

# Levenberg-Marquardt Based Training Algorithm for Neural Network Modelling of Automobile Exhaust Thermoelectric Generator

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**Abstract** -- Automobile exhaust thermoelectric generator (AETEG) is a nonlinear system based on thermoelectric materials and thermoelectric modules (TEMs) for its performance is directly affected by several factors such as temperature difference, electric topology, circuit load and so on. Considering the nonlinear mapping and self-study ability of neural networks, a kind of revised back propagation (BP) neural network is applied to set up its output characteristic model in this paper. In the modelling process, the rotating speed and output power of engine, as well as the output currents of AETEG are defined to be the input variables of neural networks, while the output voltage of AETEG is treated as its output variable, and the neural network is trained with Levenberg-Marquardt (LM) algorithm. The approach is validated with operation data from an AETEG test platform developed by our group, both simulation and experiment results regarding root mean square error (RMSE) and variance account for (VAF) indicate that the proposed AETEG model has compact structure and fast convergence rate, it can precisely predict AETEG's output characteristic without regard to the complicated mechanism, which makes it convenient to optimize the real-time energy control of AETEG in the powertrain system and vehicle electric bus in further.

**Keywords** -- automobile exhaust thermoelectric generator (AETEG); output characteristic; neural networks; Levenberg-Marquardt (LM) algorithm; modelling

## I. INTRODUCTION

The utilization efficiency of fossil fuel energy in internal combustion engine of traditional vehicle is below 30%, while nearly 40% of the rest one is directly wasted from exhaust [1]. Recovering and reusing some of the exhaust gas heat based on thermoelectric generators (TEGs) of low temperature and middle temperature has been a novel research focus so far [2], for the automobile exhaust thermoelectric generator (AETEG) can improve the overall efficiency of internal combustion engine by applying the generated power in the vehicle's electric bus, and decreasing fuel consumption and reducing environmentally harmful emissions [3]. To realize the energy control and system optimization of AETEG, the research on its model is in the initial stage, up till now, several mechanism models and theoretical analysis based on Seebeck effect, Peltire effect and Thomson effect have been the research focus [4-10], even though they are basically consistent with the experimental results and suitable for the performance researching and designing of AETEG, they can't roundly describe the complicated heat transmission process among heat exchanger and TEMs, for there are lots of partial differential equations with parameters difficult to be measured and defined especially

when there are lots of TEMs of different characteristic connecting together, and there are some assumed conditions which contributes to its weakened self-adaptive capacity, they are not convenient for the engineering application and energy control to some extent.

The output characteristic of AETEG is soft, i.e. its output voltage drops as its output current increases, its output performance is directly affected by several factors such as temperature difference, output currents and so on. Moreover, different operations of engine and output currents, different output voltage for AETEG, there is an evident non-line relationship among them. Due to the advantages of non-line mapping and self-study ability, a neural network model is put forward in this paper, its input variables are the rotating speed and corresponding output power of engine, the output currents of AETEG, while the output variable is the AETEG output voltage. Finally, it is validated with simulation and experiments based on an AETEG test platform.

## II. DESIGN OF AETEG

### A. Schematic diagram

The typical schematic diagram of designed AETEG is shown in Fig.1. It includes engine, heat exchanger, TEMs, cooling units, muffler, three-way catalytic converter and so on. There are two groups of thermoelectric modules sandwiched between heat exchanger and cooling units 1 and 2, respectively. For the cooling unit 1 and cooling unit 2, the single-column cold source structure [11] is adopted, i.e. the cold sides of four TEMs of each row in a layer are fixed with a common small cooling box, 8 cooling boxes of each layer compose a whole cooling unit. The

operation conditions of engine are controlled by a dynamometer, i.e. different absorbed power and rotation speed, different heat exhausted from the outlet gas of engine. The inlet of heat exchanger is connected to the outlet of engine, due to the non-uniform temperature distribution of heat exchanger, once its surface temperature approaches the maximum operating temperature of a certain single thermoelectric module, the exhaust gas is directly bypassed to the catalytic converter to protect them.

