A Method of Searching Optimal Path in Urban Rail Transit Network Based on Residual Intervals Capacity

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Abstract — In this paper, we study the problem of searching Origin-Destination (O-D) optimal paths in urban rail transit network. Considering the passengers’ excepted departure time and residual intervals capacity, we construct the passenger travel network of the urban rail transit. And we analyze the topology of passenger travel network of the urban rail transit according to different residual intervals capacity. We present an approach for searching optimal paths in passenger travel network of the urban rail transit network based on travel time, transfer number and residual intervals capacity, aiming at minimizing the overall time associated with required paths. The proposed approach has been developed for analyzing the urban rail transit network in Beijing in the paper.

Keywords - urban rail transit; travel network; residual capacity; optimal path problem

I. INTRODUCTION

With the constant improvement of the urban rail transit network, more and more passengers choose urban rail transit to travel in city. Urban rail transit plays a role of the urban transportation systems. In order to analyze passenger flow distribution in urban rail transit network, it is of paramount importance to study the problem of searching Origin-Destination (O-D) optimal paths in urban rail transit network.

Rational passenger travel network is backbone of optimal path searching problem. In previous studies, a schedule-based public transit network which each node in the network has a list of scheduled departure times for transit vehicles was described based on the timetable [1-3]. Optimal path searching is one of the prominent research topics in the field of transportation organization. Dijkstra’s algorithm is widely applied in optimal path searching problem [4-5]. Some optimized Dijkstra’s algorithm combined with other methods to search the optimal path is proposed. In [6] an optimized Dijkstra’s algorithm combined with Floyd-Warshall algorithm to search the all-pairs shortest path in a large-scale transportation network. An optimal path searching approach which was basically based on Dijkstra’s algorithm and the tree-building algorithm used to search optimal paths [7]. In the transportation field, many researchers presented the various path searching algorithms for different types of networks to solve shortest paths problem [8-9].

The rest of the paper is organized as follows. In the next section, the topology features of urban rail transit network are analyzed. In Section 3, the residual capacity of the path as the foundation of path searching procedure is developed, and travel network topology is analyzed. The optimal paths searching algorithm based on residual intervals capacity is presented in Section 4. An illustrative case is presented to demonstrate the validity of passenger travel network construction method and the optimal path searching algorithm in Section 5. In the last section, the conclusions and recommendations are presented for future research.

II. TOPOLOGY FEATURES OF URBAN RAIL TRANSIT NETWORK

Urban rail transit network is undirected network, station is abstract to the node of undirected network, adjacent station section is abstract to the edge of undirected network. Topology of urban rail transit network is shown in Figure 1.

![Figure 1. Topology of urban rail transit network.](image-url)
in urban rail transit network, has a great impact on travel route choice for passengers, as shown in Figure 3.

Besides, different lines have several same stations in complex urban rail transit network. For example, passengers can transfer any transfer station from SIHUI to SIHUI East in Beijing Subway Line 1, the line topology feature as shown in Figure 3. If we don’t treat this similar situation in urban rail transit network, there will be two same edges between two nodes, which is a ring in the network. This situation is inconformity the definition of undirected connected graph. So, the similar situation needs to be treated separately.

III. PASSAGEN TRAVEL NETWORK MODEL

A. Description of Passenger Travel Network

Passenger travel network model of urban rail transit consist of nodes and directed arcs between adjacent nodes. Let \( G = (V, E, V', E', C, N) \) denote the passenger travel network of urban rail transit (Figure 4), where \( V = \{ v_i | i = 1, 2, 3, ..., n \} \), \( E = \{ e_{ij} = (v_i, v_j), \forall v_i, v_j \in V, i \neq j \} \), \( V' = \{ v'_i | i = 1, 2, 3, ..., n \} \), \( E' = \{ e'_{ij} = (v'_i, v'_j), \forall v'_i, v'_j \in V', i \neq j \} \), \( E'' = \{ e''_{ij} = v'_i \rightarrow v'_j, \forall v'_i, v'_j \in V', i \neq j \} \), \( C = \{ e_i = c(e_i), \forall e_i \in E_i \}, N = |e_i| = |n_i| = n(e_i), \forall e_i \in E \).

