Cycle Slip Detection Based on Carrier Phase Difference Series for Dual-Frequency GPS Receivers

Zhige Jia¹, Zhengsong Chen¹, Dijin Wang², Pengfei Yu²

¹ Key Laboratory of Earthquake Geodesy, Institute of Seismology, China Earthquake Administration, Wuhan, Hubei 430071, China
² Wuhan Base of Institute of Crustal Dynamics, China Earthquake Administration, Wuhan, Hubei 430071, China

Abstract —The GPS position based on carrier phase measurement is produced and developed for the exaltation of positioning accuracy. However, in practice, carrier phase observations often includes various errors besides essential parameters. Sometimes even satellite signals may be temporarily interrupted and lead to cycle slips. Detection and correction of cycle slip are specific issues in GPS carrier phase measurements. As a gross error, cycle slips decreases badly positioning accuracy but can’t removed by differencing approach. For all this, based on the feature of cycle slips and time series analysis theory, a simple and convenient method of detecting and correcting cycle slips is presented in this paper for high accuracy position solution. The key of this method is to build difference series between carrier phase observations of satellites and stations, which is used to eliminate various errors implied in observation data, and highlight the features of cycle slips. Based on the time-series analysis theory, the model of difference series of carrier phase observation data is built, cycle slips can be quickly detected by the stability of this time series. At last, the experiment results show that the new method can not only be suitable both for static and dynamic measurements of dual-frequency GPS receivers, but also can eliminate most measurement errors of dual-frequency GPS receivers, improve the efficiency of detection and correction of cycle slips less than 1 cycle.

Keywords - GPS; Dual-frequency receiver; Carrier phase difference; Time series analysis; Cycle slip

I. INTRODUCTION

Detection and correction of cycle slip are specific issues in GPS carrier phase measurements [1]. Whatever for static or dynamic observations, cycle slips often occurs, and belong to random error and often lead to a same error of integer ambiguity is contained in the subsequent phase observation values, and decreases badly positioning accuracy [2]. For now, there are several methods available for the cycle slip detection and repairing, such as combined pseudorange and phase method, ionospheric residual method, polynomial fitting method, filter method and so on [3,4]. Ability of combined pseudorange and phase method to detect cycle slip depends on the precision of pseudo-range measurement, and often needs P-code pseudoranges. The privacy of P-code restricts application range of this method [5]. Ionospheric residual method needs dual-frequency carrier phase observation values and is suitable for single-frequency receivers. And Inference of ionospheric residual method is not perfect and can’t avoid the multi-valued resolutions of cycle slip, so which can only repair cycle slips less than 5 cycles [6]. Polynomial fitting method is suitable for single-frequency receivers and can only repair cycle slips larger than 5 cycles, so which can’t satisfy the need of high-accuracy positioning[7]. Kalman filter method is of the recursive version of linear minimum variance estimate, and more suitable for static measurements [8, 9].

Above all, most of existing methods are suitable to detect the greater cycle slips and could not meet the requirements both on static and dynamic measurements. In practice, carrier phase measurement is the signal phase changes on the transmission path. Carrier phase observations often include various errors besides essential parameters, such as the atmosphere delay errors, multi-path effect. If there are cycle slips with greater than 10 cycles in carrier phase observation values, which can be find and deleted in the preprocessing stage. But if cycle slips are less than 10 cycles, especially about 1–5 cycles, a half cycle or a quarter cycles, which are hardly finding [10]. In this situation, even if there are several weeks of irregular variation in phase observation values, it’s difficult to decide whether cycle slip exists. For all this, for detecting cycle slip, especially little cycle slip less than 1 cycle, we must first remove various errors far less than 1 cycle, such as clock error, ionospheric delay error.

Based on the feature of cycle slips in carrier difference phase observations, this paper presents a quick method of cycle slip detection, even less than 1 cycle by difference series between carrier phase observations of adjacent epochs. Thus the key of the presented method is to build difference series of carrier phase observations in order to eliminate various errors implied in observation data and highlight the features of cycle slips. Then cycle slips are detected and corrected, so that the subsequent positioning solution can obtain high precision positioning result. The presented method is suitable both for static and dynamic measurements of dual-frequency GPS receivers.

