ZHIPING LI et al: ASIAN TABLE TENNIS SIMULATION MODEL STUDY

Asian Table Tennis Simulation Model Study

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Abstract — As sport has flourished in recent years, table tennis has specially attracted increasing attention. Asia has always been the most developed region in world table tennis, and also takes the lead in the level of table tennis in the world. Table tennis simulation model is established with Asian table tennis characteristics together with the scientific description and interpretation on table tennis. This provides powerful theoretical support and references for Asian table tennis athletes’ training and table tennis development. Angle and intensity’s mathematical model is obtained through analyzing table tennis operational process and joint force and angle in horizontal and vertical direction. we then expand the model, considering loop drive in table tennis, and establishing table tennis dynamics mathematical simulation model. Since only gravity, air drag and Magnus force (a spinning ball curves away from its principal flight path) are considered in this model, more consideration of buoyancy and additional mass force suffered in table tennis operational process is taken. Improved table tennis simulation model is established that can simulate table tennis operational process to scientifically predict the result, so that perfect system is formed and evidence is provided to make training and selection more reliable. It is believed that such research result is important theoretically and of practical significance and value in training and selection. Moreover, it improves Asian table tennis level and provide scientific guidance for the development of the game.

Keywords - mathematical model; table tennis simulation; operational process;loop drive

I. INTRODUCTION

Asian table tennis has always taken the lead in the world; nearly swept most of championship and runner-up in world table tennis championship, and in the ranking of world table tennis of international table tennis federation, Asian players have nearly made up 60%, especially based on main Asian power as China, Japan and South Korea. Asian table tennis is characterized as smaller figure and stronger flexibility by comparing with Europe and America. Table tennis exercising is characterized as high speed, good flexibility and more tactical strategies, and tends to be quite difficult. To grasp such event and win glory for the state in Olympic Games, athletes should undergo long-term hard training. Among them, loop drive is a relative critical part in training. What cannot ignore in loop drive operational process is Magnus effect found by G Magnus, a German in 1952; with main principle a phenomenon arises in by high speed rotated table tennis in air stream [1-7].

The development of table tennis is closely linked to various disciplines, such as materials science, physics and mathematics, all of which promotes table tennis scientific development. Main operations of table tennis are service and receiving, in which process intensity and service angle of athlete is mainly considered, and table tennis intensity and angel models are therefore established. After that, the model is expanded, considering loop drive suffered force in operational process, and table tennis dynamics simulation model is established [8-10]. On this basis, having model improved, adding forces suffered in loop drive operational process into consideration, and improved table tennis dynamics simulation model is then established. Finally, it gets the conclusion that table tennis flight distances can be effectively controlled and ball can be dropped into opponent table at high speed by increasing loop drive [11-14].

As far as table tennis such a specific sports event is concerned, it has unique nature although possessing common training process and rules as other athletic sports. Therefore, to find training process and rules conform to such special features from numerous factors, and constantly study and explore in these main factors that is a process to let table tennis move towards success. Rapidly improve and increasingly develop table tennis levels, and apply more and more modern science and technology in table tennis would be beneficial to table tennis performance, so to speak. This may also increase the dependence on modernized cutting-edge technology and means, ad require lots of sports researchers to provide scientific and theoretical basis for athlete training and selection by virtue of all kinds of research results.

II. MODEL ESTABLISHMENT

Development of table tennis is closely linked to mathematics, kinematics and mechanics. Provided that mechanics and mathematics knowledge is properly applied in table tennis, direction of the ball can be
changes as one’s thoughts and so skills in table tennis can be improved to a certain extent. Furthermore, main operations of table tennis are service and receiving, in which process intensity and service angle of athlete is mainly considered. In the following, it makes analysis of suffered intensity and angle in horizontal and vertical direction during table tennis operational process, and establishes table tennis intensity and angle’s mathematical models.

A. Table tennis differential equation

Establish space coordinate system (o-xyz) as shown in figure, table midpoint coordinate origin o, from the perspective of hitter, right hand direction is positive direction of x axis, forward is positive direction of y axis, upward is positive direction of z axis, as shown in Figure 1.

