore, wavelet function is directly taken as input function of next model and does not select more appropriate model with target towards different characteristics of series after decomposition.

This paper adopts wavelet transform to fully extract and separate various hidden circulation and nonlinearity in financial time series. Meanwhile, we also applies characteristics of wavelet decomposition series and reduction law of decomposition data with increasing measurement to neural network and modeling of autoregressive moving average model. The wavelet reconstruction technology is used to integrate forecast results in various scale regions to form final forecasting of time series. In case study, we utilize wavelet transformation to decompose the Shanghai composite index and obtain scale transformation series and various wavelet transformation series. Then, ARMA model is used to fit scale transformation series and neural network is used to fit wavelet transformation series. Finally, the wavelet reconstruction technology is used to sum the results of various models. The analysis shows that the forecasting method proposed in this paper are feasible and effective in approximately periodic, nonlinear and unsteady time series forecasting.

II ELMAN WAVELET NEURAL NETWORK

The network structure of Elman wavelet neural network [7] is depicted as figure 1. It has four-layer neurons. The input of correlation layer neuron is the output of hidden layer neurons, and the output of correlation layer and input layer are input to hidden layer at the same time. Correlation layer neuron is also a kind of neuron which stores previous output of hidden layer neuron. This type of recursive memory brings dynamic performance to the network.

In this figure, the Elman wavelet neural network has n input neurons, one output neuron, m correlation layer neurons and m hidden layer neurons. $x_j(t) \in R^n$ is n-dimension input vector of neural network at *hour* t. $y(t) \in R$ is one-dimension output vector of neural network at *hour* t. They comprise the training vectors group. $H_i(t) \in R^m$ is hidden layer output and $x_c(t) \in R^m$ is correlation layer output.

We set the feedback gain as α , that is, the output of correlation layer at each hour is:

$$x_{ck}(t) = \alpha H_k(t-1) \tag{1}$$

The dynamic equations of the neural network are:

$$y(t) = \sum_{i=1}^{m} W_i^1(t-1) H_i(t) \tag{2}$$

$$H_i(t) = \phi(\frac{h_i(t) - b_i(t)}{\alpha_i(t)}) \tag{3}$$

$$h_i(t) = \sum_{j=0}^{n} W_{ij}^2(t) x_j(t-1) + \alpha \sum_{k=1}^{m} V_{ik}(t-1) \tag{4}$$

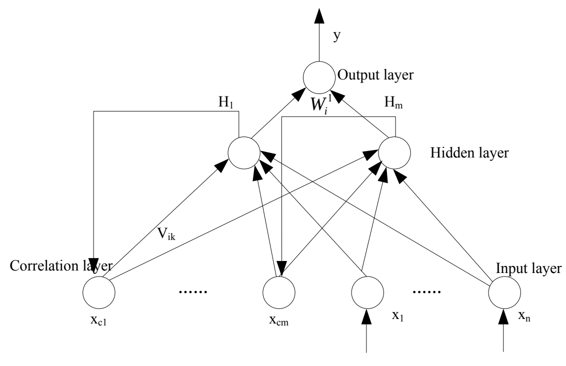

Figure 1. Structure of Elman Wavelet Neural Network

$W_i^1(t)$ is connection weight between input layer neuron j and hidden layer neuron i . $V_{ik}(t)$ is connection weight between correlation layer k and hidden layer neuron i . $H_i(t)$ is the output of i . ϕ is wavelet function. We set α_i as expansion coefficient and b_i as pan coefficient. Let $X = \frac{h_i(t) - b_i(t-1)}{\alpha_i(t-1)}$, then its mother function is:

$$\phi(X) = \cos(1.75X)e^{-X^2/2} \tag{5}$$

III. ANALYSIS AND FORECASTING OF FINANCIAL TIME SERIES

A) Overall Idea

Traditional linear measurement model only extracts rules of overall and the global time in time series and it hardly presents influence of any short-time incident and accident. However, the nonlinear method like neural network is opposite since it focuses more on describing local and short-term factors influential action [8]. In order to appropriately discover the hidden global rules in time series, it must adjust connection coefficients in neural network as much as possible. Neural network is very effective to describe nonlinear relationship between variables but there is no theoretical guidance for network design in practical application [9]. Furthermore, when hidden high nonlinear relationship is directly used in practical learning of neural network and approximate time series, it usually needs large quantity of input units so network training always falls into awkward situation with local minimization and slow convergence speed, which results in consuming lots of time and resources.

