Tuning the LC-Parameters of Metamaterial Unit Cells Using Genetic Algorithm

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Abstract — In recent years, composite right-left hand transmission line (CRLH-TL) metamaterials have been the subject of research interest for many applications. In this paper, a simple and efficient genetic algorithm method is adopted for tuning the LC-parameters of composite right-left hand transmission line (CRLH-TL) metamaterial unit cell. The genetic algorithm is used with the pseudo-inverse method to get optimum solutions of the LC-parameters. Three different designs are implemented to evaluate the proposed algorithm for composite right-left hand transmission line metamaterial unit cell. The first design represents symmetric T-network at the frequency of 10 GHz, the second design represents symmetric π network at the frequency of 5.5 GHz, and the third design represents an asymmetric network at the frequency of 5.5 GHz. In this work we report exactly balanced designs obtained with simple computations, and the characteristics of each metamaterial unit cell are analyzed.

Keywords - genetic algorithm; optimization; composite right-left hand transmission line metamaterials

I. INTRODUCTION

Composite right-left hand transmission line (CRLH-TL) metamaterials are effectively homogeneous electromagnetic materials to achieve unique properties, which are not ordinarily available in nature [1]. Metamaterials have been widely used in optical, emerging microwave circuits and antenna applications [2][3]. Metamaterials have brought new notions in electromagnetic propagation characteristics and new applications, due to an unusual characteristics of left-handed metamaterials (LHMs) in comparison with traditional natural materials, such as simultaneously negative permittivity and permeability, anti-parallel phase and group velocities, negative refractive index, nonlinear dispersion relation, and a zero propagation constant at a resonant frequency [4][5][6][7][8].

The design of composite right-left hand transmission line (CRLH-TL) metamaterial unit cell is carried out with the analysis based on equivalent LC circuits. An effective design of CRLH-TL structures requires an accurate LC-parameters extraction. The famous technique to extract the LC-parameters for metamaterial unit cell is known as: “method of separate extraction of inductances and capacitances”, [1]. This method is capable of giving an exact solution, but it suffers from two disadvantages. The first-one is related to the separate extraction of the inductances and capacitances from the individual shunt stub and an interdigital capacitor (IDC), and the second-one relates to the standard forms of the unit cells.

This paper is based on pseudo-inverse method for extracting the LC-parameters of CRLH-TL metamaterial unit cell [9]. This method is capable of extracting the inductances and capacitances together from the S-parameters of the unit cell; therefore, it is considered to be a simple method for extracting the LC parameters as compared to [1]. Also this method is appropriate for unit cells with arbitrary structures. In contrast, the disadvantage of this method is that it gives only approximate solutions; therefore, it achieves unbalanced design for metamaterial unit cell. To solve this problem, genetic algorithms (GAs) allow for tuning the LC-parameters that are extracted by pseudo-inverse method and achieve the balanced design of metamaterial unit cell (i.e. optimize the series and shunt frequencies) at a target or transition frequency. GAs are applied with three different examples at two different frequencies (5.5, and 10 GHz).

The paper is organized as follows: metamaterials unit cell, extraction of the LC-parameters using pseudo-inverse method and the details of the problem-formulations presented in section II. Section III discusses the genetic algorithms. Section IV gives the results and discussion. Finally, section V provides the conclusions.

II. CRLH-TL METAMATERIAL AND THE PROBLEM FORMULATION

The basic construction of metamaterial unit cell consists of a specific (per-unit length) series impedance constituted by a right hand (RH) specific inductance in series with a left hand (LH) capacitance of unit length times capacity. And per-unit length shunt admittance constituted by a RH per-unit-length capacitance in parallel with (LH) unit-length times inductance [1].

\[ Z(\omega) = j\left(\frac{\omega L}{\omega C}\right) \]
\[ Y(\omega) = j\left(\frac{\omega C}{\omega L}\right) \]

Where

- \( Z \) --- Impedance (Ω/m),
- \( Y \) --- Admittance (S/m),
- \( L' \) --- Right inductance (H/m),
- \( C' \) --- Right capacitance (F/m),
- \( L_L \) --- Left inductance (H/m),

\[ Y' = \frac{1}{Y} \]
\[ C' \quad \text{Left capacitance (F.m)}, \]
\[ \omega \quad \text{Angular frequency } = 2\pi f. \]

