A Study on Fuzzy Control Learning Algorithms Based on Cascade Observers

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Abstract - The design of a fuzzy learning controller does not depend on the model of the controlled object. But it depends very closely on the experiences and knowledge of control experts or operators. However, it is difficult to design a high-level fuzzy learning controller. Moreover the fuzzy learning controller is not easy to control its learning rate and adjustment of its parameters, which makes the structure of the self-adaptive fuzzy learning controller difficult. Self-adaptive learning algorithms can effectively compensate for the decline of control performance caused by the imperfection of the rule base. We consider the two kinds of forms of self-adaptive fuzzy learning control. The methods are applied to the control of the small-scale learning machine and achieve good performance and effect.

Keywords - model free; cascade observer; fuzzy learning controller; self-adaptive; learning machine

I. INTRODUCTION

At present, many scholars have studied the design of the course channel controller for small unmanned learning machines under the condition of the model having been established. In fact, the dynamical models of course channels of small unmanned learning machine with different types and different parameters are different. And the establishment of the course channel model is cumbersome and complex, so it is necessary and urgent to research the model free control method independent of mathematical models. As one of the most important methods in the model free control field, the intelligent control method has garnered widespread attention from scholars [1-3]. But on account of being in the stage of theoretical research without many practical applications, the intelligent control method is tried in the design of course channel controller for learning machines in this paper.

II. PROBLEM DESCRIPTION

In this paper, we study the system aiming at the following class of nonlinear systems [4-5]:

\[ \dot{x} = f(x) + bu(t) + d(t) \]

where \( f(x) \) and \( d(t) \) denote the unknown nonlinear function and the complex time-varying interference; \( u(t) \) is the control input:

\[ x = [x_1, x_2, L, x_n]^T = [y, y L, y^{(e-1)}]^T. \]

Expand the SISO method to the following multi-input and multi-output model later.

\[ y_i^n = f_i(X, t) + b_i u(t) + d_i(t), \quad i = 1, L, p \]

where \( X = [X_1^T, X_2^T, L, X_p^T]^T \) is the output differential vector; \( X_i = [y_i, y_i^{(0)}, L, y_i^{(n-1)}]^T \) is the \( i \)th output differential vector; \( y_i \) and \( u_i \) respectively express the \( i \)th output and the \( i \)th input; \( d_i(t) \) denotes the unknown complex time-varying interference. It assumes that \( f_i(\cdot) \) is the unknown time-varying bounded smooth nonlinear function and satisfies an increasing condition, namely Lipchitz.

A. Cascade Observer

In the actual system, through sensors we can only obtain the position signals, but not the high order differential signals, so it is necessary to obtain it through mathematical methods. Tracking-differentiator is a useful tool to obtain high-order differential signals. Studies on this field are much more, and there is mainly high-gain observer method, etc. A cascaded observer design method proposed in this section is used to estimate high order differential signals [6-7]. The purpose of the design is to ensure the convergence of the observer cascade method, i.e.

\[ \hat{x}_1 = x_1 \rightarrow \hat{x}_2 = \hat{x}_1^C \rightarrow \hat{x}_2 = \hat{x}_1 \rightarrow \hat{x}_3 = \hat{x}_2 \rightarrow \hat{x}_3, \]

\[ \hat{x}_2 = \hat{x}_2^C, \quad L, \]

\[ \hat{x}_n+1 = \hat{x}_n \rightarrow \hat{x}_{n+1} = \hat{x}_n^C = \hat{x}_n = x_1^{(n)} \]
Based on this concept, the designed cascade observer is shown as follows:

\[
\dot{\hat{x}}_i = \hat{x}_{i+1} + l_i (y - \hat{x}_i) + \hat{p}_i \text{sgn}(y - \hat{x}_i), \quad i = 1, 2, 3, n \tag{3}
\]

\[
\dot{\hat{x}}_i = l_{ni+1} (\hat{x}_{i+1} - \hat{x}_{ni+1}) + \hat{p}_{ni+1} \text{sgn}(\hat{x}_{i+1} - \hat{x}_{ni+1})
\]

Utilizing the following conditions:

\[
|\hat{x}_{i+1} - \hat{x}_i| \leq \rho_i, \quad i = 1, 2, 3, n + 1, \quad \text{and} \quad \text{sgn}(a) = \begin{cases} 
1 & \text{if } a > 0 \\
0 & \text{if } a = 0 \\
-1 & \text{if } a < 0 
\end{cases}
\]

