Controller Adjustment Method by Frequency-Domain Fictitious Reference Iterative Tuning using Sliding DFT

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Abstract — In this paper, we propose a method for online controller adjustment by fictitious reference iterative tuning (FRIT) of a frequency domain using sliding discrete Fourier transform (DFT). We have extended the FRIT to correspond time delay and characteristic variation of a control object. A sliding DFT was used to correspond to the time delay of the controlled object. In addition, we have optimized the method using a recursive least-squares algorithm to determine the controller gains online. In this study, we utilized an algorithm that was designed in a two-degree-of-freedom (2DOF) control system. In simulation and actual device, we have experimentally verified the effectiveness of the proposed method. The proposed method can correspond to the time delay of a controlled object. In addition, it can also correspond to characteristic variations of a control object.

Keywords- Direct controller tuning; fictitious reference iterative tuning; sliding discrete Fourier transform; recursive least squares; adaptive algorithm

I. INTRODUCTION

Currently, control engineering practices are used in various industrial fields. A typical method is the proportional-integral-derivative (PID) controller. Because three control parameters constituting a PID controller are simple and easy to understand, the control structure has many industrial uses. However, there is a need for trial and error and skilled techniques to obtain an ideal control characteristic in a normal PID controller.

To improve industry-adjustment methods that may save time and costs of controller operations, appropriate design is required. Although this may be achieved using complex mathematical models, the development of a method for adjusting PID parameters (such as gain) using output data directly is desirable.

One such method utilizes fictitious reference iterative tuning (FRIT) [1], which is performed on the basis of input and output data by experiments only once and thus does not require iterative experimentation. This method is intuitive and easy to understand, and thus, many studies have been reported in the literature [2],[3]. However, because the standard FRIT is performed offline, control performance is degraded when characteristics of a controlled object change after controller design, thus requiring changes after each readjustment. Moreover, because this is performed in a standard FRIT, the time domain corresponding to the control object, including the dead time, is insufficient.

In this study, we utilized an algorithm that was designed in a two-degree-of-freedom (2DOF) control system [4]. To correspond to characteristic variations of a control object, the proposed method performs self-tuning online [5],[6] using a recursive least squares (RLS) algorithm. In addition, to correspond to the time delay of the controlled object, using a sliding DFT [7], input and output data were obtained in the frequency domain.

II. FRIT IN A 2DOF

The proposed controller is a method of using the input and output data to adjust several parameters, such as the PID gain, directly without using a mathematical model of the controlled system. The method utilizes FRIT, which is executed in an intuitive and easy to understand manner based on the input data obtained from a single experiment.

Consider the 2DOF control system illustrated in Fig. 1, where P(z) is the plant discretized with a zero-order hold, T_d(z) is the reference model, u(k) and y(k) are the input and output of the plant, respectively r(k) is the reference signal, C_f(z) is the feedback controller, and C_f(z,p) is the feedforward controller.
Let the tunable parameter of the feedforward controller be $\rho$. The transfer function of the system shown in Fig. 1 is

$$
W(z) = \frac{Y(z)}{R(z)} = \frac{P(z)(C_d(\rho, z)+T_d(z)C_{fb}(z))}{1+P(z)C_{fb}(z)},
$$

(1)

where $T_d(z)$ and $C_{fb}(z)$ are assumed to be the known parameters. Assuming that the system is consistent with a reference model $T_d(z)$ as follows:

$$
T_d(z) = \frac{P(z)(C_d(\rho, z)+T_d(z)C_{fb}(z))}{1+P(z)C_{fb}(z)},
$$

(2)

The output of the plant, $y(k)$ may be defined as,

$$
y(k) = T_d(z)r(k).
$$

(3)

By performing a one-shot experiment using an initial parameter of the feedforward controller $\rho_0$ in the 2DOF control system illustrated in Fig. 1, the initial data for the input and output $u_0(k), y_0(k) (k=1,\cdots,N)$ are obtained.

