Research of Oil Film Characteristics of Hydrostatic Turntable under Rotation and Tilt

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Abstract — In view of the influence of rotation and tilt to the lubrication performance of the multi-pad closed hydrostatic turntable which will cause the working table surface deformation and oil film failure when design is not correct. The present study is aimed to establish the oil film Reynolds equation which is solved by the finite difference method and successive over-relaxation iteration algorithm. The oil film characteristics of the circular oil pad and the annular oil pad is numerical analyzed. The oil film characteristics that turntable guide surface and the sealing edge is not parallel is described by the rotation speed and inclination angle. Oil film pressure distribution and velocity contours were obtained by different rotation speed and inclination angle. Oil film thickness and bearing capacity of hydrostatic turntable are obtained with different speeds and loads by experiments. This study provides theoretical basis for the pressure bearing analysis, design and experiment of hydrostatic turntable.

Keywords - Hydrostatic turntable; Reynolds equation; Tilt; Rotation speed

I. INTRODUCTION

Hydrostatic turntable is an important key part of ultra precision and heavy-duty CNC machine tools. It plays a supporting role and makes the workpiece high precision rotary motion. Because of the potential characteristics of low friction high load-carrying capacity and high stiffness, hydrostatic turntables are widely used in heavy machinery, aerospace, marine and other fields. With the development of the research on hydrostatic bearing technology in the depth and width and workpiece machining quality is increasingly high. Due to speed improvement of the hydrostatic turntable and tilt caused by manufacturing error, installation error and structural deformation, workpiece machining error is not negligible.

The bearing capacity of hydrostatic bearing is decreased due to tilt, which may result in the oil film failure. T.J.Prabhu studied the static and dynamic characteristics of rectangular oil pad and annular and conical hydrostatic thrust bearings under rotation and tilt. The influence of bearing capacity, flow, stiffness and damping characteristics of hydrostatic bearing are obtained respectively [1-4]. Nakamura T studied the dynamic and static characteristics of aerostatic rectangular double-pad thrust bearings with compound restrictors in theory and experiment when coupled loads or offset loads are applied and clarified the usefulness of aerostatic thrust bearings with compound restrictors by comparison with the characteristics of conventional aerostatic thrust bearing with feedhole restrictors [5,6]. Van Beek A studied on the effect of the bearing shape function on the self-aligning action of tilted hydrostatic thrust bearings of finite length [7]. Yadav S K studied on the influence of the static and dynamic characteristics of the hydrostatic tilted thrust bearing of various recess shapes [8]. Liu Zhifeng described the nonparallelism degree of turntable guide surface and the sealing edge by inclination angle and got the working parameters of the oil pad and the changing rule of bearing performance with the tilting angle [9]. The rapid increase of the friction power and the increase of the centrifugal force and the temperature caused by speed improvement lead to the decrease of the bearing capacity of hydrostatic bearing. Especially, as the speed increases, the temperature rise is very obvious. Many scholars have done many researches in this field. Zhang Y Q established oil film viscosity-temperature equation with heavy hydrostatic bearing as the research object and revealed the oil film temperature influence rule of hydrostatic bearing on the viscosity and rotation speed. The results showed that the viscosity and rotation speed had a great influence on the hydrostatic bearing oil film temperature rise, but the effect regularity varied [10]. Chen D J established the thermal mechanical error model of high precision machine tool spindle, obtained the variation law of motion error caused by thermal effect during machining process and calculated and analyzed the flow state and thermal characteristics of hydrostatic spindle [11]. Yu X D established a mathematical model of the clearance oil film's three-dimensional flow and boundary condition for the problem of the jumbo size hydrostatic thrust bearing's lubrication in heavy duty vertical CNC equipment, obtained the relationship between rotational speed and temperature of clearance oil film, and gained the regularities of temperature field distribution.
Scholars have made some studies on rotation and tilt of the hydrostatic bearing respectively, which makes the hydrostatic technology become more mature and perfect. However, the research on the influence of the oil pad of constant pressure oil supply with the tilt and rotation is not enough. The Reynolds equation of the hydrostatic turntable is established and is solved by the finite difference method in this paper. The oil film characteristics that turntable guide are established and is solved by the finite difference method.