Note: \(\rho_i, \quad i = 1, 2, 3, n + 1\) is the upper limit value and a constant. This will be proved in theorem 1. Theorem 1 shows that \(\rho_i\) has a boundary. \(\hat{\rho}_i\) denotes the estimated value of \(\rho_i\). \(l_i\) is a positive value. When select \(l_i, s_i, \hat{l}_i\) is even larger than \(l_{ni+1}\). The reason is that the comparison of estimating the previous step exceeds the cascade structure of estimating the latter[8-9]. Such gain selection needs a smaller gain value to increase with the state order, which is different from the high-gain observer requiring a higher gain value to increase with the state order.

\[
\hat{x}_i = \begin{cases} 
\gamma_i |\hat{x}_{i+1} - \hat{x}_i| & \text{if } \hat{p}_i \leq \bar{\rho}_i \\
\gamma_i \left(1 + \frac{\hat{\rho}_i - \rho_i}{\delta_i}\right) |\hat{x}_{i} - \hat{x}_i| & \text{other} 
\end{cases}, \quad i = 1, 2, 3, n + 1 \tag{4}
\]

where \(\delta_i\) is a positive constant, that plays a big role for reducing the self-adaptive gain \(\hat{\rho}_i > \rho_i\) and preventing the divergence of \(\hat{\rho}_i\). \(\bar{\rho}_i\) can be set to any positive, and the choice should be rational, because they affect the transient response of the cascade observer[10-11]. The formula (4) guarantees \(\hat{\rho}_i\) bounded.

Theorem 1: The designed cascade observers (3) and (4) can guarantee the asymptotic stability of the estimation error of the differential, i.e., \(\lim_{t \to \infty} \sum_{i=1}^{21} (\hat{x}_i(t) - \hat{x}_i(t)) = 0\).

Prove: In order to test the stability analysis of the proposed observer, we have considered the Lyapunov function.

\[
V = \frac{1}{2} \sum_{i=1}^{n+1} (\hat{x}_{i+1} - \hat{x}_i)^2 + \frac{1}{2} \sum_{i=1}^{n+1} \frac{1}{\rho_i} 
\]

Where \(\rho_i = \rho_i - \hat{\rho}_i\). Following the trajectory of the system, i.e. the differential of \(V\), we can get

\[
\dot{V} = \frac{1}{2} \sum_{i=1}^{n+1} l_i (\hat{x}_{i+1} - \hat{x}_i)^2 + \frac{1}{2} \sum_{i=1}^{n+1} \frac{1}{\rho_i} 
\]

Put formula (5) into formula (6), and we can get

\[
\dot{V} = \sum_{i=1}^{n+1} \left[ (\hat{x}_{i+1} - \hat{x}_i)(\hat{x}_{i} - \hat{x}_i) + \hat{p}_i \text{sgn}(\hat{x}_{i+1} - \hat{x}_i) \right]
\]

Then, as \(V(0)\) and \(l_i\) are constants:

\[
\dot{V} \leq -\frac{1}{2} \sum_{i=1}^{n+1} l_i (\hat{x}_{i+1} - \hat{x}_i)^2
\]

From formula (8), we can confirm that \((\hat{x}_{i+1} - \hat{x}_i)\) and \(\rho_i\) are bounded. Similarly, \(\hat{\rho}_i\) is bounded from formula (3.4). Integrate the formula (8), and we can get:

\[
\int_0^T \sum_{i=1}^{n+1} l_i (\hat{x}_{i+1} (t) - \hat{x}_i (t))^2 dt \leq -V(t) + V(0) \leq V(0) 
\]

Then, as \(V(0)\) and \(l_i\) are constants:

\[
\frac{1}{2} \sum_{i=1}^{n+1} (\hat{x}_{i+1} (0) - \hat{x}_i (0))^2 + \sum_{i=1}^{n+1} \frac{1}{\rho_i} (0) 
\]
\[
\sum_{i=1}^{n+1} (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i) \in L_2.
\]

As \( \rho_i, \hat{\rho}_i, (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i) \),
and
\[
\hat{x}_{i+1} - \mathbf{f}_{i+1} \mathbf{e}
\]
have been proved bounded, we can get:
\[
\sum_{i=1}^{n+1} \frac{d}{dt} (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i) \in L_\infty.
\]

Through using Barbalat's lemma:
\[
\lim_{t \to \infty} \sum_{i=1}^{n+1} (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i) (t) - \hat{\mathbf{x}}_i (t) = 0.
\]

Theorem 2: (non-adaptive stability) for the system (1), the cascade differential tracker of \( \gamma_i = 0 \) is given to guarantee that the differential estimation error is globally uniformly bounded.