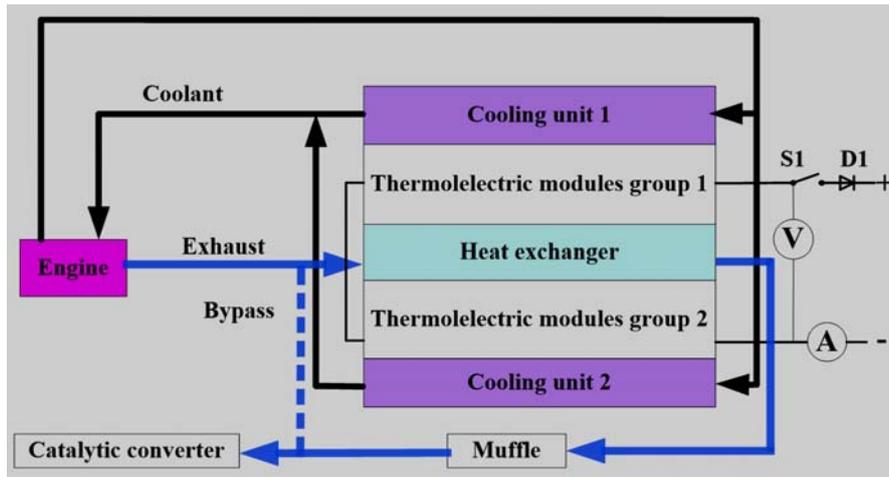


Fig.1. Schematic of the AETEG designed

*B. Distribution of thermoelectric modules*

As shown in Fig.2, there are 32 independent single TEMs fixed above each layer surface of the heat exchanger, they are arranged with 4 rows and 8 columns from the exhaust inlet to exhaust outlet direction [12].

During thermoelectric modules group 1, the number of single thermoelectric module is from 1 to 32, while the number of single thermoelectric module in TEMs group 2 is from 33 to 64, all the TEMs are connected in series as the whole output of AETEG.

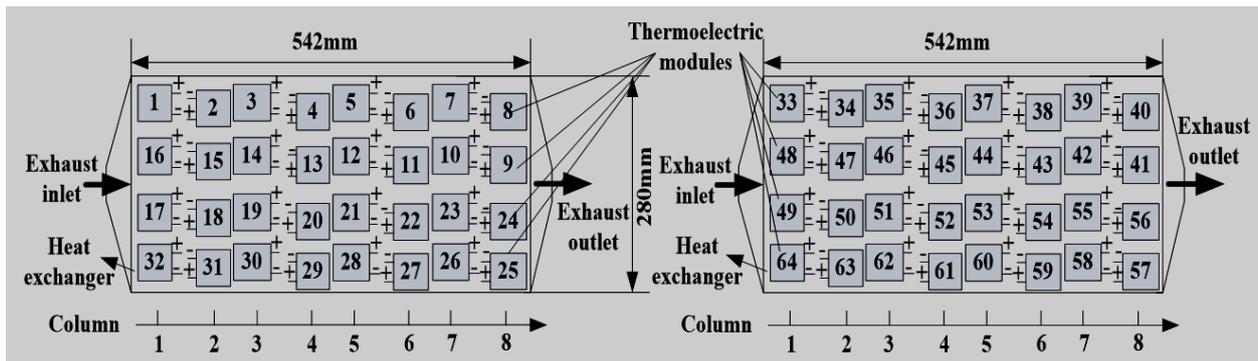


Fig.2 Number and distribution of thermoelectric modules in group 1 and group 2

*C. Output characteristic of TEMs and AETEG*

As stated in [13-15], the performance of a single TEM is proportional to the temperature difference between its hot and cold source, it can be maximized by raising the hot source temperature appropriately based on the engine

coolant. Figure 3 illustrates the voltage–current–power (V–I–P) characteristic of AETEG with different temperature difference noted  $\Delta T_1$ ,  $\Delta T_2$ ,  $\Delta T_3$ , respectively ( $\Delta T_1 < \Delta T_2 < \Delta T_3$ ). Compared with the output performance V–I–P characteristics of single TEM, both of them have the same soft output characteristic.