The rest of this paper is organized as follows. Section 2 gives building of difference series of carrier phase observation data and its applications on test cycle slips and PCOs. Section 3 discusses the time series analysis model of
carrier phase differences and its stability test, type
identification, order determination, parameter estimation and
a executable algriothm. Section 4 presents two examples to
verify the presented method of the time series analysis
model of carrier phase differences obtained by dual-
frequency GPS receivers. Lastly, Section 5 concludes the
paper.

II. DIFFERENCE SERIES OF CARRIER PHASE
OBSERVATIONS

Based on the feature of carrier difference phase of Carrier
L1 and L2, most of errors inherent to GPS positioning can be
eliminated with differencing approach. For carrier phase
observation of dual-frequency receivers, there are many
modes of differences between satellites, receivers and
observation of dual-frequency receivers, there are many
be eliminated with differencing approach. For carrier phase
measurements from satellite

A. Base observation equation of carrier phase measurement

Carrier phase measurement is the signal phase changes on
the transmission path. The measured carrier phase is the
difference between a receiver-generated replica of nominal
phase and the received carrier phase as determined by the
receiver’s phase-lock loop [11].

Reference [12] gives the base observation equation of
carrier phase measurement. Analogously, the carrier phase
measurements from satellite \(i\) observed at antenna \(P\) on
frequency \(m\) and epoch \(t_1\) are modeled as:

\[
\phi_{\text{r},P}(t_i) = \lambda_{\text{m}}(r_i^P(t_i) - c(\delta_P + \delta' +) - IT_i^P(t_i)) + d_i(t_i) + d'(t_i) + e_{\text{r},P}(t_i)) + N_{\text{r},P}(t_i)
\]  

with wavelength \(\lambda_{\text{m}}\), speed of light \(c\), distance \(r_i^P(t_i)\) from
the satellite \(i\) to the receiver antenna \(P\), receiver clock
offset \(\delta_P\), satellite clock offset \(\delta'\), ionosphere and
troposphere delays \(IT_i^P(t_i)\), receiver hardware delay \(d_i(t_i)\),
satellite hardware delay \(d'(t_i)\), carrier phase integer
ambiguity \(N_{\text{r},P}(t_i)\), residual measure errors \(e_{\text{r},P}(t_i)\) due to
carrier phase multipath, thermal noise.

B. Single-difference between GPS stations

The distance of two adjacent stations is often short, so the
difference of the troposphere and ionosphere effects
between stations is sufficiently small to be neglected [13].
The same satellite \(i\) is observed by two receivers \(P\) and \(Q\),
the differences of the error associated with satellite clock
offset and hardware delay are small enough that we also can
neglect them. Hence, the single-difference carrier phase
from satellite \(i\) observed at two adjacent receivers \(P\) and \(Q\)
on frequency \(m\) and epoch \(t_1\) is:

\[
SD_{\text{r},P,Q}(t_i) = \lambda_{\text{m}}(r_i^Q(t_i) - r_i^P(t_i)) - c(\delta_P - \delta_Q + \delta') - IT_i^Q(t_i) + d_i(t_i) + d'(t_i) + e_{\text{r},P}(t_i)) + e_{\text{r},Q}(t_i) + N_{\text{r},P}(t_i)) + N_{\text{r},Q}(t_i))
\]  

where \(r_i^P(t_i) = r_i^Q(t_i)\), \(\delta_{\text{r},P} = \delta_P - \delta_Q\),
\(d_{\text{r},P}(t_i) = d_i(t_i) - d_Q(t_i)\), \(e_{\text{r},P}(t_i) = e_{\text{r},P}(t_i) - e_{\text{r},Q}(t_i)\) and
\(N_{\text{r},P}(t_i) = N_{\text{r},Q}(t_i) - N_{\text{r},Q}(t_i)\).

