Sensitivity Analysis of Machine Tool Geometric Errors Based on Screw Theory

Yanan GOU, Hong MIN

College of Mechanical and Electrical Engineering, Zaozhuang University, Zaozhuang, Shandong 277000, P.R. China

Abstract — Geometric errors have a major effect on machining accuracy of multi-axis machine tool, and because the complex intercropping among them, how to control these geometric errors and then improve the machining accuracy is a recognized difficult problem. In this paper, a method for volumetric machining accuracy sensitivity analysis of machine tool is proposed based on product of exponential (POE) screw theory and error variance method. In this paper, a typical 3-axis machine tool with high precision as is selected as an example, and there are three screws to represent the six basic error components of each axis in an original way according to the geometric definition of the errors and screws. This type of POE model is precise and succinct enough to express the relation of each component. The error variance method is adopted to identify the key geometric errors which have greater influence on machining accuracy by sensitivity analysis. Finally, according to the analysis results, suggestions with good guidance are provided to adjust and modify machine tool components to improve the machining accuracy economically.

Keywords - machine tool; screw theory; geometric error; error variance; sensitivity analysis

I. INTRODUCTION

The fast development of industry asks for more requirements for precision components. Meanwhile, the demand for high precision machine tools has been increasing rapidly with the growing of the geometric complexity of work piece surfaces. Improving the accuracy of machine tools, which has a great influence on the machining accuracy, is becoming more and more important. The main method is to compensate a variety of errors, among which the geometric errors are the major part. The first and the most important step to compensate the geometric errors is establishing the precise and systematic geometric error model.

In the past decades, plenty of research studies have been focused on the geometric error modeling methods. The robust, easy, and precise error models can lay the foundation for the later error compensation. And much work has been done to analyze the influence of geometric errors in the three-axis machine tools[1]. Okafor and Ertekin provided the detailed analysis of the basic geometric error components for three-axis machine tools which has 21 errors including linear errors, angular errors, and squareness errors[2]. At the same time, they established the geometric error model using homogeneous coordinate transformation based on the rigid body kinematics. Lei and Hsu[3, 4] proposed a geometric error model to measure position errors in machine tools accurately using a 3D probe ball. Jung and Choi[5] parameterized the geometric error model and determined model parameters based on four diagonal measurements by approximating error components with polynomial functions. Lin and Shen[6] proposed a matrix summation approach, which divided global geometric errors into six components. This approach made the model manageable and enabled an ease in understanding of the model due to a clear link to physical parameters. Fan et al. adopted the multi-body system to develop a universal kinematics error modeling method and built the precision machining condition equation to obtain the inverse changed cutter path (ICCP) using the method[7]. Bohez et al. fitted the error components by the cubic polynomials to establish the first order mathematical model. And they also proposed the corresponding solution method for the error compensation[8]. Chen et al. developed the volumetric error modeling of machine tools according to rigid body kinematics and homogeneous transformation matrix; they also carried out the sensitivity analysis of all the error components for machine design[9].

The majority of the research studies were developed on the basis of the multi-body system, and the D-H method was used to obtain the homogeneous transformation matrix [10–14]. However, the D-H method needs local coordinate system to assign to each axis, which makes the modeling process very tedious, especially for complex structure such as five-axis machine tools. In addition, it is not robust enough that the little change of structure may lead to the failure of the model. To overcome these drawbacks, the product of exponential (POE) method has been adopted to establish the model.

There are many inter-coupling[15] geometric errors in multi-axis machine tool, therefore how to determine the different influence of geometric errors on the machining accuracy is currently a difficult problem of machine tool design. Sensitivity analysis exactly is one approach to identify and quantify the relationships between input and output uncertainties [16-17]. Different strategies have been applied in the literature[18], which are typically classified into two main categories: global sensitivity analysis (GSA)[19-21], which is applied to understand how the model response varies with the model parameters to determine interaction strengths among the parameters (Monte Carlo analysis; Fourier Amplitude Sensitivity Test (FAST), variance-based sensitivity analysis, However, LSA only can inspect one point at a time, and the sensitivity index of a specific parameter is dependent on the central values of the other parameters[22]. In this paper, a new
method is provided that is proposed to identify the key geometric errors and crucial components that have great influence on the machining accuracy which is error variance.