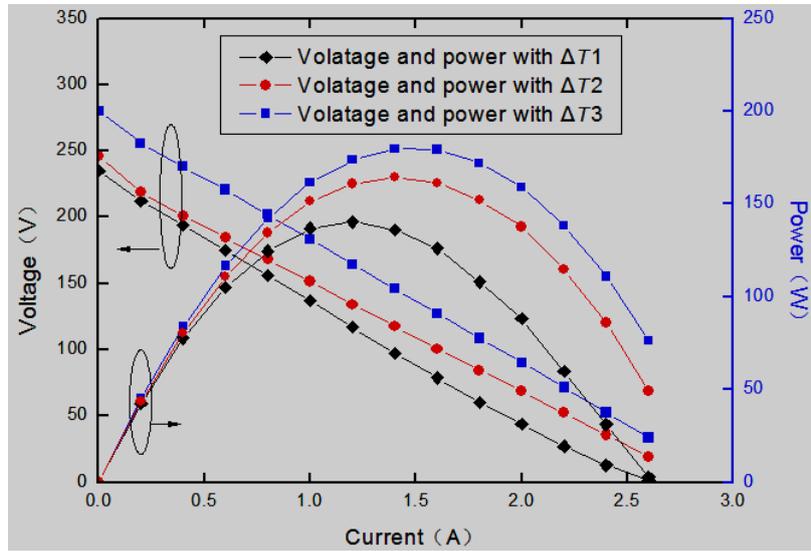


Fig.3. Output characteristic of AETEG with different temperature difference

Usually, the engine coolant temperature maintains around 90°C, thus, the temperature difference of AETEG is directly affected by the surface temperature distribution of heat exchanger once engine coolant is applied to control the cold source temperatures of AETEG. For the herring-bone structure flow structure of heat exchanger [11], once its dimensions and the installed pressure are fixed, its surface temperature distribution is directly

controlled by the working conditions of engine such as rotation speed, torque and power. Therefore, the output characteristic influence factors of AETEG and the corresponding process can be described as Figure 4, for the engine power is a product function of its speed and torque, the output voltage of AETEG can be treated as a nonlinear function of engine speed, engine power and the output current of AETEG.

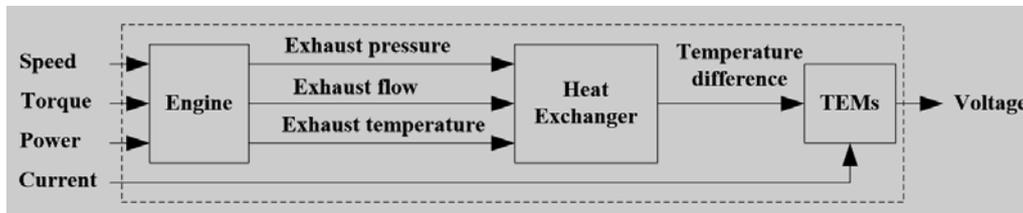


Fig.4. Output characteristic influence factors of AETEG

### III. NEURAL NETWORKS MODEL OF AETEG

#### A. Identification Structure of neural networks model for AETEG

For the output characteristic influence factors of AETEG stated above, the nonlinear function of AETEG can be described with neural networks due to its self-study nonlinear mapping capability. Considering the relationship between engine power with its speed and torque, it can be concluded that different rotation speed and power of engine on AETEG test platform, different temperature difference and performance for TEMs. Once

the external load changes, i.e. the output current of AETEG varies, its output voltage and power fluctuates accordingly. Thus, in the neural networks model of AETEG shown in Fig.5, engine speed (denoted S), engine power (denoted P), and the output current (denoted I) of AETEG are treated as its input variables, while the real output voltage (denoted as  $V_{out}$ ) of AETGE is the output variable. The identification process of neural networks is to ensure the error  $e$  between the real output voltage of AETEG and the predictive value of neural networks model (denoted  $V_{ann}$ ) is as much small as possible based on the different tested variables of V, P, I and  $V_{out}$ .