The fictitious reference signal is calculated by

$$
\tilde{r}(\rho,k) = \frac{u_0(k)+C_{fb}(\rho, z)y_0}{C_d(\rho, z)+T_d(z)C_{fb}(z)}.
$$

(4)

Application of the $\tilde{r}(\rho,k)$ term to (2), produces the relationship as follows:

$$
T_d(z)\tilde{r}(\rho,k)
$$

$$
= \frac{P(z)(C_d(\rho, z)+T_d(z)C_{fb}(z))}{1+P(z)C_{fb}(z)} \times \frac{u_0(k)+C_{fb}(\rho, z)y_0}{C_d(\rho, z)+T_d(z)C_{fb}(z)}
$$

$$
= \frac{P(z)u_0(k)+P(z)C_{fb}(\rho, z)y_0}{1+P(z)C_{fb}(z)}.
$$

(5)

From the relation

$$
y_0(k) = P(z)u_0(k)
$$

(6)

and (2), we also see that (5) can be written as

$$
T_d(z)\tilde{r}(\rho,k) = y_0(k).
$$

(7)

Based on the fictitious reference signal, the controller parameters are tuned such that the following loss function is minimized as follows:

$$
J = \|y_0(k)-T_d(z)\tilde{r}(\rho,k)\|^2.
$$

(8)

### III. FRIT IN THE FREQUENCY DOMAIN

We propose a control system design method by FRIT, which calculates the controller parameter using a sliding DFT, and an RLS algorithm in the frequency domain to obtain an appropriate response even if the parameter of interest is unknown.

The feed-forward controller is defined as

$$
C_d(z) = \frac{\rho_1 z^2 + \rho_2 z + \rho_3}{z^3 + a_{w1}z + a_{w0}} = a(z)^T \rho,
$$

(9)

Where

$$
\rho = [\rho_1 \ \rho_2 \ \rho_3]^T,
$$

(10)

$$
a(z) = \frac{1}{z^2 + a_{w1}z + a_{w0}}[z^2 \ \ 1]^T.
$$

(11)

From Fig. 1, the fictitious reference trajectory of the 2DOF system can also be written as

$$
\tilde{r}(\rho,k) = C_d(\rho, z)^{-1}u_0(k)\chi.
$$

(12)

When we apply $\tilde{r}(\rho,k)$ to (12), we have

$$
J = \|C_d(\rho, z)^{-1}u_0(k) - T_d(z)u_0(k)\|^2.
$$

(13)

From (13), the new loss function is defined as

$$
\hat{J} = \|C_d(\rho, z)Y_0(k) - T_d(z)u_0(k)\|^2.
$$

(14)

For convenience, the loss function of (14) may be replaced with

$$
U(j\omega)\otimes T_d(j\omega)U_d(j\omega)
$$

$$
= C_d(\rho, z)Y_0(j\omega) = \phi(j\omega)^TW,
$$

(15)

where

$$
\phi(j\omega)^T = Y_0(j\omega)a(z)^T,
$$

(16)

$$
W = \rho.
$$

(17)
Then, determine the parameter $\rho$ so as to minimize the cost function
\[ J = \| \Psi - \Phi \|^2, \] (18)
where
\[ \Psi = [U(j\omega_1) \cdots U(j\omega_d) U(-j\omega_1) \cdots U(-j\omega_d)]^T, \] (19)
\[ \Phi = [\varphi(j\omega_1) \cdots \varphi(j\omega_d) \varphi(-j\omega_1) \cdots \varphi(-j\omega_d)]^T. \] (20)

In this study, we obtained the estimated values $\hat{W}$ that minimize the loss function using an RLS algorithm.