The rest of this paper will be arranged as follows. Section 2 and 3 deal with the modeling of the volumetric machining accuracy with consideration of geometric error by POE screw theory. In Section 4, a method is proposed to identify the key geometric errors and crucial components that have great influence on the machining accuracy by error variance. The conclusions are presented in Section 5.

II. TWIST AND POE MODEL

This section briefly reviews the theory of POE model. Any motion of one rigid body can be divided into two parts: a rotation about an axis and a translation along this axis, which are just like a twist. So twist $S$ can express any motion of rigid body. And the twist can be written as

$$\dot{S} = [\hat{\omega} \quad \nu]$$  \hspace{1cm} (1)

Where, $\nu = [v_1, v_2, v_3]^T$ and $\hat{\omega}$ is a skew-symmetric matrix. If $\omega = [\omega_1, \omega_2, \omega_3]^T$, Error! Reference source not found., can be expressed as

Error! Reference source not found.

One six-dimensional vector $S$ can also represent the twist, termed the twist coordinates

$$S = [\omega \quad \nu]^T = [a_1, a_2, a_3, v_1, v_2, v_3]^T$$  \hspace{1cm} (3)

Commonly, a twist is associated with a screw denoting the general motion of rigid body, and the screw contains an axis, a pitch, and a magnitude. $\omega$ is the directional vector of the axis indicating the rotation, and it also can express the rotational velocity. $\nu$ is the position vector of the axis relative to the reference frame, and it also can express the translational velocity.

The composite motion of the rigid body contains rotation and translation. Assume the vector between the rigid body coordinate system and reference coordinate system is $q$, and the homogeneous transformation matrix of the rigid body is

$$T = e^{i\theta} = \begin{bmatrix} I_{3 \times 3} \quad \nu \theta \\ 0 \quad 1 \end{bmatrix}$$  \hspace{1cm} (5)

If $\omega \neq 0$, which means that the rigid body also has rotational motion, and the homogeneous transformation matrix is

$$e^{i\theta} = \begin{bmatrix} e^{i\omega} \left(1 - e^{i\nu} \frac{\hat{\omega} \times \nu}{||\nu||^2} + \frac{e^{i\omega} \nu}{1} \right) \\ 0 \end{bmatrix}$$  \hspace{1cm} (6)

Where, $e^{i\omega}$ can be expanded by trigonometric series method

$$e^{i\omega} = I + \hat{\omega} \nu$$  \hspace{1cm} (7)

In all of the above, if the screw $S$ is unit, motion of rigid body can be transformed by Eq. 6 and Eq. 7, and Eq. 8 can be described as follows

$$T = e^{i\theta} = \begin{bmatrix} I_{3 \times 3} \quad \nu \\ 0 \quad 1 \end{bmatrix}$$  \hspace{1cm} (8)

When $||\nu|| \neq 0$, the rotation angle $\theta = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}$. Otherwise, the translational distance Error! Reference source not found. A given location point has different coordinates in different coordinate systems, and the transformation matrix of these coordinate systems is used to express the relationship between the coordinates. Screw is similar to a point. The ad joint matrix, in different coordinate frames for the screws, is used to obtain the different expressions.

The transformation matrix of rigid body motion twist $\dot{S}$ can be expressed by Eq. (2) and the ad joint matrix of transformation matrix is given as follows

$$\text{Adj}(e^{i\theta}) = \begin{bmatrix} R \\ q \quad R \end{bmatrix}$$  \hspace{1cm} (9)

The properties of ad joint matrix can be shown as follows.

$$\dot{S}_1 = \text{Adj}(e^{i\theta}) \dot{S}_2 = e^{i\theta} \dot{S}_2(e^{i\theta})^{-1}$$  \hspace{1cm} (10)

Moreover, the POE screw theory modeling can be adopted to express the forward kinematics of an open chain.
robot. For an n-DOF (degrees of freedom) robot, the forward kinematics can be written as.

\[ T = e^{\hat{\theta}_1} \cdot e^{\hat{\theta}_2} \cdots e^{\hat{\theta}_n} \cdot T(0) \]  

(12)

Where, \( T(0) \) represents the initial transformation matrix. The Eq. (12) which represents the POE modeling, can also be used in error modeling of the machine tools.