C. Double-difference between GPS satellites

Double-difference between two GPS satellites can be
used to remove receiver-generated common-mode errors

\[
\frac{1.4 \times 10^{-7} \times (r_i^Q(t_i) - r_i^P(t_i))}{\lambda_{\text{i}}} < 0.015\ , \text{ the carrier phase value}
\]  

of \(L_1\) is about 0.012 cycle. Hence, for near relative
positioning, the effect of \(r_i^Q(t_i) - r_i^P(t_i)\) on single-difference
observation value does not need to take into account.
from the single differences between two receivers [14]. For instance, the error associated with the receiver clock (Error! Reference source not found.) can be approximately eliminated by differencing the phases measured from satellite \( i \) with that from satellite \( j \) at a given measurement interval. As shown in Fig. 1, the double-difference carrier phase from satellites \( i \) and \( j \) observed at two antennas \( P \) and \( Q \) on frequency \( m \) and epoch \( t_1 \) takes the form:

\[
DD_{\text{PQ,A}}^{\text{i,j}}(t_1) = \lambda_n^i e_{\text{PQ,A}}^{\text{i,j}}(t_1) + N_{\text{PQ,A}}^{\text{i,j}}(t_1)
\]

(4)

Analogously, the double-difference carrier phase for two antennas \( P \) and \( Q \) on continuous epoch \( t_2 \) is:

\[
DD_{\text{PQ,A}}^{\text{i,j}}(t_2) = \lambda_n^i e_{\text{PQ,A}}^{\text{i,j}}(t_2) + N_{\text{PQ,A}}^{\text{i,j}}(t_2)
\]

(5)

III. ANALYSIS ON INFLOW PERFORMANCE OF THE OIL WELL

A. Detection of cycle slips

Systematic errors implied in GPS observations must almost eliminated by proper data-processing in order to test cycle slips. Observation value series without systematic errors is more suitable for testing cycle slips. If there don’t exist cycle slips, testing series will change gently with time. Based on the gross error detection theory, observations are grouped a proper testing series, and cycle slip presents as gross error of this testing series. The testing of cycle slip is to compare the difference between change of testing series and evaluation criteria [15]. If the difference is in the range of index, then there don’t exist cycle slip, otherwise cycle slip exists and the position and size of which must be determined and used for correcting baseline solution data.

For dual-frequency carrier phase observation values, the double-differences of carrier phase observations of \( L_1 \) and \( L_2 \) on epoch \( t_1 \) are:

\[
DD_{\text{L1,A}}^{\text{i,j}}(t_1) = \lambda_n^i e_{\text{L1,A}}^{\text{i,j}}(t_1) + N_{\text{L1,A}}^{\text{i,j}}(t_1)
\]

(6)

\[
DD_{\text{L2,A}}^{\text{i,j}}(t_1) = \lambda_n^i e_{\text{L2,A}}^{\text{i,j}}(t_1) + N_{\text{L2,A}}^{\text{i,j}}(t_1)
\]

(7)

In theory, \( DD_{\text{L2,A}}^{\text{i,j}}(t_1) = DD_{\text{L1,A}}^{\text{i,j}}(t_1) \), thus

\[
N_{\text{L2,A}}^{\text{i,j}}(t_1) - \frac{\lambda_n^i}{\lambda_i} N_{\text{L1,A}}^{\text{i,j}}(t_1)
\]

Equation (4) can then be simplified as:

\[
N_{\text{L1,A}}^{\text{i,j}}(t_1) - \frac{\lambda_n^i}{\lambda_i} N_{\text{L2,A}}^{\text{i,j}}(t_1) = \Omega(t_1) + u(t_1)
\]

(11)