When the initial stages and parameters of the plant change suddenly, the parameter of the feedforward controller tends to change significantly, causing a reduction of stability and degradation of the system control performance. To suppress such a change in the control parameters, the parameter is updated through the equation as follows:
\[ \rho(k) = W(k) = (1 - \alpha)W(k-1) + \alpha \hat{W}(k-1), \] (21)
where $\alpha$ is a positive number of a sufficiently small size. Using (21), is subjected to a low-pass filter $\alpha/(z + \alpha - 1)$, and the parameter of the feedforward controller changes slowly.

Fig. 2 shows a block diagram of the proposed method.

IV. EXPERIMENTAL RESULTS

In this section, the effectiveness of the proposed method is explained. A square-wave signal was assumed as the input. The controlled plants were discretized by a zero-order hold model with sampling time $T_s = 1$ ms. The parameter of a sliding DFT was $N = 512$. There were five frequency points of odd higher harmonics. The initial value of the covariance matrix of the RLS algorithm was $1000*I$. The control system was simulated using MATLAB/Simulink.

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A. First-order plant including time delay

This section describes the simulation results of the target-value responses for a first-order plant. Consider the first-order plant including time delay as follows:
\[ P(s) = \frac{K}{1 + Ts} e^{-st}, \] (22)
where $K = 741$, $T = 13$, and $L = 0.05$. The reference model is given as follows:
\[ T_d(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}, \] (23)
where $\omega_n = 10$ and $\zeta = 1.0$. Furthermore, the PID gain of the feedback controller was
\[
\begin{align*}
K_p &= 0.4 \\
K_i &= 0.2 \\
K_d &= 0.01.
\end{align*}
\] (24)

The parameters of feed-forward controller $d_{ref1}$ and $d_{ref2}$ are coefficients of a denominator polynomial of the discretized reference model. The initial value of the feedforward gain was $\rho_0 = [0 \ 0 \ 0]^T$. The parameter of the low-pass filter was $\alpha = 10^{-4}$. An input-side step disturbance of magnitude $-0.1$ was applied at time $t = 132$ s. The simulation results for the output responses are shown in Fig. 3, and the expanded responses in $t = 150–200$ are shown in Fig. 4. The transition of the feedforward gain is shown in Fig. 5.

On the basis of the results, the initial output of the plant differed greatly from the reference model output. However, the output of the plant approached to the reference model output as time increased, and at approximately, $t = 100$ s, the output of the plant followed the reference model output. Fig. 4 shows that the output of the plant followed the reference model output even after applying the disturbance.
B. Second-order plant

This section describes the simulation results of the target-value responses for a second-order plant. Consider the second-order plant as follows:

\[ P(s) = \frac{b}{s(s+a)} , \]  

(25)

where \( a = 9.66 \) and \( b = 82.2 \). The reference model is given as follows:

\[ T_d(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} , \]  

(26)

where \( \omega_n = 10 \) and \( \zeta = 1.0 \). Furthermore, the PID gain of the feedback controller was

\[
\begin{align*}
K_p &= 5.5 \\
K_i &= 0.98 \\
K_d &= 6.8
\end{align*}
\]  

(27)

The parameters of feed-forward controller \( a_{ref} \) and \( a_{ref0} \) are coefficients of a denominator polynomial of the discretized reference model. The initial value of the feedforward gain was \( \rho_0 = [1 \quad -2 \quad 1]^T \). The parameter of the low-pass filter was \( \alpha = 10^{-4} \). An input-side step disturbance of magnitude \(-0.1\) was applied at time \( t = 67 \) s. The simulation results for the output responses are shown in Fig. 6. The transition of the feedforward gain is shown in Fig. 7.

\[ \rho_1 \]

Figure 3. Output responses from Section IV (A).

\[ \rho_2 \]

Figure 4. Expanded output responses from Section IV (A).

\[ \rho_3 \]

Figure 5. Transition of the feedforward gain from Section IV (A).

\[ \rho_4 \]

Figure 6. Output responses from Section IV (B).

\[ \rho_5 \]

Figure 7. Transition of the feedforward gain from Section IV (B).
Fig. 6 shows that the output of the plant followed the reference model output after \( t = 45 \) s. The proposed method appears to have excellent performance for disturbance rejection.

