

Depth Maps Enhancement Using Nonlocal Self-similarity Based on the Sparse Representation

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Abstract — Depth maps obtained by the RGB-D camera common contain noise in addition to the general Gaussian noise such as uncertain pixels, incredible singular pixels, as well as some black areas composed of pixels with no depth values. In this paper, we purpose a depth image restoration model based on the sparse representation theory to enhance the quality of depth maps. To this model, we consider the non-local similarity and singular pixels of depth map as constraints and fusion detail information of the corresponding high resolution color image. Eventually, the restoration for depth map is converted into the optimization model problem, and the process the optimization solution is also given. Finally, several experiments on public dataset demonstrated that the purposed model is validity for depth images enhancement.

Keywords - Depth maps; Non-Gaussian noise; Singular pixel; non-location Self-similarity; Sparse representation; Image restoration

I. INTRODUCTION

In recent years, with the development of computer vision, the traditional two-dimensional imaging technology cannot meet people's needs, while more and more technologies based on three-dimensional imaging in order to enrich people's lives promote the development of science and technology. On the other hand, many applications such as, human-computer interaction, three-dimensional television, scene reconstruction, gesture recognition, and virtual reality also put forward higher requirements for obtaining three-dimensional information. Therefore, we need measure the depth data for a scene or object to get the real three-dimensional image,

Currently, consumer-level 3D scanning equipment such as Microsoft's Kinect and Time-of-Flight (ToF) and others although capture object's depth data in real time, depth maps initially obtained by these devices usually contain a lot of noise. Beside the image stationary Gaussian noise in general images, more styles of noise belong to non-Gaussian noise. For example, black dots shown in subfigure (a) of Figure 1 are some uncertain or inaccurate pixels, some pixels nearby the edge shown in (b) of Figure 1 belong to singular pixels, and other "black hole" areas without valid depth values such as many black block areas shown in (c) of Figure clearly. So, original depth maps cannot be directly used as data source for numerous 3D application caused by noise mentioned before. Therefore, mainstream image denoising algorithms which it is effective to deal with the general intensity images almost cannot work well for depth maps with the characteristics of diversity of different style noise. Now lots

of adopted strategy for depth image denoising just apply directly image denoising algorithms worked well for intensity images or improve algorithms partly to restore depth maps. These methods can be simply classified into three categories: filtering methods, probability statistics-based methods and sparse representation methods.

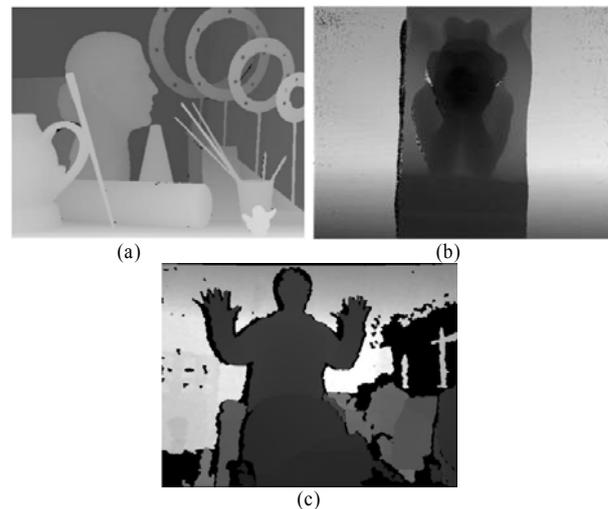


Figure.1 Different noise contained in depth maps

Depth maps enhancement based on filters can get good results because it makes an assumption that incredible pixels and noise can be separated in the frequency domain. Huhle, et al. [1] proposed a two-step method filter to remove singular pixels and smooth depth maps based on color

similarity of non-local means; Liu et al. [2] proposed an extensible and fast matching method which designed and guided the filter to fill the depth map "empty" area by using color images information; Kopf et al. [3] proposed framework for depth maps denoising based on the joint bilateral filtering. Although all algorithms discussed above have been widely used for simple and easy to implement and even it can achieve acceptable results in some situation with low demand, there are some over-smooth areas in the restored depth map during the process due to lack of sufficient additional information.