Data processing flow is divided into four phases which are wavelet de-noising, wavelet decomposition, decomposition series forecasting and original series forecasting. The first phase is wavelet de-noising whose purpose is to eliminate noise in time series. The main object of wavelet decomposition is to obtain wavelet decomposition series and final scale decomposition sequence of time series in various transformation domains. The third phase is to utilize Elman neural network model for modeling and forecasting wavelet decomposition series in various transformation regions. Meanwhile, it adopts ARMA model to model and forecast the final scale decomposition series. The fourth phase aims to adopting wavelet reconstruction technology to combine forecasting series in various transformation domains to generate the forecasting of short-term time sequence in the system. The measures taken in this paper integrate wavelet analysis, Elman neural network and ARMA so it is called WEA model for short. The detailed model flow is shown as figure 2.

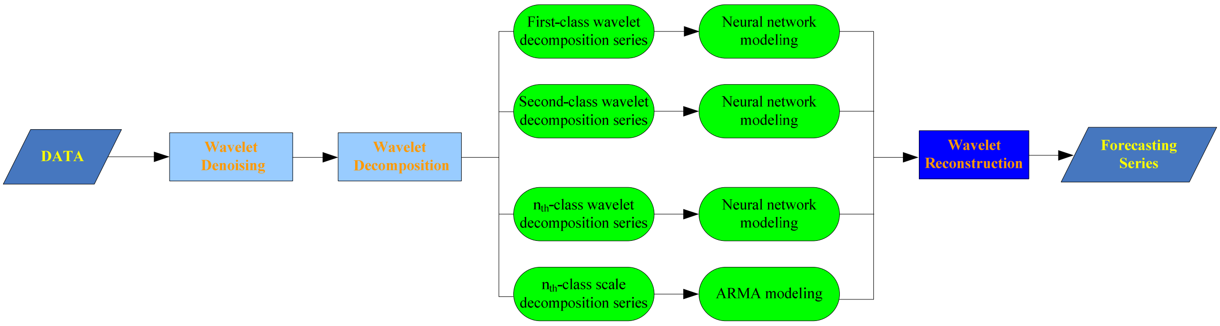

Figure 2. Diagram of Data Mining Model

B. *Wavelet De-noising*

Since time series data of stock index has abnormal data, such data need to be deleted in short-term time series of stock index future to guarantee forecasting accuracy. Similarly, it will be used to be compared between forecasting sequence and genuine sequence, providing data for discovering abnormal data and enriching the means to identify manipulation behavior of stock index future. Wavelet de-noising refers to use prior knowledge of specific problems to construct corresponding regulations, since wavelet coefficient of signal and noise exist different property mechanism in different scales. Other mathematical methods are applied to process wavelet coefficient of noisy signal in wavelet region. The basic steps include:

(1) Wavelet decomposition. Wavelet function and decomposition layer number are selected for multi-scale orthogonal decomposition to obtain detail coefficients in each layer and the coefficient at the lowest layer.

(2) Threshold processing. Choose threshold rule to obtain a threshold value based on signal for threshold process in each layer coefficient.

(3) Signal reconstruction. The coefficient at the lowest layer and the coefficient after process are performed signal reconstruction to obtain signal after noise reduction.

One-dimensional signal model containing noise can be expressed as:

$$e(k) = f(k) + \varepsilon(k) \tag{6}$$

In this equation, $f(k)$ refers to signal containing noise, $f(k)$ refers to information of signal and $\varepsilon(k)$ refers to noise in signal.

We supposes that $\varepsilon(k)$ is Guass white noise N(0, 1). In practical application, useful signal is usually expressed as low frequency signal or some stable signals but noise signal is usually expressed as high frequency signal [10]. Above features of practical noisy signals provide preconditions to implement wavelet analysis and delete noise. When wavelet is used to decompose the signals, signal energy relatively focuses on some positions but noise distribution is usually very extensive. Thus, according to characteristics of abruptness and instantaneity, signal is shown as some large coefficients. However, most of small coefficients are generated by increasing noise and signal energy. Therefore, the de-noising process can be performed based on following methods: At first, signal is performed wavelet decomposition; then, we use high frequency and low frequency of wavelet threshold to divide signals, and wavelet is used to process wavelet coefficients after decomposition. High-frequency coefficients will become zero while low-frequency coefficients will be kept or reduced; thus, after processing, most noise in signal is eliminated. Then, signal is performed wavelet reconstruction and it can reach the purpose of deleting noise.。

C) *Relative Setting of Wavelet Decomposition*

During multi-resolution decomposition and reconstruction of financial data, the first problem is selection of wavelet basis function and wavelet decomposition levels. Wavelet basis function must be based on research purpose and combine research objective characteristics to be determined. According to characteristics of financial time series, the wavelet in this paper has symmetry, smoothness and two-parameter's compactly supported bi-orthogonal wavelet of high order vanish moments and linear phase, to ensure that wavelet transform series and scale transform series in original time series no to have necessary distortion during neural network input and ARMA modeling.