Prove: as \( \gamma_i = 0 \), and the parameter estimated value \( \hat{\rho}_i \) is a constant. For the non-adaptive Lyapunov function, we hold that
\[
V_{\hat{\rho}} = \frac{1}{2} \sum_{i=1}^{n+1} (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i)^2
\]
From the formula (7), we can obtain:
\[
\frac{1}{2} \sum_{i=1}^{n+1} (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i)^2 + \sum_{i=1}^{n+1} \hat{\rho}_i (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i)
\]
where set \( \dot{\chi} = 2V_{\hat{\rho}} = \sum_{i=1}^{n+1} (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i)^2 \),

\[
l_0 = (n+1) \min_{i \in \mathbb{N}} l_i \quad \text{and} \quad \beta_0 = (n+1) \max_{i \in \mathbb{N}} \beta_i.
\]

Then use
\[
\sum_{i=1}^{n+1} (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i) \leq \sqrt{n+1} \left( \sum_{i=1}^{n+1} (\mathbf{f}_i \mathbf{e} - \hat{\mathbf{x}}_i)^2 \right)^{1/2} = \sqrt{n+1} \chi^{1/2}.
\]

By Schwarz inequality, we can get:
\[
\frac{d\chi}{dt} \leq 2\sqrt{n+1} \beta_0 \chi^{1/2} - 2l_0 \chi
\]
By replacing, solve this type of equation. In fact, set
\[
\eta = \chi^{1/2}, \quad \text{and we can get}
\]
\[
\frac{d\eta}{dt} = \frac{1}{2} \chi^{-1/2} \frac{d\chi}{dt}
\]
Rewrite (12), and we can get
\[
\chi^{-1/2} \frac{d\chi}{dt} = \frac{2}{2\sqrt{n+1} \beta_0 - 2l_0} = \frac{2\sqrt{n+1} \beta_0 - 2l_0 \eta}{2\sqrt{n+1} \beta_0 - 2l_0}
\]
Using the formula (14) into the formula (13), we can get:
\[
\hat{\mathbf{f}} + l_0 \eta = \sqrt{n+1} \beta_0
\]
Solve the linear equation (16) to get the solution of \( \eta \), and it is available that
\[
\eta(t) = \sqrt{n+1} \beta_0 \left( 1 - e^{-l_0 t} \right) + \eta(0) e^{-l_0 t}
\]
\[
= \sqrt{n+1} \beta_0 + e^{-l_0 t} \left( \eta(0) - \sqrt{n+1} \beta_0 \right)
\]
Return \( \chi \) by the replacement \( \chi = \eta^2 \), then we can get:
\[
\chi(t) = \left\{ \sqrt{n+1} \beta_0 + e^{-l_0 t} \left( \eta(0) - \sqrt{n+1} \beta_0 \right) \right\}^2
\]
\[
\leq \left\{ 2\sqrt{n+1} \beta_0 \left( \eta(0) - \sqrt{n+1} \beta_0 \right) + (n+1) \left( \beta_0 \right)^2 \right\}^2
\]
This proves that the solution of \( \chi \) is globally uniformly non-adaptive bounded, i.e., when \( t \to \infty \),
\[
\chi(t) \to (n+1) \left( \beta_0 \right)^2.
\]