To the Probability Statistics Method (PSM) for depth maps restoration, a probability model need to make the problem of improving the accuracy as deal with the uncertain problem of depth measurement, and then to achieve the purpose of repairing depth maps. Jung and Ho [4] first sampled the depth map using two local interpolation algorithms and obtain the credibility from each pixel. Second, the depth map can be enhanced through using the graph cut algorithm to solve an energy function model; Aodha et al. [5] used the height field of the low-resolution depth map patches, and selected high-resolution candidate patches by solving the problem of the MRF labeling; Wang et al. [6] proposed a recovery optimization framework to fill the empty hole areas; Yang et al. [7] adopted the regression model on the RGB-D dataset to predict pixel by pixel with the similarity between the depth map and the corresponding color image. Therefore, it is hard to build a cost function for judging unreliable pixels.

Methods based on sparse representation theory assume that incredible depth pixels are sparse, and then depth maps can be enhanced by solving the optimization model. Zheng et al. [8] proposed a multi-dictionaries sparse optimization model with different structures and different atoms; Xie et al [9] combined the trained dictionary generated by local coordinate constraint with adaptive regularization filter in order to denoising and sharpen the edge of the object. Lu et al. [10] used a similar structure of RGB-D to form a low rank matrix which enhances the depth map by filling low-rank matrix. While such methods are usual more effective, but the key and difficulty is how to generate the basis of sparse representation.

By discussing the type of noise in depth maps shows that general intensity image denoising algorithms or simply improved algorithms cannot get good results for depth maps restoration. Although, a lot of denoising algorithms assisted in with detail information of the corresponding color image can further improve the restored depth map quality, restoration results are not as good as expected due to no direct constraints to noise. Therefore, in this paper, we purpose a depth restoration model based on sparse representation which constrained directly by the self-similarity and singular pixels of the depth map. To further improve the accuracy, we combine with the structural consistency between a depth map and a corresponding high-resolution color image. Finally, Several experiments show that the purposed method cannot only effectively eliminate the general Gaussian noise both in depth maps and intensity

images, but it is also effective to non-Gaussian noise contained in depth only.

The organization of the lecture is as follows. After a general introduction of the noise characteristic contained in depth maps and some mainstream methods for restoring depth maps in section I, and the purposed restoration model is described in the section II, where we discuss the principle of restoration, the non-local self-similarity and how to build optimization model in detail. The solution of the optimization model is given in the next section. Experimental results has been presented and comparison to results has also been shown in section VI. At the end of this paper, the paper is concluded and the future work is discussed in section V.

II. DEPTH MAPS RESTORATION BASED ON SPARSE REPRESENTATION

It is an ill inverse problem to restore a high-quality depth map from the depth map interfered by various kinds of noise such as system noise and singular pixels, and so on [11]. To reconstruct a higher quality depth map, we introduce the prior depth information, and model the depth map by a mathematical method to achieve the purpose of denoising.

A. Image sparse representation

Image sparse representation is very popular in recent years, it has had a huge impact in the field of image quality improvement. For a given N -dimensional sparse signal $x \in \mathbb{R}^{n \times 1}$, and α is assumed as a transform coefficient on $D \in \mathbb{R}^{n \times k}$. If most of elements in the coefficient vector α is zero or close to zero, the vector x is called sparse or sparse approximation. So, x can be expressed by α , D and n :

$$x = D\alpha + n \quad (2-1)$$

Where D is Dictionary, and n is noise. Solving problem of α is to solve the formula (2-2) norm optimization problem:

$$\hat{\alpha} = \arg \min_{\alpha} \|\alpha\|_0, \text{ s.t. } \|x - D\alpha\| \leq \varepsilon \quad (2-2)$$

Where ε is the constant of controlling the approximation error. Solving L_0 -norm minimization is an NP-hard problem, For NP-hard optimization problems, there are no effective methods for optimal solution in polynomial time. The usual adopted methods are: 1) searching by unconventional evolutionary optimization algorithms (such as genetic algorithms, ant colony optimization, particle swarm optimization, etc.); 2) Using the approximate solution of the problem instead of the original optimal solution. Due to the evolutionary algorithm lacks of theoretical analysis, it is difficult to ensure the stability and reliability of the solution, so we mainly considerate the approximate solution

method. In practice, the L_0 - norm is usually replaced by L_1 -norm, and we introduce the Lagrange multiplier to convert the constrained formula (2-2) into an unconstrained problem as follow:

$$\hat{\alpha} = \arg \min_{\alpha} \left\{ \|x - D\alpha\|_2^2 + \lambda \|\alpha\|_1 \right\} \quad (2-3)$$