According to wavelet transformation theory, with increasing transformation scale, approximate wavelet decomposition of time series will have better and better stability, periodicity and linearity. Approximating sequence in the highest decomposition scale grade has perfect stability. Its numerical value not only approximates original sequence but it also has similar trend with original sequence. Thus, final scale transformation series is processed by autoregressive moving average model. Towards wavelet transformation sequence in each scale reflecting short-term law of time series, ELMAN neural network is used to be modeling and predicting [11]. By repeatedly experiment of stock price and return series, this paper believes that slow fluctuation series such as stock price should be less than 5 layers. For those series with strong fluctuation in returns, they have more high frequency factors. So we can not select higher layers, lower than three for appropriateness.

D) Modeling and Forecasting of Scale Transform Series

According to wavelet transformation theory, with increasing transformation scale, approximate wavelet decomposition of time series will have better and better stability, periodicity and linearity. Approximating sequence in the highest decomposition scale grade has perfect stability. Its numerical value not only approximates original sequence but it also has similar trend with original sequence. Thus, final scale transformation series is processed by autoregressive moving average model. For wavelet transformation sequence in each scale reflecting short-term law of time series, Elman neural network is used for modeling and forecasting.

This paper adopts AR model [12] to make modeling and analysis. It first uses $X = X - u$ to equalize this series. X_1, X_2,.., X_n denote each data in new series after the series of general signal are equalized. Then it is transfomed into the problem of $AR(P)$ fitting by base data series. The model is :

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + ... + \varphi_p X_{t-p} + \alpha_t \quad (7)$$

$$E(\alpha_t) = 0 \text{ , } E(\alpha_s X_t) = 0, s > t$$

$\{\varphi_i, i = 1,2,..., p\}$ are autoregressive coefficients and $\{\alpha_i\}$ is white noise series. The primary choice is made by its truncation and tailing. The time series $X_t (t = 1,2,..., N)$ is analyzed by correlation. The task is computing the autoregressive coefficients of $\{X_t\}$ samples according to the following equation:

$$\hat{P}_k = \frac{\hat{r}_k}{\hat{r}_0} = \frac{\sum_{t=1}^{n-k} (z_t - \overline{z})(z_{t+k} - \overline{z})}{\sum_{t=1}^{n} (z_t - \overline{z})^2} \, (k = 0,1,2,..., n-1) \quad (8)$$

From the definition of partial correlation function we know the partial correlated coefficient is the last autoregressive coefficient of $AR(p)$ model. If the autocorrelative function is truncated and partial correlation function is tailed, this model is an AR model. The primary choice of p can be determined by the number of partial correlation coefficients falling out of the random region.

The partial correlation coefficient is truncated when $k = p$ and the model is judged preliminarily as AR model. Then its order is determined by equation

$$AIC(k) = \ln \hat{\sigma}_\alpha^2 + 2k / N \quad (k = 0,1,..., L) \quad (9)$$

$$\hat{\sigma}_\alpha^2 = \hat{r}_0 - \sum_{j=1}^{k} \hat{\varphi}_j \hat{r}_j \quad (10)$$

N is the size of sample and L is the highest order given in advance. If $AIC(p) = \min AIC(k) \; (0 \le k \le L)$, the order of AR model is p.

E) Time Series Wavelet Reconstruction

Final phase of model integrates various laws relying on time length in stock index future time series and uses wavelet reconstruction technology integration to obtain final forecasting of time series of original stock index future. According to characteristics of wavelet decomposition and scale decomposition, the acquired decomposition sequence in ARMA transformation field is applied [13]. Then, the wavelet reconstruction equation is:

$$\tilde{\alpha}_{l-1,m} = \sum_n (\tilde{h}_{m-2n} \tilde{\alpha}_{l,n} + \tilde{g}_{m-2n} \tilde{d}_{l,n}) \quad (11)$$

The forecasting series re integrated in each domain obtains time series forecasting which satisfies various special requirements. $\tilde{\alpha}_{l,n}$, $\tilde{d}_{l,n}$ respectively refer to approximate forecasting sequence and wavelet decomposition forecasting sequence [14]. In addition, signal $\tilde{\alpha}_{l-1,m}$ is the approximate forecasting sequence in the first level. When it is gradually reduced from M to 1, the final composite series is forecasting series of time series.