### III. GEOMETRIC ERROR MODELING WITH POE METHOD

According to the theory of the second section, the POE model is applied into the geometric error modeling of multi-axis machine tools. Because of the geometric property, the twists and the POE models can be used to describe the motion of each axis and the geometric errors of machine tools. The squareness errors are represented in detail with the POE models. What’s more, when using the POE method, only the coordinate system is needed during modeling the geometric error, because each twist can be represented in the global coordinate frame through the adjoint matrix. This is quite different from D-H method, which needs to establish different coordinate systems for different bodies. These properties of POE model make the modeling easier and more understandable. Furthermore, POE models contain motion twists of axes, error models of basic error components, and the twists of squareness errors. The integrated POE model of geometric errors is obtained on the basis of these POE models. In order to make the modeling systematic, the order of these POE models is determined with the help of the topological structure of machine tools.

#### TABLE I. ERROR COMPONENTS OF THERE-AXIS MACHINE TOOL

<table>
<thead>
<tr>
<th>Errors</th>
<th>Number of errors</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear positioning errors</td>
<td>3</td>
<td>( \delta_x, \delta_y, \delta_z, \delta_{xA}, \delta_{yC} )</td>
</tr>
<tr>
<td>Horizontal straightness errors</td>
<td>3</td>
<td>( \delta_{x}, \delta_{y}, \delta_{z}, \delta_{x}, \delta_{y}, \delta_{z} )</td>
</tr>
<tr>
<td>Vertical straightness errors</td>
<td>3</td>
<td>( \delta_{x}, \delta_{y}, \delta_{z}, \delta_{x}, \delta_{y}, \delta_{z} )</td>
</tr>
<tr>
<td>Roll angular errors</td>
<td>3</td>
<td>( E_{xx}, E_{xy}, E_{xz}, E_{xA}, E_{y} )</td>
</tr>
<tr>
<td>Pitch angular errors</td>
<td>3</td>
<td>( E_{yx}, E_{yz}, E_{zx}, E_{xC} )</td>
</tr>
<tr>
<td>Yaw angular errors</td>
<td>3</td>
<td>( E_{zx}, E_{zy}, E_{zy}, E_{yA}, E_{zC} )</td>
</tr>
<tr>
<td>Squareness errors</td>
<td>3</td>
<td>( S_{xy}, S_{yz}, S_{x} )</td>
</tr>
</tbody>
</table>

Each component of this machine tool all of which are represented in the global reference coordinate system has motion screws and error screws, such as axes, spindle, and milling head. These screws of the parts make up the kinematic chain and are basic to the topological structure. The geometric error on the work piece obtained by the coordinate system of the working table is chosen as the reference coordinate system. The exponential matrixes of error screws and motion screws are multiplied to obtain the geometric error model by the analysis of topological structure.

In general, a 3-axis machine tool has 21 error components in total, which include linear positioning errors, straightness errors, angular errors, squareness errors and parallelism errors. These errors are listed in the Table 1.

In the above list, \( \delta \) is the linear error, and \( \varepsilon \) is the angular error; \( \delta_{y} \) is the linear error in y direction while the X-axis is in motion. As shown in Fig.1, the first subscript is the direction of error and the other is the direction of movement, namely linear axis. Similarly, for \( \varepsilon_{yz} \), the first subscript is the rotation direction of error and the other is the direction of movement. The errors are expressed as twists, and the error modeling can be established by POE.

#### A. Geometric errors modeling in each axis by Twists

For three-axis machine tool, the axes are translational. For translational axis, the motion twist can be expressed as the product of one unit twist and the displacement.

\[ S_0 = \begin{bmatrix} 0,0,0,s_x,s_y,s_z \end{bmatrix}^T \]  

(13)

In which, \( s = \begin{bmatrix} s_x, s_y, s_z \end{bmatrix}^T \) represents the unit vector in direction of motion of the translational axis.