III. THE DESIGN OF MODEL FREE CONTROLLER

Set \( y_i \) ( \( i = 1, L \) ) as the \( i \)th given input. \( \mathbf{X}_i = [y_i, y_i, \ldots, y_{i-L}, y_{i-L}^{(n-1)}]^T \) is the \( i \)th given input differential vector. \( \mathbf{X}_i = [x_i, y_i, \ldots, y_{i-L}]^T \) and \( \mathbf{X}_i = [x_i, y_i, \ldots, y_{i-L}]^T \) denote respectively the \( i \)th given input expanding differential vector and output expanding differential vector.
\( E_i = [E_{i1}, E_{i2}, L, E_{i\text{m}}]^T = X_n - X_i \)
\( = [e_i, \hat{e}_{i(1)}, L, \hat{e}_{i(n-3)}]^T \)
\( \dot{E}_i = X_n - \dot{X}_i = [E_i^T, \hat{e}_{i(n)}]^T \)  

(18)

\( E_i \) and \( \dot{E}_i \) in formula (18) denote respectively the \( i \)th error differential vector, the \( i \)th error expanding differential vector. Therein, \( e_i = y_n - y_i \). It assumes that \( y_n \) can achieve the \( n \)-order bounded differential[12-13].

Theorem 3: For system (2), the model free controller can be expressed as follows:
\[
u_i = \frac{1}{b_i} \mathbf{K}_i \dot{E}_i + \hat{u}_i \quad i = 1, L, p
\]  
(19)

where \( \mathbf{K}_i = [k_{i1}, L, k_{i1},1] \) makes:
\[s^n + k_{i1}s^{n-1} + L + k_{i1} \text{ become a Hurwitz polynomial,}
\]
\( \hat{u}_i \) is the estimated value of \( u_i \). Then the received model free control algorithm has the following properties.

1) It can realize the linear decoupling control.
2) It makes the closed-loop system asymptotically stable, and meets the following convergence:
\[
\lim_{t \to \infty} \lim_{s \to \infty} X_i = X_n
\]  
(20)

3) All system variables are bounded.

Proof: putting formula (19) into (2), it is available that
\[
y_i^{(n)} = f_j(X,t) + b_j \left( \mathbf{K}_i \dot{E}_i + \hat{u}_i \right) + d_i(t),
\]
\[i = 1, L, p
\]  
(21)

Then, we can get
\[
\left( y_i^{(n)}+k_{i1}y_i^{(n-1)}+L+k_{i1}y_i \right) = \left( y_i^{(n)}+k_{i1}y_i^{(n-1)}+L+k_{i1}y_i \right) + b_i \delta
\]
\[i = 1, L, p
\]  
(22)

where \( \delta_i = \hat{u}_i - u_i \). Based on formula (22), \( p \) linear decoupling differential equations are obtained. From the definition of scalar \( e_i \) in formula (22) and the definition of scalar \( e_i \), we can get:
\[
e_i^{(n)} = -k_{i1}e_{i(n-3)} - L - k_{i1} e_i + b_i \delta_i
\]  
(23)

i.e.,
\[
\mathbf{K}_i e_i = -k_{i1} E_{i(n-3)} - L - k_{i1} E_{i} + c_i b_i \delta_i
\]  
(24)

It is easy to get the following important equation from equation (24) and the definition of vector \( E_i \) in equation (25).

\[
\mathbf{E} = A_{im} E_i + c_i b_i \delta_i
\]  
(25)

where \( A_{im} \in \mathbb{R}^{n\times n} \) is a controllable matrix, and the matrix parameters are \( k_{im}, L, k_{il} \) and \( c_i = [0, L, 0,1]^T \in \mathbb{R}^{n+1} \). Similar to the proof of the Theorem 1, \( \delta_i \to 0 \). Moreover, from formula (25), it can be seen that \( s^n + k_{i1}s^{n-1} + L + k_{i1} \) is a Hurwitz polynomial, and we can get
\[
\lim_{t \to \infty} E_i = 0
\]  
(26)

Hence, based on the higher-order differentiator in the above section, we can obtain the model free controller of the MIMO system below.