If the signal from satellite \( i \) and \( j \) observed at receiver \( P \) and \( Q \) on epoch \( t_1 \) is continuous and there are without cycle slip in the observing process, \( N_{\text{L1,A}}^{\text{i,j}}(t_1) \) and \( N_{\text{L2,A}}^{\text{i,j}}(t_1) \) remain a constant integer over time. This is, the left of Equation (4) is a constant. If \( u(t_1) = 0 \), the computed values of \( \Omega(t_1) \) should be a constant. But because \( e_{\text{L1,A}}^{\text{i,j}}(t_1) \) and \( e_{\text{L2,A}}^{\text{i,j}}(t_1) \) are residual errors computed by double-differencing approach, values of which are very small values with the same positive or negative sign. And Error! Reference source not found. is equivalent to the difference between \( e_{\text{L1,A}}^{\text{i,j}}(t_1) \) and \( e_{\text{L2,A}}^{\text{i,j}}(t_1) \), value of which is more small, and variation from adjacent epochs is not often big. Then variation of the computed values of \( \Omega(t_1) \) from two adjacent epochs must be very small. If this value exists break at a certain epoch, a cycle slip is considered to be existing.

According to the above analysis, residual error \( \Omega(t_1) + u(t_1) \) is mainly phase measurement errors because troposphere and ionosphere effects, errors associated with satellite clock offset and hardware delay, receiver-generated common-mode errors are eliminated by differencing between carrier phase observation values of two adjacent stations and satellites. Then \( \Omega(t_1) + u(t_1) \) can be seen as a random parameter following normal distribution. This is, statistics of cycle slip detection is defined by
Based on error propagation law, \( \sigma_v^2 = 4\sigma_{\phi_1}^2 + 4\left(\frac{\lambda_2}{\lambda_1}\right)^2\sigma_{\phi_2}^2 \).

If the medium errors of \( \phi_1 \) and \( \phi_2 \) are equal to 0.01 cycle, then the medium error of \( v \) is equal to 0.023, \( \sigma_v = 0.032 \). According to the criteria of triple medium errors, the maximum error is 0.097 cycle. If \( |v| \leq 3\sigma_v = 0.097 \), then it can be determined that without cycle slip has happened from epoch \( t_1 \) to \( t_2 \). Otherwise, there exist cycle slips in Carrier \( L_1 \) or \( L_2 \), which should be corrected.

**B. Correction of cycle slips**

The value \( \hat{N} \) of cycle slip can be computed by the prediction value \( \hat{L}(n+1) \) of carrier phase measurement value \( L(n+1) \) when it exists.

\[
\hat{N} = \text{Int}[\hat{L}(n+1) - L(n+1)]
\]  
(13)

Based on the value of cycle slip \( \text{Error! Reference source not found.} \), from the \( n+1 \) epoch, the carrier phase measurement value minus \( \text{Error! Reference source not found.} \) respectively, and the fractional portion less than 1 cycle remain unchanged in order to achieve cycle slip correction. Based on the feature of cycle slip, cycle slips can be detected and corrected by the time these jumps presented.

**IV. MODELING OF DIFFERENCE TIME SERIES ANALYSIS**

Motion track of GPS satellite in space is a characteristic curve, so almost errors in the measurement process change gradually and without skipping. Change of measurement noise is not very gentle but under a certain scope. Thus normal carrier phase measurements change regularly with time, and the variation of which presents a smooth curve with time. If without cycle slip, quadruple-difference series of carrier phase measurements is almost caused by random errors of receiver oscillators, and present the statistic characteristic of accidental errors [7]. In view of nonexistence of the correlation between integer ambiguity and time, if there exists a cycle slip, which will break the smoothness of curve of carrier phase observation value. And from now on, equivalent jumps are produced in subsequent phase observation series in succession. Checking if cycle slips exist is to proving the absence of such jumps in phase observation series.

For standing out base change of cycle slips from observation value series, it is necessary to eliminate fully various other errors implied in observation series before cycle slips are detected. For the features of carrier phase observations, based on the time-series analysis theory, the model of carrier phase observation data is built for detecting cycle slips, if test value of cycle slips is produced sufficiently with the stability and the sensitivity.

**A. Time series analysis model**

As a function of time, carrier phase observation values present a smooth curve against time. Based on the time-series analysis method, the model of carrier phase observation data is built as shown in Fig. 2.

**B. Run test on stability of time series**

Data should be stable in order to build the model of carrier phase observation data, so it is necessary to test the stability of carrier phase observation data. Runs test method of statistical analysis theory only needs one group data measured actually but doesn’t need the hypothesis of data distribution law [16], so that is used to test the stability of carrier phase value measured actually.