For instance, with regard to X-axis, \( S_x \) is the unit motion screw. The symbol x represents the displacement of X-axis. \( S_y, S_z \) represent the motion screws of Y-axis and Z-axis, respectively, and the symbols y and z represent the movement distance of each axis, respectively. Then the corresponding exponential matrixes are the ideal transformation matrixes of each axis. The motion twists and exponential matrixes are as follows

\[ S_x = \begin{bmatrix} 0,0,1,0,0,0 \end{bmatrix}^T \]  

(14)

\[ S_y = \begin{bmatrix} 0,0,0,1,0,0 \end{bmatrix}^T \]  

\[ S_z = \begin{bmatrix} 0,0,0,0,1 \end{bmatrix}^T \]
Due to manufacturing and installation defects, geometric errors inevitably exist in each axis. In general, six error components can be used to describe the geometric errors of a moving axis, because a rigid body has six degrees of freedom, which include three translational errors and three rotational errors. The six error components are as modular error components using the screw theory. They defined the modular error twist, $e^{ee}_{ee}$, as follows

\[
e^{ee}_{ee} = \frac{e^{ee}_{xx}, e^{ee}_{yy}, e^{ee}_{zz}, e^{ee}_{xy}, e^{ee}_{xz}, e^{ee}_{yz}}{T}
\]

(15)

Due to the deviation between the actual axis and the ideal axis, the angle between the adjacent axes is not equal to $90^\circ$, and this causes squareness error. The explanation on squareness errors is as follows: as the Y-axis is defined to align with the reference coordinate system, there is no squareness error (for actual Y axis). The squareness error $S_{xy}$ is between X-axis and Y-axis, $S_{yz}$ is between Y-axis and Z-axis, and $S_{xz}$ is between X-axis and Z-axis. The actual X-axis and Y-axis form a plane, and it is called the reference X–Y plane. There is only squareness error $S_{xy}$ for actual X-axis; meanwhile, for the actual Z-axis, there exist two other squareness errors, which are shown in Fig.2

\[
S_{xx} = \begin{bmatrix} 0, e_{xx}, 0, 0, \delta_{xx}, 0 \end{bmatrix}^T
\]

(22)

\[
S_{yy} = \begin{bmatrix} 0, 0, e_{yy}, 0, 0, \delta_{yy} \end{bmatrix}^T
\]

(23)

\[
S_{yy} = \begin{bmatrix} 0, 0, 0, 0, \delta_{xy}, 0 \end{bmatrix}^T
\]

(24)

\[
S_{xz} = \begin{bmatrix} 0, 0, e_{xz}, 0, 0, \delta_{xz} \end{bmatrix}^T
\]

(25)

\[
e^{ee}_{ee} = e^{ee}_{yy} \cdot e^{ee}_{xx} \cdot e^{ee}_{yy}
\]

(26)

\[
T^x = e^{ee}_{xx} \cdot e^{ee}_{yy} = e^{ee}_{xx} \cdot e^{ee}_{yy} \cdot e^{ee}_{yy}
\]

(27)

B. The POE models of squareness errors

Due to the deviation between the actual axis and the ideal axis, the angle between the adjacent axes is not equal to $90^\circ$, and this causes squareness error. The explanation on squareness errors is as follows: as the Y-axis is defined to align with the reference coordinate system, there is no squareness error (for actual Y axis). The squareness error $S_{xy}$ is between X-axis and Y-axis, $S_{yz}$ is between Y-axis and Z-axis, and $S_{xz}$ is between X-axis and Z-axis. The actual X-axis and Y-axis form a plane, and it is called the reference X–Y plane. There is only squareness error $S_{xy}$ for actual X-axis; meanwhile, for the actual Z-axis, there exist two other squareness errors, which are shown in Fig.2

\[
S_{xy} = \begin{bmatrix} 0, e_{xy}, 0, 0, \delta_{xy}, 0 \end{bmatrix}^T
\]

(22)

\[
S_{yy} = \begin{bmatrix} 0, 0, e_{yy}, 0, 0, \delta_{yy} \end{bmatrix}^T
\]

(23)

\[
S_{yy} = \begin{bmatrix} 0, 0, 0, 0, \delta_{xy}, 0 \end{bmatrix}^T
\]

(24)

\[
S_{xz} = \begin{bmatrix} 0, 0, e_{xz}, 0, 0, \delta_{xz} \end{bmatrix}^T
\]

(25)

\[
e^{ee}_{ee} = e^{ee}_{yy} \cdot e^{ee}_{xx} \cdot e^{ee}_{yy}
\]

(26)

\[
T^x = e^{ee}_{xx} \cdot e^{ee}_{yy} = e^{ee}_{xx} \cdot e^{ee}_{yy} \cdot e^{ee}_{yy}
\]