Where, λ is a non-negative parameter. There are many proven methods to solve formula (2-3): Basis Pursuit algorithm (Basis Pursuit), Iterative Thresholding Algorithm (Iterative Thresholding), Splitting Algorithm (Splitting Algorithm), etc. After $\hat{\alpha}$ saluted, \hat{x} can be represented by sparse $\hat{\alpha}$ that is $\hat{x} = D\hat{\alpha}$.

Image is a special signal, image denoising process can be treated as a reconstruct process of a noise signal general approach is to segment and sample the image by block. It assumed that the image block is sparse, and the formula (2-3) can be applied to denoising for images.

B. Depth map self-similarity

During studied super-resolution image reconstruction, Glasner et al. [12] pointed out that similar patches always exist in natural images with the prevailing scale and different scales, that is, it is common that the similarity of patches are in the same neighborhood of the same image, in different regions of the same image and in different images. Many researchers have been apply the self-similarity into intensity image restoration, intensity image compression and intensity image super-resolution. However, it is difficult for the similarity of natural images to be described, and results cannot be good as desired. When block matching algorithm has been purposed, an image can be divided into a lower dimension pieces, and the entire image can be described by the description of each image block. So it is possible to restore the noise image using the non-local self-similarity and then improve image quality. Statistic experiments have been shown that [13] more than 90% of images blocks can be found more than nine blocks similar with itself if sampling the block by size of [5x5] in one image.

Although the non-local self-similarity can greatly improve the performance of image restoration, but its key point is that how to accurately describe the image self-similarity. For symbol describing convenience in this paper, the depth map is denoted as $\{x_i | i = 1, 2 \dots N\}$, where x_i represents pixels, N represents the total number of pixels, Ω_{x_i} represents the image block area around on the center of x_i , and C_{x_i} is the pixels permutations vector of Ω_{x_i} . Then x'_i , C'_{x_i} and Ω'_{x_i} represent the pixel, vector, image block in corresponding high-resolution color images of the same scene, respectively.

The color image and depth map do imaging the same scene and represent the same structure, so they have a structurally relation for each other. To further enhance the

reliability of measuring similar blocks, in addition to calculating the similarity of the depth map patches, it is also necessary to incorporate a color image similarity measure.

Assume that $\varphi(\Omega_{x_i}, \Omega_{x_j})$ and $\varphi(\Omega'_{x_i}, \Omega'_{x_j})$ represent the block similarity of depth map and corresponding color image, then we define the final similarity can be represented by formula (2-4):

$$\omega(x_i, x_j) = \varphi(\Omega_{x_i}, \Omega_{x_j}) \cdot \varphi(\Omega'_{x_i}, \Omega'_{x_j}) \quad (2-4)$$

Where $\varphi(\cdot)$ is similarity cost function of image blocks, and in this paper, $\varphi(\cdot)$ is the Euclidean distance calculation function. For each image block Ω_{x_i} , its similarity value is calculated according to the formula (2-4) and a similar block set $\{\Omega_{x_j}\}$ can be collected.

C. Singular pixel probability detection

First of all, singular pixels caused by occlusion and environmental factors must be detected using a detective algorithm, and then they would be restored by using image restoration techniques. So, for each pixel, we estimate its local probability distribution according to surrounding pixels and build a probability distribution model. Then we calculate the probability of one pixel and determine it belongs to the singular pixel or not by compared with the given threshold value. The specific process is described as follows.

