A Novel Enterprise Horizontal Merger Model Based on Tax Factors

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Abstract — In this paper we use Cournot oligopoly competition model to introduce the tax factors into SSR model, and apply game theory method to establish: i) an extension model of horizontal mergers, ii) the condition of enterprise mergers and acquisitions, and iii) analyze the social welfare effect change before and after mergers. We found that: i) when enterprises operate purely in order to obtain tax benefits, the implementation of merger was inversely proportional to its success with the industry enterprises; ii) when other synergistic effect is not zero, the larger other synergetic effect is, the more easily mergers tend to succeed. To the four different merger environments, the government should use various tax tools to influence the effective tax rate after the merger and after-tax profit margins, guiding enterprise merger behavior to social welfare maximization.

Keywords - Cournot compete; tax factors; synergistic effect; merger environment

I. INTRODUCTION

In new normal economy, the market of enterprise merger and reorganization is faced with many contradictions and problems, such as enterprises’ dominant position is not fully respected, market mechanism fails to fully exert effects and governments’ management and service are to be improved and so on. In practice, many reorganization schemes are postponed due to taxation such as the case of Latitude Communications merging Zm Software Technology Co, Ltd. and the case of reorganizing SDICZL. Only further optimization of market environment for enterprise merger and reorganization can effectively give play to merger and reorganization and promote positive effects of industrial structure adjustment. In the Thesis, factor of income tax is introduced to SSR model to discuss various environment and tax policies of enterprise merger so as to provide reference for optimization of market environment for enterprise merger and reorganization.

II. LITERATURE REVIEW

Domestic researches on merger model are mainly expanded based on foreign researches. Ma Xin’an, Feng Yun and Zhang Lieping (2000) researched merger strategies of upstream and downstream enterprises in Cournot competition; Hua Bing and Chen Hongmin (2003) found enterprises with stronger differentiated products had strong motivation of horizontal merger based on endogenic productivity and their merger can facilitate improvement of social welfare. Based on linear requirements and marginal productivity and their merger can facilitate improvement of social welfare. Based on linear requirements and marginal production cost of asymmetric constant, Da Qingli (2005) researched profitability of horizontal enterprise merger and problem of social welfare in Stackelberg oligarch monopolistic competition market in a broad sense. Based on SSR model of horizontal merger, An Guo and Wu Jiang (2007) introduced the concept of synergistic effect, established one expanded model of horizontal merged based on synergistic effect and then analyzed change of welfare for market participants in different conditions so as to acquire the critical value of synergistic effect by taking welfare change as standard and to explain contradictory phenomena with frequent merger in original SSR model with no merger motivation and market. Based on researches of An Guo and Wu Jiang (2007), factor of income tax is introduced in the Thesis to SSR model in consideration of synergistic effect and merger cost to make the model conform to practical situation. And then, in-depth analysis is carried out on change of social welfare before and after merger to summarize four kinds of merger environment and merger tax policies applicable to every kind of environment and provide reference for government to establish tax policy which is good for market environment of merger and reorganization.

III. COURNOT HORIZONTAL MERGER MODEL BASED ON TAX FACTORS

Company merger(1) may acquire financial synergy effects. In view of Stephen A. Ross (2012), merger synergy effects come from: decrease in cost after merger, increase in income, decrease in profit and decrease in tax burden. In view of special conditions in company merger tax, we express the first three merger synergy effects in S and other synergy effects except tax can be interpreted as cost deduction (income increase can be regarded as relative decrease of cost). The fourth synergy effect can be named as tax synergy effect and it is expressed as t′<t (t is the actual income tax rate before merger and t′ is the actual income tax rate after merger). This inequation means actual tax burden of enterprises implementing merger decreases and thus realize tax synergy. In actual merger, whether to acquire above-mentioned two synergy effects needs to be analyzed according to practical situation.

Next, based on competition model of Cournot oligopoly, tax factor is introduced into SSR model and the method of game theory is used to establish an extended model of horizontal merger so as to analyze conditions of enterprise
merger as well as change in social welfare effects before and after merger and derive different merger environment.

A. Game Process of Enterprise Merger

In case there are n enterprises (n≥3) in one industry and each enterprise produces homogeneous product and fixed cost of each enterprise is zero; the initial marginal cost is C; market inverse demand function of the homogeneous product is

\[ P = \beta - Q \]

among which P is product price, \( \beta \) is constant, Q is the gross output in this industry, qi is the output of the i enterprise,

\[ Q = \sum qi, i=1,2,...,n \]

and enterprise’ after-tax profit \( \pi \) is expressed in

\[ \pi = (P-C)qi(1-t) \].