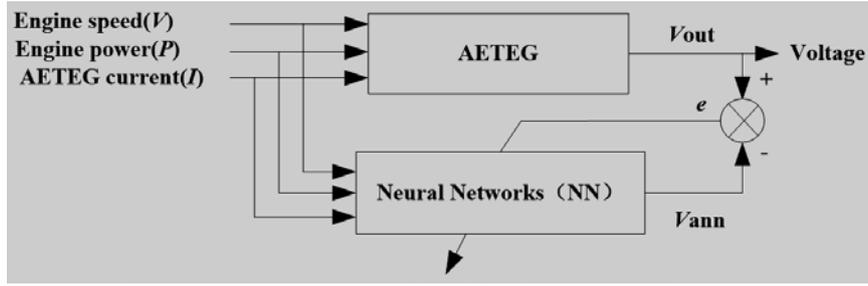


Fig.5. Neural networks model identification structure of AETEG

*B. Training Algorithm for Neural Networks Model*

The standard BP algorithm is practical to the application of forward artificial neural networks for it adjusts the weight value and threshold value to ensure the sum of square error between the output value of neural network and object value is as little as possible. However, it has disadvantage such as slow convergence, local minimal value. In this paper, Levenberg-Marquardt (LM) algorithm [16-17] is adopted to improve BP algorithm for its rapid convergence and efficient. If  $X^{(k)}$  is the  $k$ th vector comprised of weight value and threshold value, then  $X^{(k+1)}$  is calculated from Eq. (1).

$$X^{(k+1)} = X^{(k)} + \cap X \tag{1}$$

According to newton algorithm [18],  $\cap X$  is given by:

$$\cap X = -\left[\nabla^2 E(x)\right]^{-1} \nabla E(x) \tag{2}$$

where  $\nabla^2 E(x)$  is the Hessian matrix of error indicator function  $E(x)$ ,  $\nabla E(x)$  is the gradient.

We define  $E(x)$  by the following equation:

$$E(x) = (1/2) \sum_{i=1}^N e_i^2(x) \tag{3}$$

where  $e(x)$  is the training error,  $\nabla E(x)$  and  $\nabla^2 E(x)$  are calculated from Eq. (4) and (5), respectively.

$$\nabla E(x) = J^T(x)e(x) \tag{4}$$

$$\nabla^2 E(x) = J^T(x)e(x) + S(x) \tag{5}$$

where  $S(x) = \sum_{i=1}^N e_i(x)\nabla^2 e_i(x)$ ,  $J(x)$  is the Jacobian matrix given by:

$$J(x) = \begin{bmatrix} \frac{\partial e_1(x)}{\partial x_1} & \frac{\partial e_1(x)}{\partial x_2} & \dots & \frac{\partial e_1(x)}{\partial x_n} \\ \frac{\partial e_2(x)}{\partial x_1} & \frac{\partial e_2(x)}{\partial x_2} & \dots & \frac{\partial e_2(x)}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial e_n(x)}{\partial x_1} & \frac{\partial e_n(x)}{\partial x_2} & \dots & \frac{\partial e_n(x)}{\partial x_n} \end{bmatrix} \tag{6}$$

According to gauss-newton algorithm [18],  $\cap X$  can be written as follows:

$$\cap X = -\left[J^T(x)J(x)\right]^{-1} J(x)e(x) \tag{7}$$

Meanwhile, according to LM algorithm,  $\cap X$  can be rewritten as follows:

$$\cap X = -\left[J^T(x)J(x) + \mu I\right]^{-1} J(x)e(x) \tag{8}$$

where  $\mu$  is a positive variable,  $I$  is unit matrix. If  $\mu$  is equal to 0, LM algorithm is the same as Eq. (7) based on gauss-newton algorithm, once  $\mu$  is very large, LM algorithm approximates gradient descent algorithm. The computation speed of gauss-newton algorithm is extremely quick when the minimal error is closed to the target value. For LM algorithm makes full use of similar second derivative information, its computation speed is almost as hundred times as the basic gradient descent algorithm.