For a given pixel x_i in, its conditional probability is denoted as $P(x_i | C_{x_i})$, and it is subject to Gauss distribution. Ω_{x_j} and Ω_{x_i} are similar blocks and they have the same mean value μ and variance value σ . So, we calculate the μ_{x_i} and σ_{x_i} of the Ω_{x_j} instead of the mean value and variance value of Ω_{x_i} . Finally, pixel x_i conditional probability is estimated as following:

$$P(x_i | C_{x_i}) = \frac{1}{\sqrt{2\pi}\sigma_{x_i}} \exp\left(-\frac{1}{2\sigma_{x_i}^2}(x_i - \mu_{x_i})^2\right) \quad (2-5)$$

Where, the determination threshold value is set 0.5 in our project. When $P(x_i | C_{x_i}) < 0.5$, x_i is marked as a singular pixel, otherwise as a credible pixel. It is worth noting that when the pixel x_i is marked as a singular pixel, we will remove it and then re-calculate μ'_{x_i} and σ'_{x_i} . Therefore, singular pixels detection algorithm is iterative and can further improve the accuracy of determination

D. Depth map restoration model

Image restoration based on sparse representation theory, the key problem is that how to select the basis vectors under the dictionary D . Since the depth map is piecewise smooth, and total variation sparse representation model can deal with the piecewise smooth image for good performance and low computational complexity. So we choose the total variation origin model to represent of depth map initially, as following:

$$x = \arg \min_x \left\{ \|y - Kx\|_2^2 + \lambda \sum_i \|D_i x\|_1 \right\} \quad (2-6)$$

Where y represents the depth map with noise, K represents the degradation liner operator. It is known that different degradation processes have different K . In this paper, it belongs to denoising process, so K is a unit matrix. D_i is a discrete gradient operator at pixel x_i . To further improve the restored quality, we combine the self-similarity constraint discussed in Section II(B). The restoration model formula (2-6) is improved to formula (2-7):

$$x = \arg \min_x \left\{ \|y - x\|_2^2 + \lambda \sum_i \|D_i x\|_1 + \eta \sum_i \|D_i x - \mu_i\|_1 \right\} \quad (2-7)$$

Compared with the total variation restoration model, the model (2-6) is more effective for denoising Gaussian noise, but it is invalid for the non-Gaussian noise only in the depth map. As known, the singular pixels in one depth map meet sparse feature, and it is possible to be removed by sparse representation model. In this paper, singular pixels are represented as e , section II(C) shows that singular pixels are detected by iterative method. So introducing the sparse constraint item $\|e\|_1$ to ensure the singular pixel sparser as possible, then formula (2-7) can be further improved as model (2-8):

$$(x, e) = \arg \min_{x, e} \left\{ \begin{aligned} & Q \|y - x - e\|_2^2 + \sum_i \|D_i x\|_1 \\ & + \sum_i \|D_i x - \mu_i\|_1 + \gamma \|e\|_1 \end{aligned} \right\} \quad (2-8)$$

Where Q is a credible value matrix generated by the conditional probability which consists of 0 and 1. 0 represents the pixel corresponding to the x is singular pixel, and 1 represents normal pixel. The Depth error can be further reduced without singular pixels.

Sparse representation of formula (2-8) do not satisfy the nonlocal similar characteristics, but also sparse. Eventually, the restoration for depth map is converted into the optimization model problem.

III. MODEL SOLUTION

Depth maps restoration model (2-8) is solved by iterative method, and that x and e are solved alternately. Therefore, the optimization for equivalent (2-8) can be divided into solving two sub-optimization questions shown as (1) and (2):

1) e fixed, solving x :

$$x = \arg \min_x \left\{ \begin{aligned} & Q \|y - x - e\|_2^2 + \sum_i \|D_i x\|_1 \\ & + \eta \sum_i \|D_i x - \mu_i\|_1 \end{aligned} \right\} \quad (3-1)$$

1) x fixed, updating e :

$$e = \arg \min_e \left\{ Q \|y - x - e\|_2^2 + \gamma \|e\|_1 \right\} \quad (3-2)$$

To objective function (3-1), the value x is solved by using variable splitting algorithm here.

First of all, we introduce auxiliary variables Z and W , and let $z_i = D_i x$, $w_i = D_i x - \mu_i$ respectively.