![Figure 1. Space coordinate system](image)

Standard coordinate of a table tennis table shown in figure, make it dimension-less with standard table tennis data and get; net width is \( \frac{125.152}{1830} \), table length and width are respectively \( \frac{1897.175}{2740} \approx 105.152 \); Radius \( r \) non-dimensional quantity of table tennis at 40mm diameter is: \( \frac{20}{152.5} = 0.131 \) (net height)

During non-dimensional on process, gravity accelerated speed (half) non-dimensional quantity is: \( g = \frac{9.8 \times 10^3}{152.5} = 64.262 \) (net height / \( S^2 \))

Set table tennis surrounding x, y, z axis rotational angles are respectively \( x_1, y_1, z_1 \), table tennis air movement instant suffered action force is as shown in figure. In Figure, \( (x_1, y_1, z_1) \) is table tennis mass center speed component in the direction of \( (x_1 y_1 z_1) \) is table tennis surrounding \( x(y, z) \) axis rotational angular speed component; \( F_{ex}, F_{oy}, F_{oz} \) are respectively air resistance, rotational resistance and Magnus force components, according to basic assumption 5, calculate as following formula: \( F_{ex} = -k_x \rho m \dot{x}_1, F_{oy} = -k_y \rho J \dot{y}_1, F_{oz} = k_m \rho m (x_1 y_1 - y_1 x_1) \)

Similarly other components computing formulas are:
\( F_{rx} = -k_x \rho m \dot{x}_3, F_{ry} = -k_y \rho J \dot{y}_3, F_{rz} = k_m \rho m (z_3 x_3 - x_3 z_3) \)

In formula, \( m = 2.7 \times 10^{-4} \) kg is table tennis mass, \( J = \frac{2}{3} m r^2 \) is table tennis rotational inertia, coefficient \( k_v, k_a, k_m \) according to experience and by comparing with experiment, take approximately value, as shown in Figure 2.

\( k_v = 38.1, k_a \approx 0, k_m = 7.6 \)

![Figure 2. Table tennis force Schematic diagram in the air](image)

According to Newton second law, it can get table tennis differential equation during movement process.

\( x'(t) = x_1(t), y'(t) = y_1(t), z'(t) = z_1(t) \)
\( x_1'(t) = x_1(t), y_1'(t) = y_1(t), z_1'(t) = z_1(t) \)
\( x_3'(t) = k_m r(y_1(t) - z_1(t)) - k_a r x_2(t) \)
\( y_2'(t) = k_m r(z_1(t) - x_1(t)) - k_a r y_2(t) \)
\( z_3'(t) = k_m r(x_1(t) - y_1(t)) - k_a r z_2(t) - g \)
\( x_2'(t) = -k_a r x_2(t) \)
\( y_2'(t) = -k_a r y_2(t) \)
\( z_2'(t) = -k_a r z_2(t) \)

In formula \( x_2, y_2, z_2 \) are respectively table tennis...
mass center $x, y, z$ directions speed components; $x_3, y_3, z_3$ are respectively table tennis axis rotational angular speed components when surrounding $x, y, z; k_v, k_w, k_m$ are respectively air speed resistance coefficient, rotational resistance coefficient and Magnus force parameter, and take values by formula(3).

Establish table tennis drop point time and kinetic energy integration optimization mathematical planning model that similar to time optimization control model, as shown in Figure 3.

![Coordinate system schematic diagram](image)

Figure 3. Coordinate system schematic diagram

$$f = k \frac{t_f}{0.09443} + (1-k) \frac{46.652}{T(f)}$$

$$\min \ s.t \ g_1 = x_3(t_{mov}) - (1 + r) \geq 0$$

$$g_2 = x_3(t_f) \geq 0 \quad g_3 = 9 - x_3(t_f) \geq 0$$

$$g_4 = x_3(t_{mov}) + 5 \geq 0 \quad g_5 = 5 - x_3(t_f) \geq 0$$

$$g_6 = v_{max} - \left( \sqrt{v_0 \cdot v_0} + \frac{2 r^2 \omega_0 \cdot \omega_0}{3 \sqrt{v_0 \cdot v_0}} \right) \geq 0$$

$$g_7 = 1 - \frac{2 r^2 \omega_0 \cdot \omega_0}{3 \sqrt{v_0 \cdot v_0}} \geq 0$$, And $h_1 = v_0 \cdot \omega_0 = 0$

Among them, k value range is 0~1, when $k = 1$, it is time optimization control, when $k = 0$, it is kinetic energy optimization control. Take $k = 10\%$

Table tennis initial position and maximum swinging speed values are as following:

$x_3(0) = 0, x_3(0) = -10, x_3(0) = 1,$

$x_3(0) = 0, x_3(0) = 0, x_3(0) = 0$

$v_{max} = 200$

Model solution result:

$t_f = 0.11964s \quad t_{mov} = 0.06551s$

$v_0 = [0.153, 0.013, 44.049], \quad \omega_0 = [-752.455, 0, 0]$

$\alpha = \frac{d}{r} = 0.4132, \quad \{x, y, z\}_v = \{0.8, 2319, 0.1311 = r\}$

$z(t_{mov}) = 1.78478, \quad T(t_f) = 43.188$

Time and kinetic energy integrated optimization loop drive initial angular speed is:

$$\omega_n = \frac{752.455}{2\pi} = 119.757/s$$.