IV. THE EXPERIMENT RESULT ANALYSIS

*A. Data source*

For the neural networks model of AETEG, 618 groups of different experimental data are obtained from the AETEG test bench shown in Figure 6 as the training sample, whereas another 198 different groups are selected as the testing data. As to the AETEG test bench, the

engine capacity is 1997cc, its maximum power is 108 kW (6000r/min), its maximum torque is 200NM (4000r/min), the maximum absorbed power of adopted dynamometer is 160kW, its maximum rotate speed is 6000r/min, and its maximum absorbed torque is 600NM. The heat exchanger is made of brass, its interior thickness is 3mm, and its heat conduction area is 542mm×280mm. For each single thermoelectric module implemented in AETEG test bench, its maximum operation temperature is 350°C, and its dimensions are 56mm×56mm×6mm.

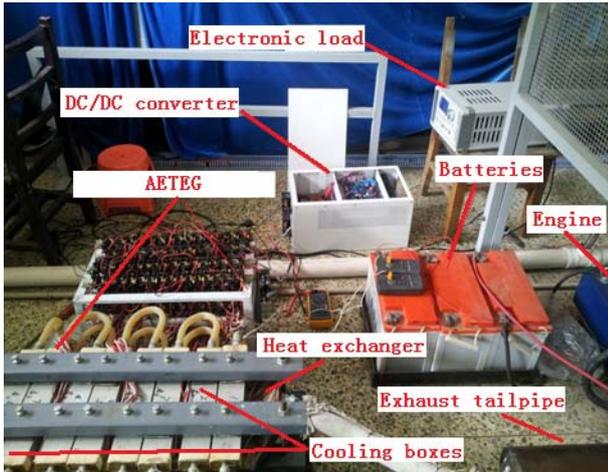


Fig.6. AETEG test bench

During the experiments, the engine speed and engine power are adjusted by controlling the dynamometer operation condition, whereas the output current of AETEG is controlled with DC/DC converter by controlling its duty cycle. For the necessity and length limitation of paper, only some typical important experimental data are provided in Tab.1 as the training sample.

TABLE 1. PARTIAL TRAINING SAMPLE FOR AETEG NEURAL NETWORKS MODEL

$S(r/min)$	$P(kW)$	$I(A)$	$V_{out}(V)$
2500	1	0	104.5
2600	2	1.2	53.8
2700	2	0.4	113.2
2800	4	0.8	100
2800	4	0	150
2900	6	0.6	125.5
3000	8	0.2	162
3000	8	1	108
3200	10	0	197.5
3200	10	1.4	100

Due to the different order of magnitudes for the input and output variables, to enhance the study efficiency of

neural networks model of AETEG, both the input and output variables are normalized by the following expression:

$$\bar{x}_i = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}} \quad (9)$$

where  $\bar{x}_i$  is the normalization value of input and output variable,  $x_i$  is the real value,  $x_{\max}$  and  $x_{\min}$  are the maximum and minimum value of variable  $x$ , respectively.

### B. Training for Neural Networks

To evaluate the estimated performance of neural networks model, the root mean square error (RMSE) is adopted in this paper, it is defined by [19]

$$RMSE(y, y_m) = \sqrt{\frac{1}{N} \sum_{i=1}^N (y(i) - y_m(i))^2} \quad (10)$$

where  $y$  is the target value of neural networks model (i.e.  $V_{out}$ ),  $y_m$  is the output value of neural networks model,  $N$  is the sample data number. The smaller RMSE is, the closer  $y_m$  is to  $y$ .

To describe the approximation degree between target value of neural networks model and the output value of neural networks model, variance account for (VAF) is given by [20]

$$VAF(y, y_m) = \left[ 1 - \frac{\text{var}(y - y_m)}{\text{var}(y)} \right] \times 100\% \quad (11)$$

where  $\text{var}()$  is the variance operation, the large VAF means the output value of neural networks model approximates the real output value of AETEG.

In the modelling process, the adopted neural networks function is newff(), the transmission function of hidden layer is tansig, the training function is trainlm. For the generalization of neural networks will be reduced when the nerve number of hidden layer increases, while the precision will be lowered when the nerve number of hidden layer decreases, the nerve number of hidden layer changes from 4 to 30 based on traversing method until the minimum RMSE value is obtained as the optimal object.