Then the optimization of formula (3-1) can be split into three sub-optimizations of the following formulas (3-3), (3-4) and (3-5):

$$Z = \arg \min_z \left\{ \sum_i \|z_i\|_1 + \beta_1 \sum_i \|D_i x - z_i\|_2^2 \right\} \quad (3-3)$$

$$W = \arg \min_w \left\{ \sum_i \|w_i\|_1 + \beta_2 \sum_i \|w_i - (D_i x - \mu_i)\|_2^2 \right\} \quad (3-4)$$

$$x = \arg \min_x \left\{ \begin{aligned} & Q \|y - x - e\|_2^2 + \sum_i \|z_i - D_i x\|_2^2 \\ & + \beta_2 \sum_i \|w_i - (D_i x - \mu_i)\|_2^2 \end{aligned} \right\} \quad (3-5)$$

Detailed solution steps are given as follows.

1) x and w fixed, solving z :

Derivative of z in formula (3-3), and make the result as 0:

$$\sum_i \text{sgn}(z_i) + \beta_1 \sum_i (z_i - D_i x) = 0 \quad (3-6)$$

Then we get z_i :

$$z_i = \begin{cases} \max \{ (D_i x - 1/\beta_1), 0 \} & D_i x \geq 0 \\ \min \{ (D_i x + 1/\beta_1), 0 \} & D_i x < 0 \end{cases} \quad (3-7)$$

Similarly, we can also get w_i as follow:

$$w_i = \begin{cases} \max \{ (D_i x - \mu_i - 1/\beta_2), 0 \} & D_i x - \mu_i \geq 0 \\ \min \{ (D_i x - \mu_i + 1/\beta_2), 0 \} & D_i x - \mu_i < 0 \end{cases} \quad (3-8)$$

2) z and w fixed, solving x :

$$\frac{\partial}{\partial x} = 2Q(x - y - e) + \sum_i 2\beta_1 D_i^T (D_i x - z_i) + 2\beta_2 \sum_i D_i^T (D_i x - w_i - \mu_i) = 0 \quad (3-9)$$

we get the value of x :

$$x = \left[Q + \beta_1 \sum_i D_i^T D_i + \beta_2 \sum_i D_i^T D_i \right]^{-1} \cdot M \quad (3-10)$$

Where

$$M = Q(y - e) + \beta_1 \sum_i D_i^T z_i + \beta_2 \sum_i D_i^T w_i - \beta_2 \sum_i D_i^T \mu_i$$

Depth Maps Restoration Based on the Sparse Representation of Nonlocal Self-similarity shown as Algorithm 3.1

Algorithm 3.1 Depth Maps Restoration Using Nonlocal Self-Similarity Based On The Sparse Representation

1. Parameters Initialization: $k = 0, z = 0, w = 0, e = 0$
2. Repeat times k
 - i. Solving the z : $z_i^{(k+1)} = F(D_i x^{(k)}, 1/\beta_1)$;
 - ii. Solving the w :
 $w_i^{(k+1)} = F(D_i x^{(k)} - \mu_i^{(k)}, \beta_3/\beta_2)$;
 - iii. Solving the e : $e_i^{(k+1)} = F(y - x^{(k)}, \tau)$;
 - iv. Updating $\mu_i^{(k+1)}$;
 - v. Updating Q ;
 - vi. Solving the $x^{(k+1)}$
3. $k = k + 1$
4. End
5. **Output:** High-quality depth map \mathcal{X} after restoration

where block size is $[5 \times 5]$, threshold value to determinate \mathcal{X}_i as a singular pixel is 0.5, the number of iterations is 8.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

In order to verify the effectiveness of the proposed algorithm objectively and comprehensive, we did the experiments on public data sets, and compared our results with other mainstream depth restoration methods (dictionary

learning based methods [14] and joint bilateral filter method [15]). We randomly selected two disparity images from public Middlebury data sets where we directly treated disparity images as depth maps. Black areas and the depth of the image edge areas, seen from Figure 4-1. (b) and Figure 4-2.(b), belong to non-Gaussian noise. And we have added the Gaussian noise with the mean of zero and the standard deviation of 5 into original depth maps which simulates the real Gaussian noise (shown in Figure 4-1. (b) and Figure 4-2. (b)). Figure 4-1. (d) and Figure 4-2. (d) show our results. Some parameter values in our experiments have been given in the algorithm 3.1, the other parameter values are set as follows: $\gamma = 0.3$, $\eta = 0.4$, $\beta_1 = \beta_2 = 0.5$.