IV. CASE STUDY AND ANALYSIS

This paper simulates the stock index transaction of China financial futures exchange and the applied data sample is one-minute high frequency data from Oct 31, 2013 to Jan 31, 2014. 1426 sample points are proposed totally and *sqtwolog* closed value estimation criterion is used to decompose it to the 5_{th} layer in multi-resolution. The results of wavelet de-noising are shown as figure 3 and 4. It is obvious that most small fluctuations have been removed but main fluctuation features of signal are saved. Therefore, the de-noising effect is satisfied.

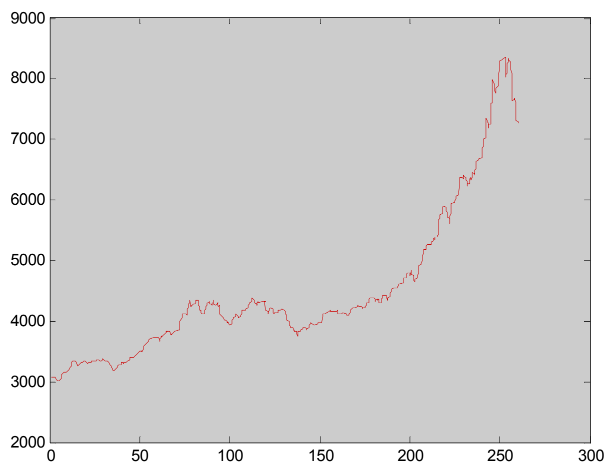

Figure 3 Transaction index before denoising

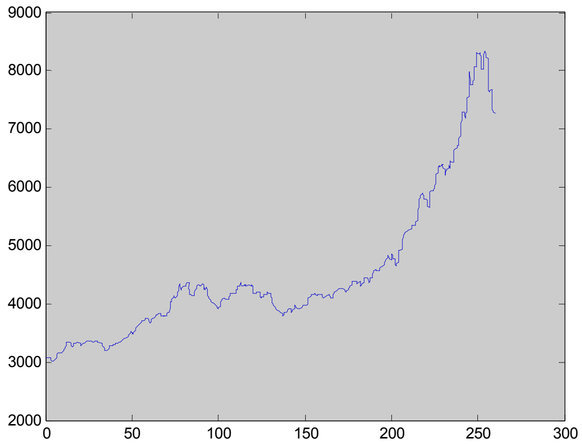

Figure 4 Transaction index after denoising

Figure 5 is the interpolation result of series after decomposition of bi-orthogonal wavelet. In figure 5, the top refers to original time series of stock index futures, which is expressed by S. In addition, the sequence from up to down is successively scale transformation time as at the 5_{th} scale grade and wavelet transformation time series at the 5^{th}, 4_{th}, 3_{rd}, 2_{nd} and 1_{st} scale grade d_5, d_4, d_3, d_2, d_1 are scale grades. Practically, the first scale grade matches scale 2 and it followed by 4, 8, 16 and 32. In figure 7, when scale is small, wavelet transform sequence shows violent change and it reflects bad data dependence, that is, the dependence time is very short. However, the change range is very little and it shows that this component does not largely influence change tendency of original time series. It is merely a local influence factor. Thus, we can expect that forecasting data of this time series has a little influence on overall forecasting of original time series. With increasing scales, the time sequence of wavelet transformation becomes more and more smooth. Wavelet transform at the 5_{th} scale grade is flat. It shows its influence on original time sequence is very stable and influencing time is longer than that of small scale. In addition, scale transform at the 5_{th} scale is also very smooth and it keeps the completely same change tendency with original time series. Meanwhile, its numerical value approximates to original time series and it fully indicates that scale transformation series has long-term fundamental effect on original time sequence. Time series wavelet transform qualities of stock index allow using previously stated WEA model to effectively analyze and forecast specific time series.