III. MODEL EXTENSION

Development of table tennis is closely linked to mathematics, kinematics and mechanics. Provided that mechanics and mathematics knowledge is properly applied in table tennis, direction of the ball can be changes as one’s thoughts and so skills in table tennis can be improved to a certain extent. Furthermore, main operations of table tennis are service and receiving, in which process intensity and service angle of athlete is mainly considered. In the following, it makes analysis of suffered intensity and angle in horizontal and vertical direction during table tennis operational process, and establishes table tennis intensity and angle’s mathematical models. Then, taking consideration of buoyancy and additional mass force, it establishes improved table tennis dynamics simulation model on the basis of such model and gets the conclusion that table tennis flight distances can be effectively controlled by loop drive at stronger rotating speed and ball can be dropped into opponent table.

A. Model establishments

This table tennis simulation analog model mainly considers major influential factors of table tennis during movement process as gravity, air resistance, Magnus force, as shown in Figure 4.

![Force analysis figure](image)

Figure 4. Force analysis figure

Gravity is G, direction is downward. Air resistance direction is opposite to table tennis movement direction,
its size is up to air density $\rho$, windward area $A$, line speed $v$ and resistance coefficient $C_d$ etc., according to fluid dynamics formula, calculate as following:

$$F_d = \frac{1}{2} C_d \rho A v^2$$

Among them, resistance coefficient $C_d$ is got by wind tunnel experiment. This paper assumes resistance coefficient is only up to table tennis translational speed and angular speed, adopt following method to calculate:

Computational formula is as follows:

$$F_m = \frac{1}{2} C_m A \rho \left( \frac{\vec{\omega} \times \vec{v}}{\vec{\rho}} \right)^2$$

where:

$\vec{v}$ is table tennis speed vector during flight process,

$\vec{\omega}$ is table tennis angular speed vector during analysis process,

$F_m$ is Magnus force

In formula, $C_m$ is Magnus force lift force coefficient obtained by experiment measuring. $C_m$ can be worked out according to formula as shown in the following:

$C_m = \frac{1}{2.022 + 0.981 \omega}$

According to Newton second law $F = ma$ list out following equation, get table tennis mathematics and dynamics simulation model.

$$\begin{bmatrix}
\cos \alpha \cos \varphi & \cos \beta \cos \theta & 0 \\
\cos \alpha \sin \varphi & \cos \beta \sin \theta & 0 \\
\sin \alpha & \sin \beta & 1
\end{bmatrix}
\begin{bmatrix}
\frac{dv_x}{dt} \\
\frac{dv_y}{dt} \\
\frac{dv_z}{dt}
\end{bmatrix}
= m
\begin{bmatrix}
\frac{dv_x}{dt} \\
\frac{dv_y}{dt} \\
\frac{dv_z}{dt}
\end{bmatrix}$$

This model carries out improvement of table tennis dynamics simulation model, adding buoyancy and additional mass force on such model, as shown in Figure 5.

B. Establishment of table tennis dynamics improved simulation model

On the basis of model one, multiple action forces table tennis model is established to further perfect and supplement [7]. The expression of gravity $F_g$ is as following:

$$F_g = mg = \frac{1}{6} \rho d^3 \eta$$

where, $\rho_d$ is table tennis density, $d$ is table tennis diameter, standard diameter is 40 mm, mass is around 2.7 g.

Buoyancy force $F_b$ is table tennis discharged force in the air with equal volume of air action force:

$$F_b = m_{ai} = \frac{1}{6} \rho a d^3 \eta$$

where, $\rho_a$ is air density.

For induction magnetic field generated additional mass force, it is represented as:

$$F_c = \frac{1}{12} \rho m a d^3 \eta = \frac{1}{2} m' \frac{dv}{dt}$$

$\frac{1}{2} m'$ is always recording as $m'$, is called additional mass.
Table tennis resistance in the air is represented as:

\[ F_D = C_D \frac{1}{8} \pi \rho_d d^2 V^2 \]

where, \( C_D \) is resistance coefficient, \( V \) is absolute speed. The expression of resistance coefficient \( C_D \) is:

\[
C_D = \frac{24}{R_a} \quad R_a < 1
\]

\[
C_D = \frac{24}{R_a} (1 + \frac{R_a}{1000}) \quad 1 < R_a < 1000
\]

\[
C_D = 0.44 \quad (1000 < R_a < 2 \times 10^4)
\]

where, \( R_a \) is Reynolds number.