(27)

B. The POE models of squareness errors

Due to the deviation between the actual axis and the ideal axis, the angle between the adjacent axes is not equal to $90^\circ$, and this causes squareness error. The explanation on squareness errors is as follows: as the Y-axis is defined to align with the reference coordinate system, there is no squareness error (for actual Y axis). The squareness error $S_{xy}$ is between X-axis and Y-axis, $S_{yz}$ is between Y-axis and Z-axis, and $S_{xz}$ is between X-axis and Z-axis. The actual X-axis and Y-axis form a plane, and it is called the reference X–Y plane. There is only squareness error $S_{xy}$ for actual X-axis; meanwhile, for the actual Z-axis, there exist two other squareness errors, which are shown in Fig.2

\[
S_{xy} = \begin{bmatrix} 0, e_{xy}, 0, 0, \delta_{xy}, 0 \end{bmatrix}^T
\]

(22)

\[
S_{yy} = \begin{bmatrix} 0, 0, e_{yy}, 0, 0, \delta_{yy} \end{bmatrix}^T
\]

(23)

\[
S_{yy} = \begin{bmatrix} 0, 0, 0, 0, \delta_{xy}, 0 \end{bmatrix}^T
\]

(24)

\[
S_{xz} = \begin{bmatrix} 0, 0, e_{xz}, 0, 0, \delta_{xz} \end{bmatrix}^T
\]

(25)

\[
e^{ee}_{ee} = e^{ee}_{yy} \cdot e^{ee}_{xx} \cdot e^{ee}_{yy}
\]

(26)

\[
T^x = e^{ee}_{xx} \cdot e^{ee}_{yy} = e^{ee}_{xx} \cdot e^{ee}_{yy} \cdot e^{ee}_{yy}
\]

(27)
The actual exponential matrix of X-axis, can be written as
\[ e^{xS_x} = \begin{bmatrix} 1 & 0 & 0 & x\cos(S_{xy}) \\ 0 & 1 & 0 & y\sin(S_{xy}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (30)

Further, the adjoint matrix can be used to represent the coordinate transformation. According to the Eq. (11), the reference coordinate system rotates through an angle around the ideal Z-axis, and this process can be denoted as
\[ S_x = \text{Adj}(e^{-S_x}) \cdot S_z \] (31)
\[ S_z = [0,0,1,0,0,0]^T \] (32)

Where, the second subscript r represents that the screw rotates around the axis denoted by first subscript which means the nominal axis.

Similarly, the ideal unit motion screw for Z-axis is $s_z$, the actual unit motion screw is $s_x$, and exponential matrix is $e^{xS_x}$. These can be written as follow
\[ s_z = [0,0,0,0,0,1]^T \] (33)
\[ s_x = [0,0,0,\sin(S_{xy}),0,0,\cos(S_{xy})]^T \] (34)
\[ e^{xS_x} = \begin{bmatrix} 1 & 0 & 0 & -z\sin(S_{xy}) \\ 0 & 1 & 0 & z\cos(S_{xy}) \sin(S_{xy}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (35)

As in the method similar to that of the adjoint matrix coordinate transformation, the reference coordinate system rotates through an angle around the ideal Y-axis firstly, and then rotates through an angle around the ideal X-axis. This transformation can be shown as
\[ S_{zy} = \text{Adj}(e^{-S_y}) \cdot S_z \quad S_{zx} = \text{Adj}(e^{-S_x}) \cdot S_{zy} \] (36)

In the above, $S_{zy} = [1,0,0,0,0,0]^T$ is the unit rotation about ideal X-axis; and $S_{zy} = [0,1,0,0,0,0]^T$ is the unit rotation around ideal Y-axis.

C. Modeling of volumetric error using topological structure

In this paper, the three-axis machine tool is considered as an example, and its schematic diagram is shown in Fig.3. The multi-body system (MBS) theory is applied to obtain a detailed topological structure of the machine tools, which is shown in Fig.4 and it can be applied to the geometric errors POE model.

Under ideal conditions, errors do not exist. The order of modeling is in the following sequence: $X_i \rightarrow Y_i \rightarrow Z_i$.