After modeling, we randomly select 18 data within set time to forecast outside the samples region. The model adopts multi-step forecasting method for analysis. The so-called multi-step forecasting refers to directly make forecasted results based on previously modeling. Table 1 depicts predicted value of wavelet transformation sequence at each grade and the predicted value of scale transformation sequence. Meanwhile, it also presents the obtained original logarithm yield forecasting by reconstruction.

As shown in table 2, during forecasting of validation set, proposed WEA model of this paper is superior to general BP network on these two evaluation indicators. It shows that WEA model has better generalization ability. With integration of wavelet analysis, neural network and ARMA model, it can obtain perfect forecasting effect.

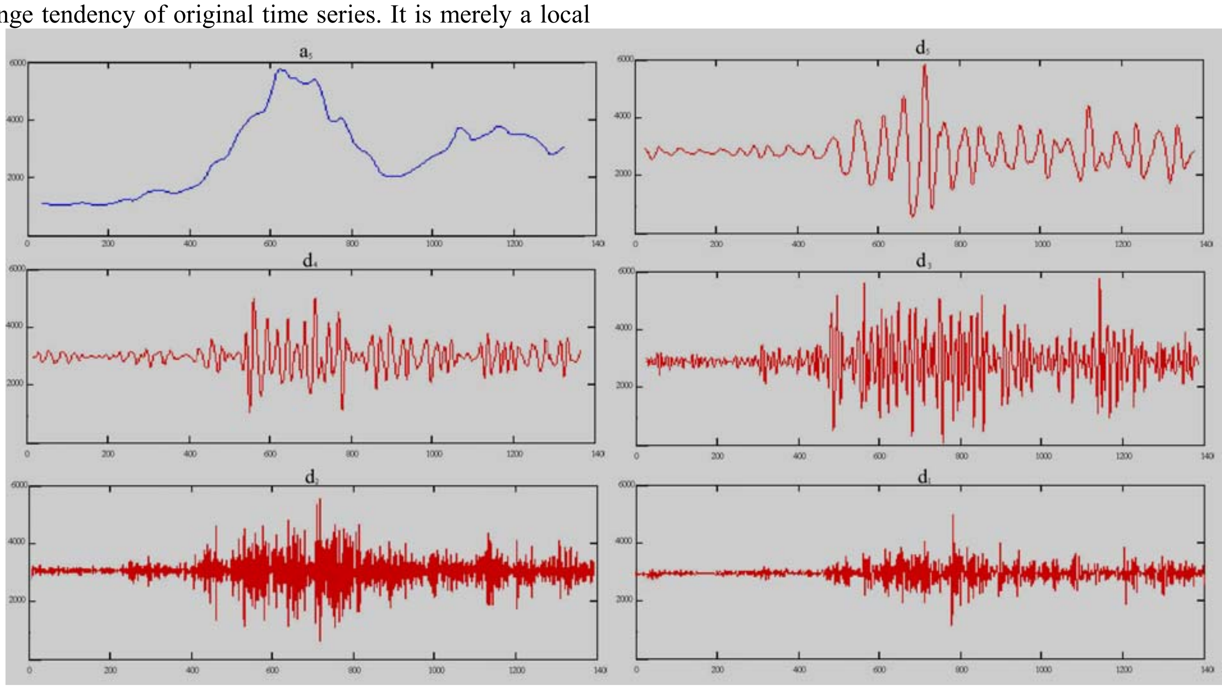

Figure 5. Redundant Wavelet and Scale Transforms of Time Series

TABLE 1. FORECASTING VALUE OF WAE MODEL

First-class transform	Second-class transform	Third-class transform	Fourth-class transform	Fifth-class transform	Scale series	Rate of return by reconstruction
0.001705	0.000274	0.002941	0.004102	0.007750	0.002433	0.019208
-0.01869	-0.00565	0.00404	0.005507	0.007789	0.00227	-0.000451
0.016230	0.002954	0.003587	0.006475	0.007782	0.002114	0.03897
-0.00378	0.007231	0.003325	0.006685	0.006518	0.001792	0.022061
0.005539	-0.0024	0.002521	0.006684	0.00554	0.00172	0.019717
-0.00672	-0.01293	0.001729	0.002589	0.00187	0.00155	0.00662
-0.01354	-0.00560	0.000241	0.004835	0.002869	0.001231	0.00990
0.013410	0.013684	-0.0022	0.003345	0.0015	0.001063	0.031545
0.017234	0.016025	-0.00412	0.001359	0.000648	0.000826	0.031975
-0.01387	-0.01252	-0.00450	-0.00085	-0.00016	0.000558	-0.03651
-0.01831	-0.02356	-0.00252	-0.00308	-0.00105	0.000325	-0.04856
0.014428	0.001728	0.001312	-0.00501	-0.00222	0.00658	0.010237
0.006552	0.0022413	0.004131	-0.00654	-0.00334	-0.0002	0.024163
0.009901	0.0012405	0.004993	-0.00745	-0.00443	-0.00047	0.001506