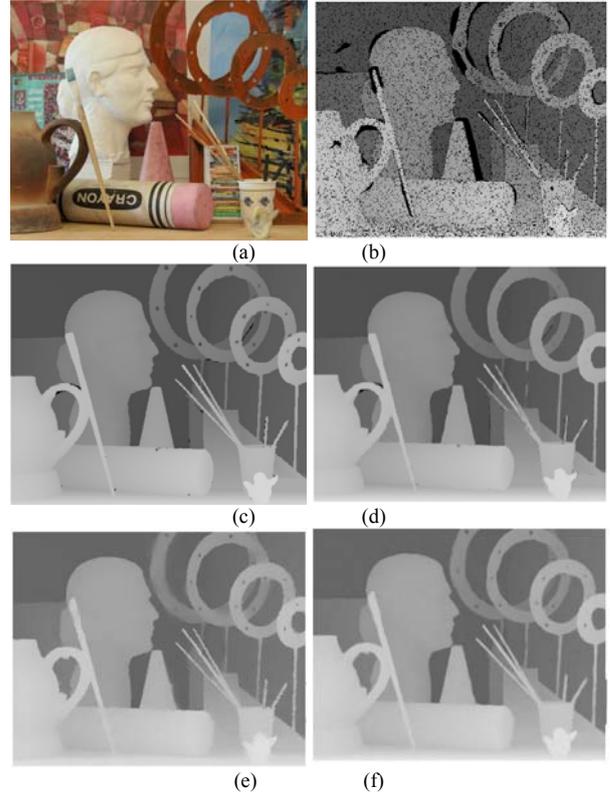


Figure 4-1 Art restoration results: (a) the corresponding color image; (b) the depth map with Gaussian noise; (c) Ground-Truth; (d) LSR; (e) JBF; (f) : Our result

Compared with the joint bilateral filtering method, the proposed method in this paper does not appear over-smooth over the phenomenon nearby edges which can be reflected at the boundary of objects in Figure 4-1 (f) and Figure 4-2 (f), respectively. It is noteworthy that it is better performance for restoring small objects. As shown in Figure 4-1 (d), (e), (f), our method can remove noise and get the smooth edge of wheat respect to the dictionary-based method which indicates that our method is effective too. Second, there are some objects which color is very close to background color, seen the leafy in the middle part of the Figure 4-2. Over-smooth and tooth profile of the leafy seen from Figure 4-2 (e) shows that the joint bilateral filtering method is due to the

high similarity color between foreground, while dictionary-based method appear discontinuous phenomenon, shown in Figure 4-2 (d). Further, Compared detail part with joint bilateral filtering method and dictionary-based method, it can be seen that the purposed method in this paper has a good performance in repairing details.

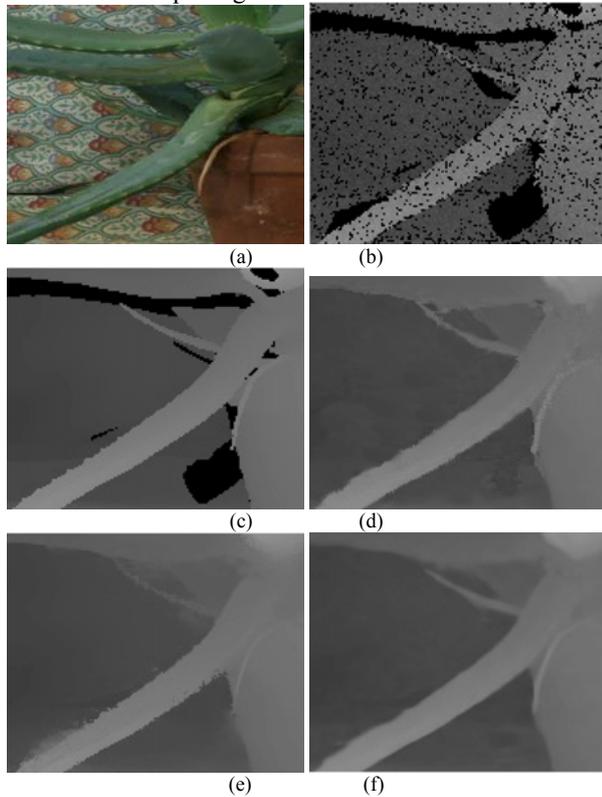


Figure.4-2 Cactus restoration results: (a) the corresponding color image; (b) the depth map with Gaussian noise; (c) Ground-Truth; (d) LSR; (e) JBF; (f) : Our result

V. CONCLUSION

In this paper, we purposed a depth maps restoration model based on sparse representation by analyzing characteristics of noise in depth maps. The purposed model can achieve the purpose of image restoration constrained by non-local self-similarity and combined with sparse representation of the singular pixels. Experiments verify that our method is more effective in removing Gaussian noise and non-Gaussian noise.