The ideal transformation matrix, namely the ideal POE model, $T_i$ is obtained as
\[ T_i = e^{-xS_x} \cdot e^{-yS_y} \cdot e^{zS_z} \] (37)

In Fig. (3), every symbol is expressed as follows: 0---bed; 1---Worktable(X-axis); 2---slide carriage (Y-axis); 3--- RAM (Z-axis); 4--- cutting tool; 5--- work-piece

Figure 3. Structure diagram of the 3-axis machining center

Figure 4. Topological graph of the 3-axis machining center

With the error twists, squareness errors, linear errors, the actual POE model $T_{pe}$ of the machine tool as illustrated in Fig. 4 can be written as follow
\[ T_{pe} = e^{-xS_x} \cdot e^{-xS_{xz}} \cdot e^{-yS_y} \cdot e^{-yS_{yz}} \cdot e^{zS_z} \cdot e^{zS_{yz}} \cdot e^{zS_{xz}} \cdot e^{zS_{z}} \cdot e^{zS_{y}} \cdot e^{zS_{x}} \] (38)
The tool tip error is the deviation between the ideal and the actual homogeneous coordinates of the tool tip. And the error transformation matrix $E$, can be written as

$$E = T_{T}^{-1} T_{f}$$

(39)

The three parts of $E$ are $E_{x}$, $E_{y}$ and $E_{z}$.

$$[E_{x}, E_{y}, E_{z}, 1]^{T} = E [0, 0, 0, 1]^{T}$$

(40)

The installation errors for the tool and workpiece are too small to be ignored and are not considered in this paper. $E_{x}$, $E_{y}$ and $E_{z}$ are shown as

$$E_{x} = \delta_{x} - \delta_{x} - \delta_{y} + \delta_{y} + \delta_{z} e_{y} e_{y} - \delta_{z} e_{y} e_{y} + \epsilon_{y} e_{y} - \delta_{y} e_{y} - \delta_{y} e_{y} + e_{y} e_{y} - e_{y} e_{y}$$

(41)

$$E_{y} = \delta_{y} - \delta_{y} - \delta_{x} + \delta_{x} + \delta_{z} e_{x} e_{x} - \delta_{z} e_{x} e_{x} + \epsilon_{x} e_{x} - \delta_{x} e_{x} - \delta_{x} e_{x} + e_{x} e_{x} - e_{x} e_{x}$$

(42)

$$E_{z} = \delta_{z} - \delta_{z} - \delta_{x} + \delta_{x} + \delta_{y} e_{y} e_{y} - \delta_{y} e_{y} e_{y} + \epsilon_{y} e_{y} - \delta_{y} e_{y} - \delta_{y} e_{y} + e_{y} e_{y} - e_{y} e_{y}$$

(43)

In the Eq. (41)-(43), $x$ represents the X-axis displacement; $y$ represents the Y-axis displacement; $z$ represents the Z-axis displacement.

IV. KEY GEOMETRIC ERRORS IDENTIFICATION

According to the POE screw theory the mean analysis modeling of volumetric error for the multi-axis machining center can be expressed in matrix as

$$F = F (E, G, P_{w}, U, U_{w}, U_{i}, G_{f})$$

(44)

Where, $F = [f_{1}, f_{2}, \ldots, f_{r}]^{T}$ represents the $r$ vectors of independent equations; $E = [E_{x}, E_{y}, E_{z}, \delta]^{T}$ represents the volumetric error vector; $G = [g_{1}, g_{2}, \ldots, g_{s}]^{T}$ represents a vector consisting of $n$ geometric errors; $G = [\Delta_{T_{y}}, \Delta_{P_{w}}, \Delta_{A_{x}}, 1]^{T}$ represents squareness errors for three axes; $P_{w} = [P_{x}, P_{y}, P_{z}, 1]^{T}$ represents the work-piece forming point vector in the work-piece coordinate system; $U = [x, y, z, B]^{T}$ represents position vector of each motion axis; $U_{w} = [x_{w}, y_{w}, z_{w}, 1]^{T}$ represents the vector of work-piece coordinate position; $U_{i} = [x, y, z, 1]^{T}$ represents the vector of tool coordinate position.