V. CONCLUSION

Financial time series has features of classical non-linearity and instability. In consideration of wavelet analysis which has good adaptability on signals, this paper adopts multi-resolution wavelet decomposition and reconstruction to decompose and reconstruct time series of the Shanghai Composite Index. By different forecasting methods on scale transformation and wavelet transformation sequence, the scale transformation sequence uses ARMA model for modeling and forecasting. However, wavelet transformation series use neural network. The advantage of traditional model method and data mining method are respectively used. So the mixed forecasting method is applied to become more accurate for forecasting. This paper predicts the test set, and compares it to general BP network forecasting, obtaining more satisfied results. After simulation verification, WEA model proposed in this paper can effectively improve forecasting index and forecasting result, which is superior to artificial neural network.

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REFERENCES

[1] Wang, L., Teo K.K., Lin Z., "Predicting time series with wavelet packet neural networks", Proceedings of the International Joint Conference on Neural Networks, Vol.16, no.14, pp.593-1597, 2001.

[2] Hu ZhiKun, Xu Fei, Gui Wei-Hua, Yang Chun-Hua, "Wavelet matrix transform for time-series similarity measurement", Journal of Central South University of Technology, vol.16, no.5, pp.802-806, 2009.

[3] ZHOU Bo, YAN Hongsen, "Financial time series multi-dimensional forecasting based on wavelet and network dynamics model", Systems Engineering-Theory & Practice, vol.33, No.10, pp.2654-2662, 2013.

[4] Rukun S. Subanar, Rosadi Dedi, Suhartono, "The adequateness of wavelet based model for time series", Journal of Physics: Conference Series, vol.423, no.1, 2013.

[5] Joo Tae Woo, Kim Seoung Bum, "Time series forecasting based on wavelet filtering", Expert Systems with Applications, vol.42, no.8, pp.3868-3874, 2015.

[6] Masuda Naoki, Aihara Kazuyuki, "Forecasting of chaotic time series with wavelet coefficients", Electronics and Communications in Japan, Part III: Fundamental Electronic Science, v 84, no. 6, pp.50-59, 2001.

[7] ZHONG Weinian, GAO Qingwei, CHENG Yanling, "The Clustering of Financial Time Serials Based On Wavelet and Multifractal", Systems Engineering, vol.27, no.3, pp.58-61, 2009.

[8] Sang YanFang, "Improved Wavelet Modeling Framework for Hydrologic Time Series Forecasting", Water Resources Management, vol.27, no.8, pp.2807-2821, 2013.

[9] Wang Zhijie, Li Weiping, "Research on the network traffic time series modeling and forecasting based on wavelet decomposition", Journal of Convergence Information Technology, vol.7, no.11, pp.124-131, 2012.

[10] Abdul Razak Noor Aina, Aripin Rasimah, Ismail Mohd Tahir, "Denoising Malaysian time series data: A comparison using discrete and stationary wavelet transforms", proceedings of International Conference on Science and Social Research,vol.18, no.30, pp. 412-415, 2010.

[11] Cao Jing, Ding WenYun, Zhao DangShu, "Time series forecast of foundation pit deformation based on LSSVM-ARMA model", Rock and Soil Mechanics, vol.35, pp.579-586, 2014.

[12] Li Hui, Pi Dechang, Jiang Min, "Forecasting chaotic time series with wavelet neural network improved by particle swarm optimization", International Journal of Hybrid Information Technology, vol.6, no.6, pp. 369-380, 2013.

[13] Chettri Sarat Kumarm, Borah Bogeshwar, "Privacy Preservation of Time Series Data Using Discrete Wavelet Transforms", Smart Innovation, Systems and Technologies, vol.28, no.2, pp.249-258, 2014.

[14] Karthikeyan L., Nagesh Kumar D., "Predictability of nonstationary time series using wavelet and EMD based ARMA models", Journal of Hydrology, vol.502, pp.103-119, 2013.