By applying the periodic boundary condition, the dispersion diagram can be computed analytically by using Bloch-Floquet theorem, and it is given by [1]

\[
\cos (\beta p) = 1 - \frac{1}{2} \left[ \omega^2 L' C' + \frac{1}{\omega^2 L' C'} - \left( \frac{L'}{L} + \frac{C'}{C} \right) \right]
\]

(2)

There are two cases of the phase constant, the first case is unbalanced case if the series and shunt resonances of CRLH-TL are different (\( f_{\text{series}} \neq f_{\text{shunt}} \)), and the second case is called balanced case if the series and shunt resonances of the CRLH-TL are equal (\( f_{\text{series}} = f_{\text{shunt}} \)). The permeability and permittivity can be determined by [8].

\[
\mu = \mu(\omega) = \frac{L'}{\omega C'} - \frac{1}{\omega^2 C'}
\]

(3.a)

\[
\varepsilon = \varepsilon(\omega) = \frac{C'}{\omega L'} - \frac{1}{\omega^2 L'}
\]

(3.b)

Bloch impedance \( Z_B \) can be represent as the ratio of voltages and currents in the network (input impedance), Bloch impedance can be determined from S-parameters [1]:

\[ Z_B = \frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2} \]

(4)

The extraction procedures can be performed as shown in the following steps:

step 1: extract the S-parameters for the metamaterial unit cell with two-port microstrip lines and S-parameters for each port transmission line only using full wave simulation HFSS[1].

step 2: convert the S-parameters to ABCD parameters and de-embed S-parameters for the two-port microstrip lines.

step 3: convert ABCD parameters to impedance (\( Z' \)) and admittance (\( Y' \)) for each unit cell [10].

step 4: extract the LC-parameters from \( Z' \) and \( Y' \) equations. The LC-parameters of CRLH-TL metamaterial unit cell can be extracted using (1) in matrix form as follows:

\[
\begin{bmatrix}
L' \\
C'
\end{bmatrix} = \left[ W \right]^{-1} \left[ Z' \right]
\]

(5.a)

and

\[
\begin{bmatrix}
C' \\
L'
\end{bmatrix} = \left[ W \right]^{-1} \left[ Y' \right]
\]

(5.b)

These parameters depend on the type of equivalent circuit of unit cell which is either symmetrical T and \( \pi \) or an asymmetrical networks. The frequency matrix \( [W] \) is not a square matrix, therefore, pseudo-inverse method is used to find the inverse of this matrix. Where \( [W]^\dagger = [W]^T \) is Moore-Penrose pseudo-inverse [10].

\[
[W]' = (W^TW)^{-1}W^T
\]

(6)

This method is capable of extracting the inductances, and capacitances together from the S-parameters of the unit cell, and this method is appropriate for unit cells with arbitrary structures. In contrast, the disadvantage of this method, is that it gives only approximate solutions. Hence, GAs are required for tuning and finding the proper LC parameters of CRLH-TL unit cell and for achieving the desired balanced design.

III. GENETIC ALGORITHM

Genetic algorithm (GA) is a branch of the artificial intelligence methods. It is an optimization technique used to solve the complex and constraint problems, especially in microstrip patch antennas and metamaterials design [11][12]. The GA process could be simplified as following:

A. Initial Population and Construction of Chromosomes

The first step is to generate an initial random population of individuals as chromosomes, where each position in the chromosome is called a gene. The initial population of this work includes the LC-parameters that are extracted using pseudo-inverse method. By using the genetic algorithms for tuning of LC-parameters, they are made to correspond to living beings and vectors represent chromosomes as follows [12][13]:

\[ d = [d_1 d_2 d_3 d_4] \]

(7)

Where \( d_i \) represents genes (\( d_1, d_2, d_3, \) and \( d_4 \)) which are corresponding to \( L_1, C_1, L_2, C_2 \) parameters respectively, and vector \( d \) represents the chromosome.

B. Fitness and Objective Function

Fitness function plays a very important role in the GA process. The best chromosome can be obtained through the maximization of fitness function, this means that the error becomes zero when \( f_{\text{series}} = f_{\text{shunt}} \), i.e. when this is fulfilled the balanced design requirement in CRLH-TL metamaterial unit cell is achieved and the program is stopped. The formulee of the important series and shunt frequencies for CRLH-TL unit cell are summarized in (8) for each chromosome [1][14].

\[ f_{\text{series}} = \frac{1}{2\pi \sqrt{L' R C'}} \]

(8.a)

\[ f_{\text{shunt}} = \frac{1}{2\pi \sqrt{L' R C'}} \]

(8.b)

And we can determine the errors for series and shunt frequencies respectively

\[ E_{\text{series}} = |f_{\text{desired}} - f_{\text{series}}| \]

(9.a)

\[ E_{\text{shunt}} = |f_{\text{desired}} - f_{\text{shunt}}| \]

(9.b)
From (9), the objective function is obtained as:

$$error = |E_{series} - E_{shunt}|$$  \hspace{1cm} (10)$$

And then the fitness function is the maximization of the objective function.

$$Fitness\ Function = 1/error$$ \hspace{1cm} (11)$$

The fitness function corresponds to each chromosome which will be used to assess it, and then they are used to compete for the next generation.