Let \( n_1 \) and \( n_2 \) denote times two signals appeared, \( R \) denotes the number of runs. Two-tailed test of significance level \( \alpha \) is performed and upper and lower bounds of probability distribution are determined. Then the stability of time series data is determined based on the value whether within or without acceptance domain.

If \( n_1 \) and \( n_2 \) exceed over 20, the sampling distribution of \( R \) meets approximation will be approximately normal distribution. At this time, the statistics \( Z \) is set as

\[
Z = \frac{R - \mu_z}{\sigma_z}
\]  
(14)

\[
\mu_z = \frac{2n_1 n_2}{n_1 + n_2} + 1
\]  
(15)
For the significance level with $\alpha = 0.05$, if $-1.96 \leq Z \leq 1.96$ (according to $2\sigma$ criterion), then the original hypothesis can be accepted and the tested series is stable; otherwise the original hypothesis will be refused.

C. Positioning solution algorithm on carrier phase difference series

After a series of above-mentioned pre-processing, positioning solution can be implemented in order to obtain high precision positioning result. Based on the time series analysis method, positioning solution algorithm of carrier phase observations is achieved as shown in Fig.3.

IV. EXPERIMENT RESULT ANALYSIS

In order to verify the presented method of the time series analysis model of carrier phase differences obtained by dual-frequency GPS receivers, precision position solution experiments are carried out in two ways: static and dynamic measurements.

A. Test of static observation data

The test was performed at the GPS Standard Baseline Site of Institute of Seismology, China Earthquake Administration. The equipment of reference and test stations include two dual-frequency GPS receivers Topcon Net G3A and two matching antennas TPSCR.G3. The baseline lengths are about 1km. Their observation interval is 30s with the elevation cut-off angle of 10°. The tests require carrying out the continuous static test lasting for 24h (one sidereal day). In this way, the observation results can be analyzed statistically. In the subsequent data processing, all collected data is obtained by receiver Topcon Net G3A with the standard RINEX format.

Differences of carrier phase observations: As mentioned, for a dual-frequency GPS receiver, because distance between the reference and test stations is about 20km, the troposphere and ionosphere delay effects does not need to take into account. the error associated with satellite clock offset and hardware delay can be eliminated by single-difference of carrier phase observations obtained by GPS receivers of the reference and test stations as the formula (3). And common-mode errors generated by receivers, such as receiver clock, can be approximately eliminated by double-difference of carrier phase observations from two different satellite signals as the formula (4). Based on the results of double-difference of carrier phase observations, cycle slip can be detected as the formula (12) and corrected as the formula (13). Then as the formula (15), antenna PCOs can be computed and corrected by triple-difference of carrier phase observations between adjacent epochs.

Conformation GPS observable into triple-difference quantities are needed in different epochs. Because of the sample interval of Topcon Net G3A is 30 seconds, so triple-differences almost agrees with the model of quadratic order polynomial, and the median error is about 1.6cm, which is far less than the carrier wavelength of 19 or 24cm. Fig.4, and Fig.5 shows the single- and double-differences of L1 carrier phase observations, respectively.
Detection of cycle slips: For present frequency-domain or time-domain methods, integer week ambiguity is rarely considered, but directly utilize common schema provided by TEQC software[17]. With this in mind, we have focused our attention on detection and elimination of cycle slip error by epoch subsection of simultaneously stationary observation with one sidereal day as Fig.3.

With observations of satellite G7, G18 on the first 360 epochs, for example, carrier phase difference of each epoch with no cycle slip is drawn in Fig.6. In our study, the above step can not find cycle slips. For verifying the presented method, one 1 cycle slip, one 0.5 cycle slip and one 0.1 cycle slip are added into the first 101 epochs observation time series of satellite G1 respectively. Double differences of L1 carrier phase observations with cycle slips are drawn in Fig.7~9.