\( V \) represents set of all nodes \( v_i (1 \leq i \leq n) \) in passenger travel network of urban rail transit \( G \), the number of nodes is \( n \); \( E \) represents set of all arcs \( e_{ij} \) in passenger travel network of urban rail transit \( G \), where \( e_{ij} = (v_i, v_j), v_i, v_j \in V, v_i \) is adjacent to \( v_j \); \( V' \) represents attribute of node \( v'_i \) in passenger travel network of urban rail transit \( G \). Node attribute contains passenger transition node \( v'_i \) and train service node \( v'_j \).

Passenger transition node contains origin node \( v_{b0} \) and destination node \( v_{b1} \), \( v_{b0} \in V, v_{b1} \in V' \): Train service node contains train departure node \( v_{a0} \) and train arrival node \( v_{a1} \), \( v_{a0} \in V, v_{a1} \in V' \). \( E'' \) represents attribute of arcs \( e_{ij} \) in passenger travel network of urban rail transit \( G \). Arbitrary arc between two train service nodes is arc of directed travel network. Arcs attribute contains boarding arcs \( (v_{b0}, v_{a0}) \in E' \), running arcs \( (v_{a0}, v_{a1}) \in E' \), transferring arcs \( (v_{a1}, v_{b1}) \in E' \), and alighting arcs \( (v_{b1}, v_{b0}) \in E' \).

\( (v_{b0}, v_{a0}) \in E' \) represents the arc between the origin node \( v_{b0} \) and the train departure node \( v_{a0} \) for the train \( t_r \) at station \( s \), where \( E_i \) is the set of boarding arcs.

\( (v_{a0}, v_{a1}) \in E' \) represents the arc between the train departure node \( v_{a0} \) and the train \( t_r \) at station \( s \), where \( E_i \) is the set of running arcs.

\( (v_{a1}, v_{b1}) \in E' \) represents the arc between the train arrival node \( v_{a1} \) of the train \( t_r \) at station \( s \), and the train departure node \( v_{b1} \) of
the transfer train tr2 at station s, where E is the set of transfer arcs. 

\((v_{s_{tr}}, v_{s_{tr}2}) \in E_v\) represents the arc between the train arrival node \(v_{s_{tr}}\) for the train \(tr\) at station sin city and the destination node \(v_{s_{tr}2}\), where \(E_v\) is the set of alighting arcs.

\(E\) represents direction attribute of arcs \(e_t\) in passenger travel network of urban rail transit G. Every arc has directivity, \(e_t\) represents directed arc from \(v_i\) to \(v_j\), \(e_t = v_i \rightarrow v_j\).

Define the arc weight \((C, N)\), where C and N represent the generalized cost and the residual capacity of arc, respectively. In urban rail transit system, the travel costs of different paths from origin station to destination station are the same. Thus, the generalized cost can be represented as \(C_{arc} = t_{arc}\), where \(t_{arc}\) is the travel time of the arc. Residual intervals capacity is the residual passengers the train can carry.

### B. Generalized Cost and Residual Capacity of the Path

Define the arc weight as \((C, N)\), where C and N represent the generalized cost and the residual capacity of arc, respectively. 

Passengers will choose optimal path prior to departure and strictly follow optimal path till they reach the destination. In urban rail transit system, the travel costs of different paths from origin station to destination station are the same. Thus, the generalized cost of arc is defined by the travel time of arc, it can be represented as \(C_{arc} = t_{arc}\), where \(t_{arc}\) is the travel time of arc.

The generalized cost of the travel path is sum of all arcs belonging to this path, it includes train running time, stopping time and transfer time. The generalized cost of the travel path \(p\) is \(C_p = \sum_{arc \in p} C_{arc}\).