C. Selection

The selection operator distinguishes the better from the worse chromosomes using their fitness function. Each chromosome is evaluated in the fitness function and the best chromosomes are selected to survive for mating, while the worse ones are rejected. There are many selection techniques available to pick the best chromosomes, such as Roulette Wheel selection, which is used in this work, and Rank selection [15].

D. Crossover

Crossover is another process after the selection. Crossover involves exchange of the genes between two parent chromosomes to make new chromosomes. After randomly choosing a crossover point, everything proceeding to the point is copied from the first parent to the second parent. Likewise all genes following the crossover point in the second parent are copied to the first parent in the corresponding space. There are many types of crossover, such as single point crossover, which is used in this work, and two points crossover [15].

E. Mutation

Random mutation takes place after a crossover. Mutation changes randomly the new offspring (children). There are many types of possible mutations (real mutation, and binary mutation). Real mutation is used in this work according to the following equation as follows [15]:

$$a_i' = \begin{cases} r(L_0 , U_p) & \text{if } m \leq P_m \\ a_i & \text{Otherwise} \end{cases}$$ \hspace{1cm} (12)$$

Where

- $m'$ is a random number,
- $r(L_0 , U_p)$ is random number with limited range ($L_0$, $U_p$),
- $a_i$ is the value of gene before mutation,
- $a_i'$ is the value of gene after mutation,
- $P_m$ is probability of mutation equal to (0.5%-1%).

IV. Results and Discussion

Three different examples are presented to demonstrate the various capabilities of this method. The genetic characteristics are as follows: The numbers of chromosomes in the population are 100 and each chromosome includes 4 genes. The probability of crossover is 85% and probability of mutation is 0.5%. This paper does not used binary numbers to avoids coding/decoding and directly represents by real numbers to simplify computational programming and to speed up the calculations. Fig. 1 shows the flow chart illustrating of genetic algorithm for tuning the LC parameters. Based on this optimization strategy, we can use the GA for achieving various design requirements. For example, we can use the GA for tuning the LC parameters of CRLH-TL unit cell to attain the balanced design ($f_c = f_{series} = f_{shunt}$).

![Flow chart of the GA for tuning the LC-parameters of metamaterial.](image-url)
A. CRLH-TL Symmetric T-Network

Fig. 2(a) shows the 2D geometry of CRLH-TL metamaterial unit cell which operates at the frequency of 10 GHz. The unit cell consists of interdigital capacitor and two shorted stub which are placed on both sides of unit cell. This unit cell is implemented on the substrate of Roger RT 5880 with dielectric constant of $\varepsilon_r=2.2$, thickness of substrate of $h=1.575$ mm; and loss tangent $\tan \delta=0.001$. Fig. 2(b) represents the equivalent T-network for the unit cell. After the implementation and running the unit cell the dispersion diagram is obtained as shown in Fig. 2(c). From this figure, it is obvious that the first of the two curves (blue curve) represents the dispersion diagram depending on the LC-parameters that are extracted by pseudo-inverse method. It is noticed that the result is unbalanced design, this means that it is an approximate solution of this method. On the other hand, the second curve represents dispersion diagram depending on the tuning LC-parameters by GA and achieve the balanced design for unit cell at the frequency of 10 GHz. Fig. 2(d) shows the reflection coefficient of the unit cell; it is noticed that $S_{11}$ is of -34 dB at center frequency. The simulated 10 dB return loss bandwidth of the proposed metamaterial extends from 6.3 GHz to 18 GHz. This means that the bandwidth and fractional bandwidth are 11.7 GHz and 117% respectively. Fig. 2(e) shows the phase of unit cell and is equal to zero at transition frequency. The Bloch impedance is 50 ohm at transition frequency as shown in Fig 2(f). Another check for GA is applied on the permittivity and the permeability of unit cell, as shown in Figs. 2(g and h). It is noticed that the permittivity and the permeability are zero values at transition frequency when using the GA. On the other hand, they have different values (zero value at 11 GHz for permittivity and zero value at 9 GHz for permeability) when using pseudo-inverse method. Table (I) shows the LC-parameters that are extracted by two methods, namely, pseudo-inverse and GA. Table (II) Dimensions of 2D symmetric T network (in mm) refers to Fig. 2(a).
Figure 2. Metamaterial unit cell and its characteristics a) 2D geometry of CRLH-TL metamaterial unit cell at frequency 10GHz b) equivalent T-model for unit cell c) dispersion diagram d) magnitude of reflection coefficient $S_{11}$ e) phase of $S_{21}$ for unit cell f) Bloch impedance g) real values of permittivity h) real values of permeability.