As shown in Fig.6, double-difference variation of carrier phase observations presents a certain periodicity and smoothness. The maximum of differences between adjacent epochs is 0.105 cycle, and consistent with the previous theoretical derivation.
Fig. 8. Reconstruction of L1 carrier phase observations with 0.5 cycle slip.

Fig. 9. Reconstruction of L1 carrier phase observations with 1 cycle slip.

Fig. 7, Fig. 8 and Fig. 9 show the detail signals which is reconstruct by double differences of L1 carrier phase observations, respectively. The results of Fig. 7, Fig. 8 and Fig. 9 show the existence of cycle slips destroy continuity of the original curve, and lead to obvious jumps on the curve. Based on the time these jumps presented, cycle slips can be detected and corrected for subsequent accuracy positioning solution.

B. Test of dynamic observation data

Dynamic data is observed by two TPS EUROCARD dual-frequency receivers. Static observation is performed on the reference station and dynamic observation is performed by the vehicle GPS way. The baseline lengths are about from 2km to 20km. Their observation interval is 5s with the elevation cut-off angle of 10°. Because the computing process on convergence of GPS satellite, detection of multi-path effect, correction of antenna PCOs of dynamic observation data is similar to the static observation data, which are omitted on account of space limitation. Here we’re only looking at detection of cycle slips of dynamic observation data.

With observations of satellite G1, G15 on the first 360 epochs, for example, carrier phase difference of each epoch with no cycle slip is drawn in Fig. 10. Multiple different-sized cycle slips are added into observation time series respectively, the values and times of which are shown in Tab. I. shows cycle slips. Double differences of L1 carrier phase observations with cycle slips are drawn in Fig. 11–Fig. 16.

<table>
<thead>
<tr>
<th>Epoch</th>
<th>101</th>
<th>201</th>
<th>301</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite</td>
<td>G1</td>
<td>G1</td>
<td>G1</td>
</tr>
<tr>
<td>Cycle slip</td>
<td>0.1,0.5,1</td>
<td>0.1,0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

As shown in Fig. 10, there is without an obvious periodicity for double-difference variation of carrier phase observations because of dynamic observation environment. The maximum of differences between adjacent epochs is 0.09 cycle. When there have no cycle slips in the data, the detail signal which is reconstruct by double differences is stationary.

Fig. 10. Reconstruction of L1 carrier phase observations with no cycle slip.
Fig. 11. Reconstruction of L1 carrier phase observations with 1 cycle slip.

Fig. 12. Reconstruction of L1 carrier phase observations with 0.5 cycle slip.

Fig. 13. Reconstruction of L1 carrier phase observations with two 0.5 cycle slips.

Fig. 14. Reconstruction of L1 carrier phase observations with 0.1 cycle slip.

Fig. 15. Reconstruction of L1 carrier phase observations with two 0.1 cycle slips.
The results of Fig.11~Fig.16 show the existence of cycle slips destroy continuity of the original curve, and lead to obvious jumps on the curve. Double-difference variation of carrier phase observations are sensitive to even small cycle slip, such as 0.1 cycle slip. So cycle slips can be detected and corrected by the time jumps presented.

VI. CONCLUSION

In this paper we have presented a simple and convenient method of detecting and correcting cycle slips is presented in this paper for high accuracy position solution. If there is no cycle slip in carrier phase difference of each epoch, double-difference variation of carrier phase observations should present a certain stationary curve. The benefit of the proposed method is that it is able to eliminate various errors implied in observation data, such as satellite clock offset, the atmosphere delay errors, and even correct PCOs of GPS antenna, and to highlight the features of cycle slips. Once other various errors are eliminated, the time series of differences can be regarded as a stationary process. Thus, we can detect quickly less than 1 cycle based on the variation characteristics of difference series between carrier phase observations of adjacent epochs and time series analysis theory. Based on the feature of cycle slips, cycle slips can be determined and corrected by the time these jumps presented. The experiment results show the new method is suitable both for static and dynamic measurements of dual-frequency GPS receivers.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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Fig.16. Reconstruction of L1 carrier phase observations with three 0.1 cycle slips.