Figure 7 shows the root mean square error with different nerve number of hidden layer based on LM algorithm, it illustrates that the training error decreases when the nerve number of hidden layer increases, while the testing error firstly decreases until the nerve number of hidden layer increases to 17, then it increases accordingly. Considering the simplification and precision of the neural networks model, the final nerve number of hidden layer is selected to be 17.

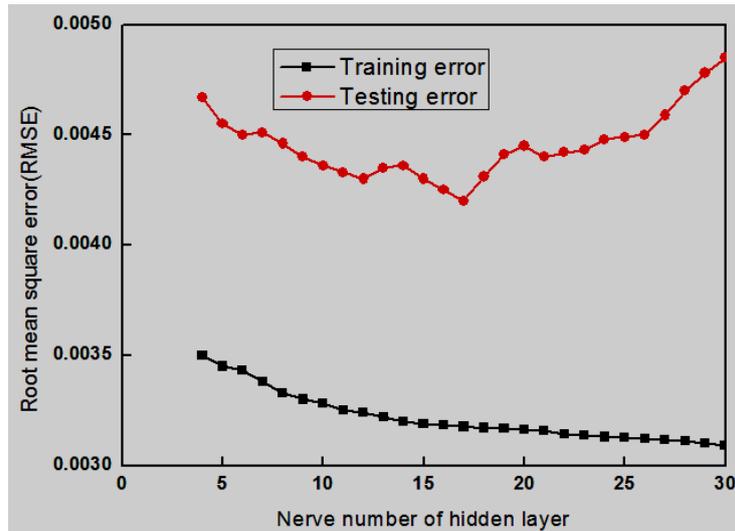


Fig.7. Root mean square error with different nerve number of hidden layer

*C. Performance analysis*

Based on the AETEG neural network model established above, the comparison result between the 198 groups of testing data which are different from the training data and the predicted value of neural network model is presented in Figure 8, the corresponding absolute

error and relative error are given in Figure 9 and Figure 10, respectively. As shown in Figure 9, the absolute error range between the expected output (i.e. 198 groups of testing data) and predicted output of AETEG is from -4V to 4.6V, while the maximum relative error shown in Figure 10 is 16.67%, the average relative error is 1.41%.