Residual intervals capacity is the residual passengers the train can carry. Specially, assume \(N = \infty\), which is interval residual capacity of boarding arcs, transfer arcs and alighting arcs; assume \(C = 0\), which is generalized cost of boarding arcs and alighting arcs.

In passenger travel network, path residual capacity is the minimum residual capacity of all arcs belonging to this path, \(N_p = \min\{N_{arc}[arc \in p]\}\), where \(p\) represents travel path \(p\) of OD in passenger travel network of urban rail transit.

The arc, which has the minimum residual intervals capacity, determines residual capacity of travel path of some OD in passenger travel network of urban rail transit, as shown in Figure 5.

A path from origin node \(v_o\) to destination node \(v_o\) is consist of some arcs. Path \(p_1\) is consist of some arcs, \(p_1 = \{(v_{o_{i_1}}, v_{o_{i_1}^d}), (v_{o_{i_1}^d}, v_{o_{i_2}^d}), (v_{o_{i_2}^d}, v_{o_{i_2}^d}), (v_{o_{i_2}^d}, v_{o_{i_2}}), (v_{o_{i_2}}, v_{o_2})\}\), corresponding value of generalized cost and residual capacity of each arc are shown in Figure 4.

Generalized cost of path \(p_1\) is the sum generalized cost of all arcs in this path, which can be represented as

\[C_p = 0 + C_{p_{11}} + C_{p_{12}} + C_{p_{2}} + 0 = C_{p_1} + C_{p_{12}} + C_{p_{2}}\]

Residual capacity of path \(p_1\) is the minimum residual capacity of all arcs in this path. Assume residual capacity of arc \((v_{o_{i_1}^d}, v_{o_{i_2}})\) is less than arc \((v_{o_{i_2}^d}, v_{o_{i_2}})\), that is \(N_{p_{11}} < N_{p_{12}}\), then, residual capacity of path \(r_t\) can be represented as

\[N_{p_{1}} = \min \{+\infty, N_{p_{11}} + +\infty, N_{p_{12}}, +\infty\} = N_{p_{12}}\]

Choose all trains, which meet the passengers’ requirement of start time and end time from train timetable, including transfer trains. Construct railway passenger travel path network by setting the minimum transfer time, as shown in Figure 4.

### C. Impact of Residual Capacity Changes to Travel Network Topology

Arc between the nodes in the train network is effective when residual train capacity isn’t zero, the train can transport passengers to the destination. Running arcs \((v_{o_{i_2}^d}, v_{o_{i_2}})\) can effectively transport passengers to the next node, as shown in Figure 6.

Residual intervals capacity will reduce greatly when passengers in the train network increase. Specially, the interval can’t transport passengers when residual intervals capacity is zero, running arcs fail, passenger travel network topology will change greatly.

Figure 6. Network topology when the train has residual capacity

Figure 7. Network topology when the train doesn’t have residual capacity
IV. OPTIMAL PATH SEARCHING ALGORITHM BASED ON RESIDUAL INTERVALS CAPACITY

In the passenger travel network \( G = (V, E) \), the running arc is disabled between \( v_i \) and \( v_j \) when \( C_{ij} = \infty \). The optimal path from \( v_i \) to \( v_j \) is the minimal generalized cost of all path sets \( P = \{p_1, p_2, \ldots, p_s\} \), it can be represented as

\[
p_{\min} = \{p_i | \min \{C_{p_i}\}\} = \{p_i | \min \{\sum_{k \in P} C_{mk}\}\}
\]

Search the optimal path by Dijkstra’s algorithm and then assign passenger flow to this path according to the capacity of the optimal path until a running arc of this path is disabled. A new passenger travel network is formed after the disable arc is deleted. Continue to search the optimal path and assign the remaining passengers until all passengers are assigned or all paths have no residual capacity. Traveling Optimal Path Searching Algorithm is shown as:

Input: \( G(V,E) \), distribution number of passenger flow \( P \).
Output: The set of the path \( L(i_1, i_2, \ldots, i_k) \), the generalized cost \( C(l_k) \) of path \( l_k \) and passenger volume \( P(l_k) \) of path \( l_k \).