II. GOVERNING REYNOLDS EQUATION

The Reynolds equation is a differential equation for the fluid pressure in the fluid lubricant film, which is based on the equation of fluid continuity and viscous flow. The continuous equations, momentum and energy equations are obtained by the three basic physical laws of mass conservation, Newton's second law and energy conservation. The continuous equations and momentum equations of compressible fluid in cylindrical polar coordinates are as follows:

\[
\begin{align*}
\frac{\partial p}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \rho v \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \rho w \frac{\partial p}{\partial z} \right) &= 0 \\
\frac{\partial u}{\partial t} + \frac{1}{r} u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + w \frac{\partial u}{\partial z} &= -\frac{\partial p}{\partial r} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} \right) \\
\frac{\partial v}{\partial t} + \frac{1}{r} u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + w \frac{\partial v}{\partial z} &= -\frac{\partial p}{\partial \theta} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
\frac{\partial w}{\partial t} + \frac{1}{r} u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \theta} + w \frac{\partial w}{\partial z} &= -\frac{\partial p}{\partial z} + \frac{\mu}{\rho} \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^2 w}{\partial z^2} \right)
\end{align*}
\]

where \( u, v \) and \( w \) are the radial, tangential and axial components of the velocity. \( p \) is density ( \( \text{kg/m}^3 \)). \( t \) is time ( \( s \)). \( \mu \) is dynamic viscosity ( \( \text{Pa}\cdot\text{s} \)).

The operators \( D_t \) and \( \nabla^2 \) become

\[
\begin{align*}
D_t &= \frac{\partial}{\partial t} + \frac{u}{r} \frac{\partial}{\partial r} + \frac{v}{r} \frac{\partial}{\partial \theta} + w \frac{\partial}{\partial z} \\
\nabla^2 &= \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}
\end{align*}
\]

Reynolds equation is the two order partial differential equation. In the past, it is necessary to obtain approximate solutions with many simplified processing methods, which have great errors. Due to the rapid development of computer technology, numerical solution is used to solve complex problems. For the further study of numerical solution, several hypotheses are put forward:

1. The flow is laminar: no turbulence nor vortex exists, and is Newton's fluid that is in accord with Newton's law of viscosity;
2. The inertia terms and the body forces are negligible compared to the viscous forces, i.e. \( D_u/D_t + D_v/D_t + D_w/D_t = 0 \);
3. There is no relative sliding at the interface of the lubricating film fluid, that is, the velocity of the lubricant is the same as the velocity of the surface;
4. The pressure, as well as the density and the viscosity, may be averaged along \( z \); i.e. it is stated that \( \partial p/\partial z = \partial \rho/\partial z = \partial \mu/\partial z = 0 \);
5. The rest of the velocity gradients are neglected except for \( \partial u/\partial z \) and \( \partial v/\partial z \), because the film thickness is very small along \( z \), the surface tangential velocity \( u \) and \( v \) are far greater than the normal speed \( w \).

According to the above assumptions, The Navier-Stokes equations(Eqn.(2)) are simplified as follows:

\[
\begin{align*}
\frac{\partial p}{\partial r} &= \rho \frac{v^2}{r} - \mu \frac{\partial^2 u}{\partial z^2} \\
\frac{\partial p}{\partial \theta} &= \rho \frac{v^2}{r} - \mu \frac{\partial^2 v}{\partial r^2} \\
\frac{\partial p}{\partial z} &= \rho \frac{v^2}{r} + \mu \frac{\partial^2 w}{\partial r^2} - \mu \frac{\partial^2 w}{\partial z^2}
\end{align*}
\]

Integrating the second of Eqn.(3), with the boundary conditions \( v(0) = 0 \), \( v(h) = \nu \), the tangential velocity of the fluid is obtained:

\[
v = \frac{1}{2\mu} \frac{\partial p}{\rho r h} z^2 + \frac{\nu}{h} - \frac{1}{2\mu} \frac{\partial p}{\rho r} h
\]

The tangential velocity \( V \) is brought into the first of Eqn.(3), and the integral is carried out, with the boundary conditions \( u(0) = 0 \) \( u(h) = u \), and the higher order terms are neglected. The radial velocity of the fluid is obtained:

\[
u = \frac{1}{12\mu} \frac{\partial p}{\rho r h^3} (z^3 - h^2 z) + \frac{z}{h}
\]

If the flow is steady, homogeneous and incompressible fluid, the density \( \rho \) is constant, then the Eqn.(1) can be written as:

\[
\left( \frac{\partial}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial}{\partial z} \right) r u = 0
\]