In the future, we need take full advantage of characteristics of structure of depth maps and corresponding color images and improve the similarity measurement function instead of calculating the relatively simple Euclidean distance. On the other hand, regularization parameters in our optimization model are determined according to several experiments and did not consider whether they can be generated adaptively according to actual instances which further improve the quality of the repaired

depth maps. Two points mentioned above are undoubtedly an important factor in our restoration model. Therefore, the above two aspects will be the future focus and direction of the work.

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REFERENCES

- [1] Huhle B, Schairer T, Jenke P, et al. "Robust non-Local Denoising of Colored Depth Data", IEEE CVPR Workshop on Time of Flight Camera Based Computer Vision, pp.1-7, 2008.
- [2] Liu J, Gong X, and Liu J. "Guided Inpainting and Filtering For Kinect Depth Maps", In IEEE International Conference on Pattern recognition, pp.2055-2058, 2012.
- [3] Kopf J, Cohen M. F, Lischinski D, et al. "Joint Bilateral Upsampling", ACM Transactions on Graphics, vol.26, No. 3, pp. 96, 2007.
- [4] Jung J, Ho Y. "Depth Image Interpolation Using Confidence-Based Markov Random Field", IEEE Trans. Consumer Electron, vol. 58, No. 4, pp. 1399-1402, 2012.
- [5] Aodha O M, Campbell N D F, Nair A, et al. "Patch Based Synthesis for Single Depth Image Super-Resolution", Computer Vision-ECCV 2012. Springer Berlin Heidelberg, pp.71-84., 2012.
- [6] Wang Y, Zhong F, Peng Q, et al. "Depth Map Enhancement Based on Color and Depth Consistency", Visual Computer, vol. 30, No. 10, pp. 1157-1168, 2013.
- [7] Jingyu Y, Xinchun Y, Kun L, et al. "Color-guided Depth Recovery From RGB-D Data Using An Adaptive Autoregressive Model", IEEE Transactions on Image Processing a Publication of the IEEE Signal Processing Society, vol. 23, No. 8, pp. 3443-3458, 2014.
- [8] Zheng H, Bouzerdoum A, Phung S L. "Depth Image Super-Resolution Using Multi-Dictionary Sparse Representation", IEEE International Conference on Image Processing, pp.957-961, 2013.
- [9] Xie J, Chou C, Feris R, Sun M. "Single Depth Image Super Resolution and Denoising Via Coupled Dictionary Learning with Local Constraints and Shock Filtering", in: International Conference on Multimedia and Expo (ICME), pp.1-6, 2014.
- [10] S. Lu, X. Ren, F. Liu. "Depth Enhancement via Low-Rank Matrix Completion", IEEE Conference on Computer Vision and Pattern Recognition, pp3390-3397, 2014.
- [11] Yi Zhou. "The Estimation and Procession of the Depth Map", Xi'an, Xidian University, 2008, (in Chinese).
- [12] Glasner D, Bagon S, Irani M. "Super-resolution from a single image", International Conference on Computer Vision, pp.349-356.
- [13] Yang S, Liu Z, Wang M, et al. "Multitask dictionary learning and sparse representation based single-image super-resolution reconstruction", Neurocomputing, vol. 74, No. 17, pp. 3193-3203, 2011.
- [14] Xi'en Liu. "Robust Sparse Representation Algorithm for Depth Maps Restoration", Beijing: Beijing University of Technology, 2014, (in Chinese).
- [15] Christian R, Carsten S, Dodgson N A, et al. "Coherent Spatiotemporal Filtering, Upsampling and Rendering of RGBZ Videos", Computer Graphics Forum, vol. 31, No. 2pt1, pp. 247-256, 2012.