B. CRLH-TL Symmetric $\pi$-Network

Fig. 3(a) displays the layout of the 2D zero order resonator coplanar waveguide ZOR-CPW based on CRLH-TL metamaterial unit cell. The proposed unit cell is composed of top metallic patches with octagon shape, and shorted meander lines to a CPW ground plane. This unit cell is implemented on the substrate of epoxy with dielectric constant of $\varepsilon_r=4.4$, thickness of substrate of $h=1.5$ mm; and loss tangent $\tan \delta=0.02$. The metamaterial unit cell operates at the frequency of 5.5 GHz, and Fig. 3(b) represents the equivalent $\pi$-network for the unit cell. After the implementation and running the unit cell, the dispersion diagram is obtained as shown in Fig. 3(c). From this figure, it obvious that the first of two curves (blue curve) represents dispersion diagram depending on the LC-parameters that are extracted by pseudo-inverse method. It is noticed that the result is unbalanced design. On the other hand, the second curve represents dispersion diagram depending on tuning the LC-parameters by GA and achieve the balanced design for the unit cell at the frequency of 5.5 GHz. Fig. 3(d) shows the reflection coefficient of total structure of unit cell, it is noticed that $S_{11}$ is of -23 dB at center frequency. The simulated 10 dB return loss bandwidth of the proposed metamaterial extends from 4.57 GHz to 11.23 GHz. This means that the bandwidth and fractional bandwidth are 6.66 GHz and 121.1% respectively. Fig. 3(e) shows the phase of unit cell and is equal to zero at transition frequency. The Bloch impedance is 50 ohm at transition frequency as shown in Fig 3(f).  Another check for GA is applied on permittivity and permeability of unit cell as shown in Figs. 3(g and h). It is noticed that the permittivity and the permeability are zero values at transition frequency when using the GA. On the other hand, they have different values (zero value at 5.1 GHz for permittivity and zero value at 5.8 GHz for permeability) when using pseudo-inverse method. Table (III) shows the LC-parameters that are extracted by two methods, namely, pseudo-inverse and GA. Table (II) Dimensions of ZOR-CPW design (in mm) refers to Fig. 3(a).

<table>
<thead>
<tr>
<th>Method</th>
<th>$L_s$ (nH/m)</th>
<th>$C_{\perp}$ (pF/m)</th>
<th>$L_L$ (nH/m)</th>
<th>$C_{\parallel}$ (pF/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-inverse</td>
<td>3.3109</td>
<td>0.1076</td>
<td>0.5943</td>
<td>0.3422</td>
</tr>
<tr>
<td>Tuning GA</td>
<td>2.5989</td>
<td>0.09742</td>
<td>0.6358</td>
<td>0.3983</td>
</tr>
</tbody>
</table>

Table I. The LC-parameters for two methods Pseudo-inverse and GA methods for symmetric T-network.
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Figure. 3 Metamaterial unit cell and its characteristics a) 2D geometry of ZOR-CPW based on CRLH-TL metamaterial unit cell at frequency 5.5GHz b) equivalent π-model for unit cell c) dispersion diagram d) magnitude of reflection coefficient S 11 e) phase of S 21 for unit cell f) Bloch impedance g) real values of permittivity h) real values of permeability.

TABLE III. THE LC-PARAMETERS FOR TWO METHODS PSEUDO-INVERSE AND GA METHODS FOR SYMMETRIC Π-NETWORK.

<table>
<thead>
<tr>
<th>Method</th>
<th>$L_a$ (nH/m)</th>
<th>$C_{\varepsilon}$ (pF/m)</th>
<th>$L_L$ (nH/m)</th>
<th>$C_\mu$ (pF/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-inverse</td>
<td>1.77</td>
<td>0.46</td>
<td>0.76</td>
<td>1.2</td>
</tr>
<tr>
<td>Tuning GA</td>
<td>1.77</td>
<td>0.415</td>
<td>0.813</td>
<td>1.38</td>
</tr>
</tbody>
</table>

TABLE IV. DIMENSIONS OF ZOR-CPW DESIGN (IN MM) REFERS TO FIG. 3(A).