In above sections, the volumetric errors modeling by POE screw theory. However, how to reduce the volumetric errors is also an important problem faced to machine tool manufacturers and users. The key geometric errors could be identified by sensitivity analysis. Because the size of fluctuations is decided by the errors variance, according to Eq.44, the variance can be expressed as

$$\Delta_{v} = \frac{\partial F}{\partial E} \Delta_{E} + \frac{\partial F}{\partial G} \Delta_{G} + \frac{\partial F}{\partial P_{w}} \Delta_{P_{w}}$$

(45)

As to the high precision 3-axis machining center, its variance can be abbreviated as

$$\Delta_{v} = \frac{\partial F}{\partial E} \Delta_{E} + \frac{\partial F}{\partial U} \Delta_{U} + \frac{\partial F}{\partial U_{w}} \Delta_{U_{w}}$$

(46)

In Eq. 46, the term $\Delta_{v}$ Error! Reference source not found. can be used to find the key geometric errors, and it can be unfolded as

$$\Delta_{v} = \frac{\partial F}{\partial \delta_{x}} \delta_{x} + \frac{\partial F}{\partial \delta_{y}} \delta_{y} + \frac{\partial F}{\partial \delta_{z}} \delta_{z} + \frac{\partial F}{\partial \epsilon_{x}} \epsilon_{x} + \frac{\partial F}{\partial \epsilon_{y}} \epsilon_{y} + \frac{\partial F}{\partial \epsilon_{z}} \epsilon_{z}$$

(47)

The partial derivatives $M = \frac{\partial F}{\partial E}$ Error! Reference source not found. are assigned to identify which geometric error has the greatest impact. After normalization as depicted, the key geometric errors can be identified easily.
In Eq. 48, \( m_{ni} \) means the importance sensitivity, and the total of \( m_{ni} \) is 1. In an axis, the biggest \( m_{ni} \) Error! Reference source not found. shows that the corresponding geometric error has major influences to the machining accuracy of machine tool. For example, the Fig. 5 shows the sensitivity of the errors in X-axis. It can be seen that \( \delta_{zx} \), Error! Reference source not found. have greater influence on the machining accuracy in X-axis. Similarly, Fig. 6 and Fig. 7 show the sensitivity of the errors in Y-axis and Z-axis. In Fig. 6, \( \delta_{yx} \) Error! Reference source not found., \( \delta_{yy} \) Error! Reference source not found. Error! Reference source not found. are the key errors in the Y-axis. The total Error! Reference source not found.of these five errors is 0.89, and the others are in 0.11. In Fig. 7, \( \delta_{zz} \) Error! Reference source not found., \( \delta_{zy} \) Error! Reference source not found. are the key errors in the Z-axis. The total Error! Reference source not found.of these five errors is 0.8. And the others are in 0.2.

Because the geometric errors of machine tool are caused by the geometric accuracy of the assembly components, there are mapping relationships between basic geometric errors and accuracy parameters. \( \delta_{xx} \), Error! Reference source not found., \( \delta_{xy} \) Error! Reference source not found. are resulted from the screw’s manufacturing precision that is the cumulative error of the pitch for the three axes. \( \delta_{zx} \), \( \delta_{zy} \) are the result of the straightness errors of vertical plane for the guide rails. \( \delta_{yx} \) Error! Reference source not found., \( \delta_{yy} \) Error! Reference source not found. are the result of the straightness errors of horizontal plane for the guide rails. \( \epsilon_{xx} \) Error! Reference source not found., \( \epsilon_{xy} \) Error! Reference source not found. are the result of the parallelism error for the guide rails. \( \epsilon_{yy} \) Error! Reference source not found. represents the assembly error of two axes. Till now, not only the key geometric errors are identified, but also the key components that have crucial influence to machining accuracy are gained.

V. CONCLUSIONS

Geometric errors are inevitable during the manufacturing or assembly process of the machine tool. The majority of errors can be compensated by the analysis.
of machine components, adjusting the installation process or the numerical control system and replacing defective or higher precision machine components. Particular for the precision and ultra-precision machining, the fluctuation will influence work-piece machining remarkably. How to reduce the volumetric errors is also an important problem faced to machine tool manufacturers and users.

In this paper, volumetric error modeling is established by the POE screw theory for the illustrative machining center. What’s more, to reduce the fluctuations in the machining process, and a method is introduced to identify the key errors and critical components of machine tool. Although in this paper the prediction and analysis method is proposed with illustration of a 3-axis machining center, it also be applicable to other multi-axis CNC machine tools.

REFERENCES