The Eqn.(6) is integrated in the \( z \) direction and substituting in it Eqn.(4) and Eqn.(5), Reynolds equation of a single oil pad is obtained:

\[
\frac{\partial}{\partial r} \left( r u h^2 \frac{\partial p}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( r^2 u \frac{\partial p}{\partial \theta} \right) = 12\mu w W +
\]

\[
6\mu \frac{\partial}{\partial r} \left( r u h^2 \frac{\partial p}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( r^2 u \frac{\partial p}{\partial \theta} \right) + 3\rho \frac{\partial}{\partial \theta} \left( r u h^2 \right)
\]

where \( W = \dot{h} = w(h) - \dot{w}(0) \).
In Eqn.(7), the boundary conditions of the circular oil pad are $p = 0$ when $r = R_2$, $p = p_s$ when $r = R_1$. The boundary conditions of annular oil pad are $p = 0$ when $r = R_{a1}$ and $r = R_{a2}, p = p_s$ when $r = R_{a3}$ and $r = R_{a4}$.

Without considering the $z$ movement of the hydrostatic turntable, the Eqn.(7) is non dimensional. Let $F = r/R$, $p = p/p_s$, $h = h/H$, $u = u/R_H, \mu = (H^2/p_s)$, $\tau_v = v_R, \mu = (H^2/p_s)$, $\bar{p} = \rho H^2/p_s (R^2 \mu^2)$.

where $R_j$ is the outermost radius of the oil pad. $R_2 = R_j$ when it is circular oil pad. $R_i = R_{a1}$ when it is annular oil pad. $p_b$ is oil recess pressure. $p_s$ is supply pressure, $H_i$ is the oil film thickness of initial design. $H = H_0$ when it is circular oil pad. $H = H_0$ when it is annular oil pad.

The dimensionless is obtained as follows:

$$\frac{\partial}{\partial \theta} \left( \frac{\bar{p}_i}{\tau_v} \right) + \frac{\partial}{\partial \phi} \left( \frac{\bar{h}_n}{\bar{p}} \right) = \frac{3\bar{F}}{20} \frac{\partial}{\partial \theta} \left( \frac{\bar{v}_R}{\tau_v} \right)$$

III. BEARING CAPACITY ANALYSIS

A. The Table Rotation

The circular oil pad is used to support the liquid hydrostatic rotary table, and the annular oil pad is used to preload. Two oil pad structures are shown in Fig.(1).

Bearing oil pad of the hydrostatic rotary table is uniform. In consideration of the influence of rotating speed on the turntable, the oil pad center cannot be used as the center of rotation.

According to a bearing pad for any position (as shown in Fig.(2)), Radial and circumferential velocity vector of oil film in different quadrant is calculated according to the rotational speed of the table. The oil film velocity of contact position between sealing oil edge and working table is obtained. Its dimensionless form is as follows:

$$\bar{u}_v(r, \phi) = -\frac{\Omega R \sin \phi R_i \mu}{H^2 p_s} \quad \bar{v}_v(r, \phi) = \frac{\Omega R \cos \phi R_i \mu}{H^2 p_s}$$

where $R = \sqrt{r^2 + R_j^2 - 2rR_j \cos \phi}$.
For the annular oil pad, the rotation center is in the center of the geometry, so it is not converted. The radial velocity can be neglected to zero, and the circumferential velocity is as follows.

\[ \tau_{0}(r_{y}, \varphi_{y}) = \frac{\Omega \cdot R_{y} \cdot \mu}{H_{0}^{2} \cdot p_{s}} \]

where \( \Omega \) is rotating speed of hydrostatic turntable, \( r_{y} \) and \( \varphi_{y} \) is the radial distance and angle of the oil film node to turntable center respectively.

### B. The Table Tilt

If the partial load is added to the operating table, the table will be inclined (as shown in Fig.(2)), which leads to the uneven distribution of oil film, and then affect the bearing capacity of the turntable. When the working face is inclined, the oil film thickness of the bearing oil pad is as follows:

\[ h_{i} = 1 + \frac{r \cos \varphi \tan \psi}{H_{0}} \]  

The oil film thickness of the preload oil pad is as follows:

\[ h_{p} = 1 + \frac{H_{0}}{H_{0}} - h_{i} \]

where \( \psi \) is the inclined angle of working table of hydrostatic turntable, \( H_{0}^{\varphi} \) is the initial oil film thickness of the preload oil pad.

### C. The Bearing Capacity of Hydrostatic Turntable

According to boundary conditions, the Eqns.(9), (11) and Eqns.(10), (12) are respectively substituted into the Eq. (8). Non dimensional numerical solution of oil film pressure distribution is obtained when the rotary table is under rotation and tilt.