C. Change in plant characteristics

This section describes the simulation results of the target-value responses for changes in characteristics of a second-order plant. Consider the second-order plant as follows:

\[
P(s) = \left\{ \begin{array}{ll}
\frac{b}{s(s+a)} & (t < 70) \\
-\frac{1}{200}(t-70) + 1 & \left(70 \leq t < 170\right) \\
\frac{1}{2} \frac{b}{s(s+a)} & (t \geq 170)
\end{array} \right.
\]

(28)

where \( a = 9.66 \) and \( b = 82.2 \). The reference model is given as follows:

\[
T_d(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2},
\]

(29)

where \( \omega_n = 10 \) and \( \zeta = 1.0 \). Furthermore, the PID gain of the feedback controller was

\[
\begin{align*}
K_p &= 5.5 \\
K_i &= 0.98 \\
K_d &= 6.8
\end{align*}
\]

(30)

The parameters of feed-forward controller \( d_{u1} \) and \( d_{u0} \) are coefficients of a denominator polynomial of the discretized reference model. The initial value of the feedforward gain was \( \rho_0 = [1 \quad -2 \quad 1] \). The parameter of the low-pass filter was \( a = 10^{-4} \). The simulation results for the output responses are shown in Fig. 8, and the expanded responses in \( t = 450-500 \) are shown in Fig. 9. The transition of the feedforward gain is shown in Fig. 10.

Fig. 8 and Fig. 9 shows that the output of the plant followed the reference model output, verifying that the proposed method has excellent performance for changes in the characteristics of the plant. Fig. 10 shows that the feedforward gain changed in response to changes in the characteristics of the plant.

D. DC motor (actual device)

This section describes experimental results of the target-value responses of a DC motor. The structure of the motor control system is shown in Fig. 11, and a photograph of the motor is shown in Fig. 12. The transfer function of the motor used in the experiment is

\[
P(s) = \frac{b}{s(s+a)},
\]

(31)
where \( a = 9.66 \) and \( b = 82.2 \). Equation (31) is the same transfer function as the plant of Section IV (B). The reference model is given as follows:

\[
T_d(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{32}
\]

where \( \omega_n = 10 \) and \( \zeta = 1.0 \). Furthermore, the PID gain of the feedback controller was

\[
\begin{align*}
K_p &= 5.5 \\
K_i &= 0.98 \\
K_d &= 6.8
\end{align*} \tag{33}
\]

The parameters of feed-forward controller \( a_{m1} \) and \( a_{m0} \) are coefficients of a denominator polynomial of the discretized reference model. The initial value of the feedforward gain was \( \rho_0 = [1, -2, 1]^T \). The parameter of the low-pass filter was \( \alpha = 10^{-4} \). An input-side step disturbance of magnitude \(-0.1\) was applied at time \( t = 67 \text{ s} \). The experimental results for the output responses are shown in Fig. 13. The transition of the feedforward gain is shown in Fig. 14.

Fig. 13 shows that the output of the plant well followed the reference model output. The proposed method appears to have excellent performance for disturbance rejection. Fig. 14 shows that the feedforward gain changed in response to disturbance.

V. CONCLUSIONS

In this paper, we have proposed a method of online controller adjustment by FRIT of a frequency domain using a sliding DFT. The proposed method can correspond to the time delay of a controlled object. In addition, it can also correspond to characteristic variations of a control object. In the simulation, the output was observed to follow the target value. Moreover, when a DC motor was investigated, the proposed method appears to have excellent performance. However, since the feedback controller gain is related to performance, it is necessary to select the gain appropriately. If the gain is not selected properly, or the influence of the disturbance is increased, the control system may become unstable. In future research, it is necessary to verify effectiveness for the plant including larger time delay. Moreover, there is a need to perform an experiment on an actual device that includes time delay.
REFERENCES