For spheres, Magnus force can be expressed by following formula:

\[ F_M = \frac{1}{8} \pi \rho_a d' V \omega \]

where, \( \omega \) is rotational speed.

Wee obtain dynamical equations during table tennis movement process respectively as:

Horizontal:

\[
(m + m') \frac{d^2 x}{dt^2} = -\frac{1}{2} C_l \rho x \frac{dx}{dt} \left(\frac{d^2 x}{dt^2} + \frac{1}{8} \pi \rho a \frac{dy}{dt}\right)
\]

Vertical:

\[
(m + m') \frac{d^2 y}{dt^2} = -F_x - F_z - \frac{1}{2} C_l \rho x \frac{dx}{dt} \left(\frac{d^2 x}{dt^2} + \frac{1}{8} \pi \rho a \frac{dy}{dt}\right)
\]

where \( t \) is time.

Sphere is the standard 40mm large ball, 2.7g weight; ambient air is in normal pressure and temperature; rotational speeds are calculated with different levels as 0, 40, 80, 120, 160r/s.

### TABLE 1. MODEL INITIAL PARAMETERS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial translational speed</td>
<td>( V=11 \text{m/s}, \theta=15^\circ )</td>
</tr>
<tr>
<td>Initial rotational speed</td>
<td>0, 40, 80, 120, 160r/s</td>
</tr>
<tr>
<td>Sphere</td>
<td>( m=2.7 \text{g}, d=40 \text{mm} )</td>
</tr>
<tr>
<td>Initial position</td>
<td>( x=0, y=0 )</td>
</tr>
<tr>
<td>Environment</td>
<td>( T=20^\circ \text{C}, P=1 \text{atm} )</td>
</tr>
</tbody>
</table>

Asian table tennis players have certain advantages over European and American players in table tennis due to their own physical quality, nevertheless, it is still suggested that they should strengthen development of their own advantages. By establishing table tennis model, every athlete corresponding influence factors can be found to study so that reasonable training can have targeted implemented on relevant athletes to achieve best performance, as shown in Table 1.

In view that Asian table tennis current study status lags behind that in the world as well as the demands for table tennis further development, evacuating regular factors from numerous factors, facilitating training and personnel selection of table tennis, so that evidence is provided to form perfect system and training and selection can be more reliable. Such a research result is of important theoretical guiding, practice significances and extraordinary practical values in training and the selection of athletes, which further improves Asian table tennis level, provides scientific guidance for the development of Asian table tennis, can be used to performance prediction and study in athlete training, evaluate relative technical indicators and increase scientific in training process. Moreover, the research result also can be applied in the selection of athletes, characterized as justice, equality and science, available for better predicting selection results and targeted training and selecting of relevant athletes.

### IV. CONCLUSION

Research in this paper focused on table tennis operational process mathematical model. Initially, it considered table tennis serve and receive process, carried out analysis of table tennis operational process suffered horizontal direction force, vertical direction force as well as horizontal and vertical directions joint force, established joint force angle mathematical mode. The model considered operational trajectory when table tennis didn’t rotate, and based on that it combined with practice, considering table tennis rotation operational trajectory, it was relative intuitive. After that, it extended model, considered table tennis operational process suffered gravity, air resistance, Magnus force, established table tennis dynamics simulation model by force analysis and applying fluid mechanics knowledge. Then on that basis, it considered buoyancy force and additional mass force, established table tennis dynamics simulation improved model. The improved model can simulate table tennis movement process and carry out scientific prediction on the result. The model structure operates in the manner going deeper from simple, gradually increasing influential conditions, forming progressive model that is helpful for shaping model completely and practical, as well as deepening the study of overall table tens flight process. Table tennis simulation model is characterized as concise structure, clear and all-encompassing differential process. It is easy to operate and propaganda with progressive iteration and simpler calculation.

With regard to athlete, thus, loop drive play should be grasped and rational applied in competition so that the winning probability could be increased to a certain degree, which also is of very important practical significance and extraordinary practical value, capable of further improving Asian table tennis level and providing scientific references for development thereof. Analysis of
table tennis movement process suggests that attacking techniques training and stalemate ability should be strengthened, ability of attacking and pulling in middle-long table should be improved, the proportion of physical training should be increased for the sake of improving players’ physical ability and skills and effective controlling and prepared judging table tennis flight path, as a result of the larger influence of table tennis force controlled intensity and angle on the ball’s flight process.

REFERENCES