When bearing oil pad is supplied oil for constant pressure pump and capillary orifice is used to compensate, the oil chamber pressure is:

\[ p_{0} = p_{s} - \frac{128 \cdot \mu \cdot Q}{\pi d_{c}^{4}} \]

where \( p_{s} \) is oil pressure, \( Q \) is flow rate, \( d_{c} \) is capillary throttle length, \( d_{c} \) is diameter of capillary.

The bearing capacity of the bearing circular oil pad is as follows:

\[ W = \left\{ \begin{array}{l} \frac{\pi}{h_{i}} \left( 2 \pi r dr + \pi R_{y}^{2} p_{0} \right) \quad (14) \\ \frac{\pi}{h_{y}} \left( 2 \pi r dr + \pi R_{y}^{2} p_{0} \right) \end{array} \right. \]

The bearing capacity of the preload annular oil pad is as follows:

\[ W_{y} = \frac{\pi}{h_{y}} \left( 2 \pi r dr + \pi R_{y}^{2} p_{0} \right) \]

\[ \frac{\pi}{h_{y}} \left( R_{y}^{2} - R_{0}^{2} \right) p_{0} \psi = \frac{W_{y}}{R_{c}^{2} p_{0}} \]

### IV. REYNOLDS EQUATION NUMERICAL SOLUTION METHOD

The Reynolds equation is solved numerically using five point difference scheme. The two order linear algebraic equations are obtained by finite difference of eqn.(8). Because the Reynolds equation is a self adjoint elliptic equation, the difference scheme is symmetric about the point \((i,j)\), and the coefficient matrix of the differential equation is a positive definite symmetric matrix, the super relaxation iteration method is used to solve the equation. The pressure
of oil pad is obtained:

$$
\bar{p}_{i,j} = \frac{(\Delta \phi)^2 C + (\Delta \tau)^2 D}{(\Delta \phi)^2 E + (\Delta \tau)^2 F}
$$

where

$$
C = \tau(e_{i,j})_i \bar{p}_{i+1,j} + \bar{p}_{i-1,j} \right( e_{i-1,j}
$$

$$
6\Delta \phi(\tau_{i,j} - \tau_{i-1,j})(\tau_{i,j} - \tau_{i+1,j}) +
$$

$$
2\Delta \phi((\tau_{i,j})_2 - (\tau_{i,j})_1) \right)^{10}
$$

$$
D = \frac{(\tau_{i,j})_1}{\tau_{i,j-1} + \bar{p}_{i,j+1} \right( e_{i,j}
$$

$$
6\Delta \phi(\tau_{i,j} - \tau_{i-1,j})(\tau_{i,j} - \tau_{i+1,j})
$$

$$
E = \tau(e_{i,j})_i \bar{p}_{i,j-1} + \bar{p}_{i,j} \right( e_{i-1,j}
$$

$$
F = \frac{(\tau_{i,j})_1}{\tau_{i,j-1} + \bar{p}_{i,j+1} \right( e_{i,j}
$$

By the finite difference method, the pressure distribution of the oil film is obtained, and bearing capacity and velocity contours of circular bearing pad and annular preload oil are obtained. The solving process is shown in Fig. (3).

![Fig. (3). Reynolds Equation Solving Process](image)

**V. EXPERIMENTAL RESEARCH**

The experimental apparatus of hydrostatic turntable is shown in Fig.(4), and related parameters are shown in Table 1. Test device includes a test rig, a loading device, an oil supply system, a temperature control cabinet, a control cabinet, a driving system and a test system. The bearing of the rotary table is based on capillary throttling, and loading device adopts the hydraulic loading device to realize the static loading. The eddy current displacement sensor is mounted on the turntable body. Pressure sensor is buried in the oil chamber. The test uses Belgium LMS signal acquisition data analysis and processing system. The experiment contents are: the rotating speed of the turntable is adopted for 10r/min, 60r/min, 110r/min and 160r/min. According to the partial load, deflection angle of the turntable is estimated for 0.20°, 0.20°, 0.20°, and 0.60°. Oil film thickness and oil chamber pressure
are tested by an embedded electric eddy displacement sensor and a pressure sensor. Throughout the testing process, the oil source refrigeration temperature is maintained at 23 degrees or so under the temperature control.