Step1: Set origin node \( v_o \) and destination node \( v_d \), and initialize passenger travel network.

Step2: From origin node \( v_o \), use Dijkstra algorithm to search the optimal path \( l_k \) according to the generalized cost of the arc. And calculate the generalized cost \( C(l_k) \) of this path. If \( C(l_k) \geq \text{Maxnum} \), end. Otherwise \( l_k (k = 1, 2, 3, \ldots) \):

- \( v_o \rightarrow v_1 \rightarrow \cdots \rightarrow v_l \rightarrow v_d \) and \( C(l_k) \) are deposited to the set \( L \) and the generalized cost set \( C_o \), respectively.

Step3: Sort the running arcs by residual intervals capacity. Set the minimal arc capacity is residual capacity of the path \( l_k \), namely \( N_{\text{max}}(l_k) \), and set \( P(l_k) = N_{\text{max}}(l_k) \);

Step4: Calculate remaining passenger flow \( P \leftarrow P - N_{\text{max}}(l_k) \). And update residual intervals capacity of all running arcs of the optimal path \( l_k \) : \( N[l][j] \leftarrow N[l][j] - N_{\text{max}}(l_k) \). If residual intervals capacity of running arc \( N[p][q] = 0 \), update the generalized cost of this running arc: \( C[p][q] \leftarrow \text{Maxnum} \).

Step5: Reconstruction of the passenger travel network. The running arc of a train cannot transport more passengers when the train has no residual capacity. Then the boarding arc and the transferring arc which connected to this running arc are disabled. Set \( C[l][p] \leftarrow \text{Maxnum} \), \( C[q][j] \leftarrow \text{Maxnum} \). If \( P \leq 0 \), then passenger volume of path \( l_k \) is \( P(l_k) \leftarrow P + P(l_k) \), go to Step6; otherwise, set \( k \leftarrow k + 1 \), go to Step2;

Step6: Output the set of the path \( L(i_1, i_2, \ldots, i_k) \), the generalized cost \( C(l_k) \) of path \( l_k \) and passenger volume \( P(l_k) \) of path \( l_k \), end.

V. NUMERICAL EXAMPLE

The procedure developed in this paper was tested on Beijing Subway network in Beijing, China. Passengers’ expected departure time and arrival time is 7:30 am and 8:30 am from HAIDIANWULUJU station (Line 6) to HEPINGXIQIAO station (Line 5). Suppose maximum transfer number \( m = 2 \). Travel time is sum of the train running time, stopping time and transferring time. Train timetable is detainted from Beijing Subway Web and capacity of the

Figure 8. Example of Passenger travel network.
running arcs among stations are shown in Table I. Passenger travel network is shown in Figure 8. In the travel network, suppose that 150 passengers need to travel from origin station to the destination station. Five paths are searched by used the proposed approach and passenger flow is assigned by residual intervals capacity in the travel network, which are shown in Table II. Obviously, the optimal path is the first path which generalized cost is minimal. Passengers hope to choose the first path for their journey when the residual intervals capacity is ignored. However, the residual capacity of the train T1 is not enough and only 55 travelers are assigned until all passengers can get on the train for their journey.

VI. CONCLUSIONS
Trains of urban rail transit are running accordance with train schedules strictly. Passengers choose their optimal path from origin to destination is more suitable for their travel based on train timetable. This study developed a rational approach based on train timetable and residual intervals capacity to construct the passenger travel network, and analyzed the network topology. In particular, we proposed an optimal path searching algorithm which combined the Dijkstra algorithm and the residual train capacity based on the passenger travel network, and the validity of the proposed approach was verified with a numerical example. This method can be applied to other mode of transportation systems, for example, the railway transport system.

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REFERENCES

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TABLE II. THE OPTIMAL PATHS FOR 150 PASSENGERS

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