IV. SIMULATION AND VERIFICATION

In this section, use the dynamical model of a miniature learning machine in vertical flight to verify MIMO model free control. The learning machine flight height changes with the change of the pitch angle. Neglecting the influence of ground factors, the learning machine dynamic equations are described as follows:
\[
\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 + \Delta
\]
\[
y = [y_1, y_2]^T = [x_1, x_4]^T
\]  
(27)

where \( x = [x_1, x_2, x_3, x_4, x_5]^T = [h, \theta, \omega, \dot{\theta}]^T \). \( h \) is height. \( \omega \) is the rotate speed of rotor blade, \( \dot{\theta} \) is the collective pitch pitching angle of rotor blade. \( u_1 \) is the throttle control input. \( u_2 \) is the collective pitch control input. \( g_1(x) = [0, 0, 1, 0, 0]^T \), and:
\[
g_2(x) = [0, 0, 0, 0, 1]^T
\]

\( \Delta \) is the modeling uncertainty.

\[
f(x) = [f_1, f_2, f_3, f_4, f_5]^T =
\begin{bmatrix}
x_2 \\
a_0 + a_2 x_2 + a_2 x_2^2 + (a_3 + a_4 x_4 - \sqrt{a_5 + a_6 x_4}) x_5^2 \\
a_7 + a_5 x_5 + [a_8 x_5 + a_9 x_5^3] x_5^2 \\
[a_1 + a_2 x_4 + a_3 x_5^3] x_4 + a_4 x_5
\end{bmatrix}
\]  
(28)

Formula (28) is written as the following form.
Height \( h \) is associated with the rotary speed of propeller blade and collective pitch pitching angles as well. Therefore, there are strong coupling characteristics in learning machine vertical dynamics.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_2 = a_0 + a_2 x_2 + a_3 x_2^2 + \left( a_4 + a_5 x_4 - \sqrt{a_0 + a_6 x_6} \right) x_3^2 \\
\dot{x}_3 &= f_3 + u_1 = a_0 + a_5 x_5 + \left( a_7 x_7 + a_8 \right) x_3^3 + u_4 \\
\dot{x}_4 &= x_5 \\
\dot{x}_5 &= f_5 + u_2 = a_9 + a_5 x_4 + a_6 x_6 \sin x_4 + a_7 x_7 + u_2
\end{align*}
\]

(29)

It can be seen from the learning machine vertical dynamical model (29) that throttle control \( u_1 \) acts on \( \dot{\omega} \) directly, and collective pitch control input \( u_2 \) acts on \( \dot{\theta} \) directly. The outputs of the controlled objects of learning machines are height \( h \) and collective pitch pitching angle \( \theta \). \( u_1 \) has control relations with the rotary speed of propeller blade and collective pitch pitching angles. \( u_2 \) also has control relations with collective pitch pitching angles and rotary speed of propeller blade.

One of the two designed tracking differentiators is third-order, and another is second-order. The model parameters in simulations are as follows:

\[
\begin{align*}
a_0 &= -17.67, & a_5 &= 5.31 \times 10^{-4}, \\
a_5 &= 2.82 \times 10^{-7}, & a_3 &= 5.31 \times 10^{-4}, & a_6 &= 1.632 \times 10^{-5} \\
a_7 &= -13.92, & a_8 &= -0.7, & a_9 &= a_{10} = -0.0028, \\
a_{11} &= 434.88, & a_{12} &= -800, & a_{13} &= -0.1, & a_{14} &= -65.
\end{align*}
\]

In the case (M=2), take the throttle input value of 0, and take the collective pitch control input value of 15. The learning machine height and the time domain response curve of pitch angles are shown in figure 6. It can be seen from the open-loop characteristics in figure 5 and figure 6 that whatever the simple collective pitch control input or the simple throttle control input, the height and the pitch angle all have definite influence. Hence, the control coupling of the learning machine model is equally serious.
Aiming at the small UAV course system with disturbances, the paper has considered its flight control problem, and designed the indirect fuzzy learning self-adaptive supervision course controller. First, take the model transformation for the learning machine course system, i.e., the system is transformed into a second-order nonlinear time-varying system. Next, design an indirect fuzzy learning self-adaptive course controller for the system. Since the designed fuzzy learning self-adaptive course controller depends on the selection of the fuzzy learning rule base, in order to reduce this dependence, continue to design a supervision controller in the second layer, so that the stability of the closed-loop system is guaranteed. The combination of the two methods can make the controller design possess great freedom and flexibility, which can greatly reduce the dependency of control on whether the selection of the rule base is correct or not. Finally, all designed methods are applied to the control of the small-scale learning machine course system to realize the tracking control of the learning machine course channel and achieve good control performance and effect.

ACKNOWLEDGMENT

This work was supported by Foundation of Science and Technology Development Project of Henan Province in 2015, Project Number 152400410347.

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