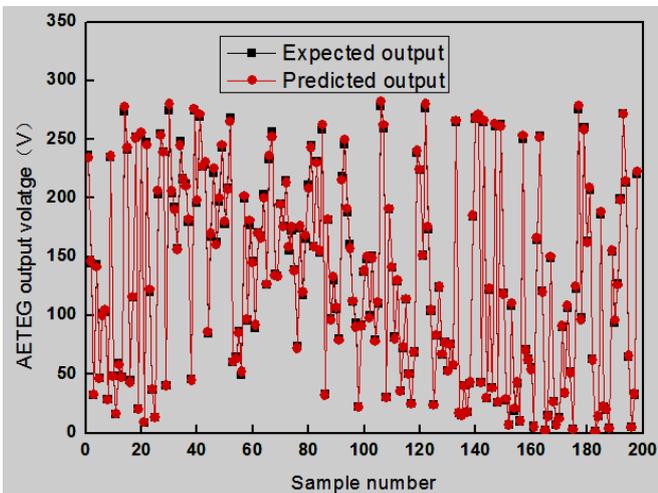


Fig.8. Predicted value of neural network model

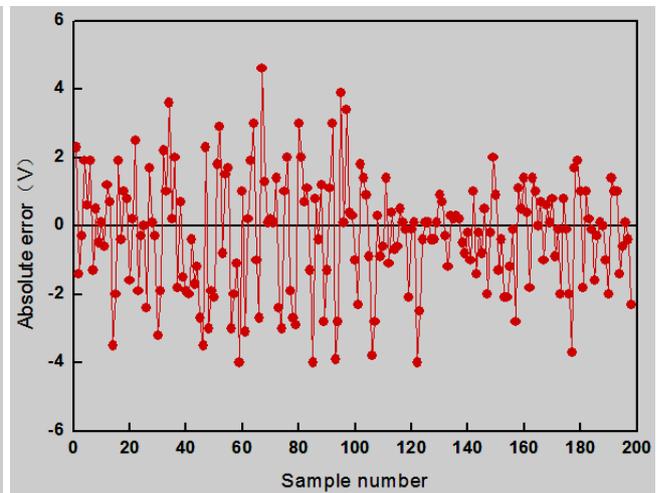


Fig.9. Absolute error between the expected output and predicted output

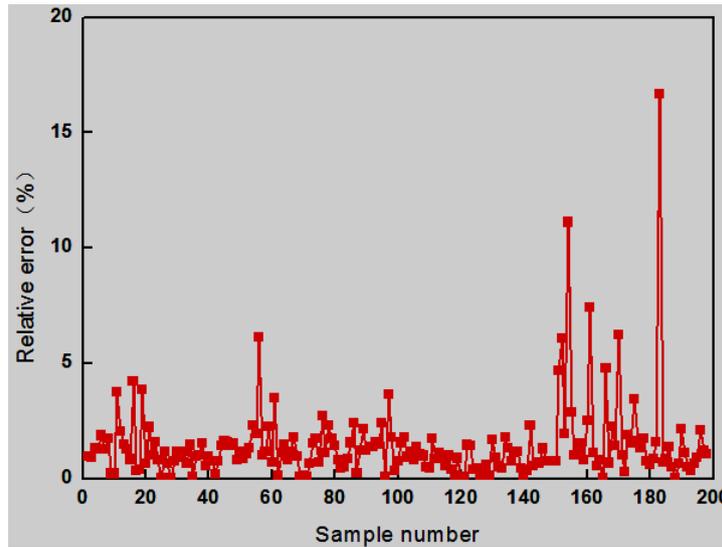


Fig.10. Relative error between the expected output and predicted output

To further analyze the performance of established neural network model, based on the training data and testing data, the traditional BP neural network model is adopted to compare their performance index, also, the nerve number of hidden layer of traditional BP neural network is 17, and the comparative results are provided in Tab.2. It shows that the RMSE of testing data based on traditional BP algorithm is 0.0049, while the one based on LM algorithm is only 0.0032 which is decreased by 34.7%, moreover, the VAF based on LM algorithm is increased by 11.6% compared with that based on traditional BP algorithm.

TABLE 2. PERFORMANCE INDEX OF NEURAL NETWORK WITH DIFFERENT ALGORITHMS

Algorithm Type	Training data		Testing data	
	RMSE	VAF	RMSE	VAF
BP	0.0049	84.43	0.0058	82.68
LM	0.0032	94.17	0.0041	92.31

From both simulation and experiment results above, it can be conclude that the AETEG neural network model set up based on LM algorithm has advanced predictive ability, its maximum predicted error is below 18%, and it has obvious advantage over the traditional BP neural network model for it has much lower RMSE and larger VAF based on different testing sample.

V. CONCLUSION

The hybrid vehicle application of AETEG in further will be a research focus even though the development of AETEG is in the initial stage, for it can obviously improve the fuel economy and exhaust emission of vehicle by recycling the exhaust waste heat. Due to the soft output characteristic, it is challenging but also

important to control and model AETEG during the application in vehicle electrical bus.

For the output of AETEG is affected by several factors, there is a nonlinear mapping between its input and output, thus, it is a good way to describe the output performance of AETEG based on the self-study neural networks model for it can not only predict the sample data, but also has good generalization performance to the data deviating from them. From both simulation and experiment results presented above, we find that the established AETEG neural network model based on LM algorithm effectively avoid the complex analytical model process and lots of nonlinear differential equations, and quickly expresses the mapping relationship between the input and output performance of AETEG, which contributes to the real-time control and simulation for AETEG. When the model is added into the MATLAB/Simulink module, the control strategy for hybrid vehicle application of AETEG can be simulated and optimized for the road testing in further.

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