VI. RESULTS AND DISCUSSION

Pressure distribution of oil film sealing edge is affected by the different rotation speed when the inclination angle of working table is $\frac{0.2H_0}{R_e}$, $\varphi = \frac{\pi}{2}$. From the Fig.(5), it can be seen that the greater the speed is, the more obvious the inner concave and the outer convex of oil pad seal oil are. Fig.(6) is speed cloud of oil seal edge under this condition, because of space limitation, cloud images are given only for 60 $r / \text{min}$ and 160 $r / \text{min}$. With the increase of rotational speed, the velocity of oil film increases.

TABLE I. HYDROSTATIC TURNTABLE AND OIL FILM PARAMETERS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of turntable $R_1$/mm</td>
<td>450</td>
</tr>
<tr>
<td>Supply pressure $p_s$/Mpa</td>
<td>1.6</td>
</tr>
<tr>
<td>Viscosity of the lubricant $\mu/(p_s \cdot s)$</td>
<td>0.04</td>
</tr>
<tr>
<td>Density of the lubricant $\rho/(kg/m^3)$</td>
<td>862</td>
</tr>
<tr>
<td>Bearing oil pad inner diameter $R_i$/mm</td>
<td>50</td>
</tr>
<tr>
<td>Bearing oil pad outer diameter $R_s$/mm</td>
<td>75</td>
</tr>
<tr>
<td>Inner shoulder radius of preload oil pad $R_{i2}$/mm</td>
<td>160</td>
</tr>
<tr>
<td>Inner radius of preload oil pad $R_{i1}$/mm</td>
<td>180</td>
</tr>
<tr>
<td>Outer radius of preload oil pad $R_{s2}$/mm</td>
<td>195</td>
</tr>
<tr>
<td>Outer shoulder radius of preload oil pad $R_{s1}$/mm</td>
<td>215</td>
</tr>
<tr>
<td>Initial oil film thickness of bearing oil pad $h_0$/mm</td>
<td>0.05</td>
</tr>
<tr>
<td>Initial oil film thickness of preload oil pad $h_{i0}$/mm</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Fig.5 Pressure Distribution of Circular Bearing Oil Pad with Inclination Angle $0.2H_0/R_e$ under Different Speeds
Fig. 6 Velocity Cloud of Circular Bearing Oil Pad with Inclination Angle

0.2H_0 / R_i under Different Inclination Angles

Fig. 7 Pressure Distribution of Annular Preload Oil Pad with Rotation Speed 110r / min under Different Inclination Angles

Fig. (7) is pressure distribution of sealing oil edge of annular preload oil pad with rotation speed 110r / min when inclination angles is 0.2H_0 / R_i and 0.6H_0 / R_i. With increase of inclination angle, the pressure pressed side is increased and the other side is reduced. As rotary center of annular oil pad is the geometric center, it is not obvious that pressure of seal oil is changed, the pressure caused by the seal oil is edge is not obvious. Due to space limitations, here is omitted.

Fig. (8) is theoretical and experimental comparison curves of recess pressure of circular oil pad with the angle variation under different rotational speeds. The pressure of oil recess decreases with the increase of inclination angle.
The curve is flat when tilt angle is small, and then the curve is reduced in a straight line. The theory simulation results match very well with the experimental results, the maximum difference is 4.46%. Oil recess pressure decreases by 19.3% through experiment test when rotating speed is 110r/min, and decreases by 17.5% through theoretic analysis. Fig.(9) is theoretical and experimental comparison curves of recess pressure of circular oil pad with rotational speeds variation under different inclination angles. With increase of rotational speed, bearing capacity decreases. The maximum difference of bearing capacity is 3.11% through experiment and theory.

Bearing capacity decreases by 6.22% through experiment test when inclination angle is 020.4 / H, and decreases by 4.85% through theoretic analysis.

VII. CONCLUSIONS

In this paper, the Reynolds equation under rotation and tilt is established by cylindrical coordinate form of fluid control equations. The influence of rotation and tilt on pressure distribution of seal oil edge is studied about circular oil pad and annular oil pad of closed hydrostatic turntable. The influence law of rotation and tilt of table on pressure and load carrying capacity of turntable is analyzed. The results show that the effect of rotational speed on bearing capacity is relatively small without considering the temperature. The bearing capacity decreases by 4.85% at the same inclination angle. Inclination degree is more affected by bearing capacity. The oil recess pressure decrease by 17.5% at the same speed. From the above we know, speed increase and working table tilt will reduce bearing capacity of the turntable, the influence of tilt is more serious.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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REFERENCES