<table>
<thead>
<tr>
<th>$\ell$ (mm)</th>
<th>$w_L$</th>
<th>$S$ (mm)</th>
<th>$s_0$ (mm)</th>
<th>$\ell_{stub}$ (mm)</th>
<th>$w_{stub}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.24</td>
<td>0.46</td>
<td>0.2</td>
<td>3.78</td>
<td>0.1</td>
</tr>
<tr>
<td>1.5</td>
<td>5.2</td>
<td>2.3</td>
<td>12</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

C. CRLH-TL an Asymmetric Network

Fig. 4(a) shows the 2D geometry of CRLH-TL unit cell metamaterial which operates at the frequency of 5.5 GHz, and Fig. 4(b) represents the equivalent an asymmetric network for unit cell. After implementation and running the unit cell, it is obtained the dispersion diagram as shown in Fig. 4(c), from this figure, it obvious that two curves: the first curve (blue curve) represents dispersion diagram depending on the LC-parameters extracted by pseudo-inverse method and it is noticed that the result is unbalanced design, while the second curve represents dispersion diagram depending on the tuning the LC-parameters by GA and gives the balanced design for unit cell at the frequency of 5.5 GHz. Fig. 4(d) shows the reflection coefficient of total structure of unit cell, it is noticed that $S_{11}$ is of -30.5 dB at center frequency. Fig. 4(e) shows the phase of unit cell and is equal to zero at transition frequency. The Bloch impedance is 51.7 ohm at transition frequency as shown in Fig 4(f).

It is noticed that from the previous three examples, the genetic algorithm can handle all of the variable parameters ($L_a, C_{\varepsilon}, L_L, C_{\mu}$). The design is based on an optimization of series frequency which depends on ($L_a, C_{\varepsilon}$) parameters, and shunt frequency which depends on ($L_L, C_{\mu}$) parameters, in order to obtain the balanced design of the unit cell. All these cases can be optimized successfully with their properties towards our design objectives. This paper includes the study of the characteristics for each unit cell such as (equivalent circuit, dispersion diagram, reflection coefficient ($S_{11}$), bandwidth, permittivity ($\varepsilon$), and permeability ($\mu$). Also the comparison between the two methods with respect to dispersion diagram, permittivity, and permeability is achieved. S-parameters are extracted using the HFSS simulation, which is exported to MATLAB. GA main program and metamaterial unit cell characteristics are calculated using MATLAB script.
Figure 4. Metamaterial unit cell and its characteristics a) 2D geometry of asymmetric metamaterial unit cell at frequency 5.5 GHz b) equivalent model for unit cell c) dispersion diagram d) magnitude of reflection coefficient $S_{11}$ e) real values of permittivity f) real values of permeability.

TABLE V. THE LC-PARAMETERS FOR TWO METHODS: PSEUDOINVERSE AND GA METHODS FOR AN SYMMETRIC NETWORK.

<table>
<thead>
<tr>
<th>method</th>
<th>$L_s$ (nH/m)</th>
<th>$C_r$ (pF/m)</th>
<th>$L_L$ (nH.m)</th>
<th>$C_i$ (pF/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudoinverse</td>
<td>1.831</td>
<td>0.487</td>
<td>0.3478</td>
<td>2.1858</td>
</tr>
<tr>
<td>Tuning GA</td>
<td>1.832</td>
<td>0.457</td>
<td>0.328</td>
<td>2.55</td>
</tr>
</tbody>
</table>
V. CONCLUSION

In this paper, the genetic algorithms have been used to tune the LC-parameters of metamaterial unit cell that are estimated by the pseudo-inverse method. Consequently, an accurate balanced design for metamaterial unit cell at a certain frequency \((f_0 = f_{\text{series}} = f_{\text{shunt}})\) can be specified. This method revises the error that may happen due to the approximation of the pseudo-inverse method. Three different types of metamaterial cell unit design (symmetric T, π, and asymmetric networks) are studied to validate the proposed method. In all the experiments, high accurate LC-Parameters of the meta-material cell units are obtained for a given resonance frequency. Additionally, it can be easily modified to fit any type of meta-material cell units design.

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REFERENCES