In case enterprises’ actual tax rate before merger is \( t(0<t<1) \), the actual tax rate after merger is \( t'(0<t'<1) \).

Before merger, n enterprises choose output at the same time to carry out Gounod output competition and the market is faced with Gounod nash equilibrium. The optimal output of each enterprise meets the following conditions:

\[ \max \pi_i = (P-C)qi(1-t) = (\beta - \sum_{i=1}^{n} qi-C)qi(1-t) \]

Above-mentioned equation specifies partial derivative of qi and sets it to be zero, then

\[ qi = \frac{\beta - \sum_{i=1}^{n} qi-C}{n} \]

And then

\[ qi = \frac{\beta - C}{n} \] (1)

At this time, maximized profit is

\[ \pi_i = (\beta - C)qi(1-t) = \left( \frac{\beta - C}{n} \right)^2(1-t) \] (2)

The merger process is as follows: m+1 enterprises in n enterprises are merged into one and m enterprises with the same quality and the rest n-m enterprises will carry out Gounod output competition and the market is faced with Gounod nash equilibrium. The optimal output of each enterprise meets the following conditions:

\[ \max \pi_i = (P-C)qi(1-t) = (\beta - \sum_{i=1}^{n} qi-C)qi(1-t) \]

Above-mentioned equation specifies partial derivative of qi and sets it to be zero, then

\[ \frac{\pi_i}{\pi} = (P-C)qi(1-t) = (\beta - C)qi(1-t) \]

And then

\[ qi = \frac{\beta - C}{n} \] (3)

At this time, maximized profit is

\[ \pi_i = (\beta - C)qi(1-t) = \left( \frac{\beta - C}{n} \right)^2(1-t) \]

B. Conditions of Enterprise Merger

As for merger enterprises, merger is feasible only if profit after merger \( \pi^* \) is more than profit before merger (m+1) \( \pi_i \). In case the function \( g(n,m,s,t,t') \) is the difference of profit before and after enterprise merger, then

\[ g(n,m,s,t,t') = \pi^* - (m+1)\pi_i \]

(1) In case \( S=0 \), the purpose of merger is to acquire tax benefits. At this time, conditions for merger are:

\[ g(n,m,0,t,t') = \pi^* - (m+1)\pi_i \]

Namely

\[ qi = \frac{\beta - C - q^*}{n - m} \] (4)

Profit of merger enterprise is

\[ \pi^* = [P-(C-S)]q^*(1-t) \]

\[ = [\beta-(q_1+q_2+...+q_{n-m-1}+q^*)-(C-S)]q^*(1-t) \] (5)

Maximum profit of merger enterprise

\[ \pi^* = \frac{\beta - C + S(n-m)}{n - m + 1} ] (1-t) \] (6)

In case of substituting Equation (2) into Equation (1), we can know output of individual non-merger enterprise

\[ qi = \frac{\beta - C - S}{n - m + 1} ] (1-t) \] (7)

Maximum profit of individual non-merger enterprise is

\[ \pi_i = \frac{\beta - C - S}{n - m + 1} ] (1-t) \] (8)

Total output of the market after merger is

\[ Q = \frac{(n-m-1)}{n - m + 1} \]

\[ = (n-m-1) \]

\[ \frac{\beta - C - S}{n - m + 1} + \frac{b - C + S(n - m)}{n - m + 1} \]

\[ \frac{b - C + S(n - m)}{n - m + 1} \]

\[ = (n-m)\left( \frac{\beta - C + S}{n - m + 1} \right) \] (9)

(10)
When other synergy effects are zero, the condition for enterprise to implement merger to acquire tax synergy effect is that the proportion of after-tax profit rate after merger and after-tax profit rate before merger is more than \( (m+1)(1-\frac{m}{n+1})^2 \).

When two enterprises are merged, \( m=1 \) and then
\[
(m+1)(1-\frac{m}{n+1})^2=2(1-\frac{1}{n+1})^2
\]
According to previously set condition \( n \geq 3 \), we can know that \( 2(1-\frac{1}{n+1})^2 \geq 1.125 \).

Therefore, when two enterprises in the industry are merged and when enterprise trying to avoid tax merges another enterprise, whether merger succeeds does not depend on absolute value of enterprises’ after-tax profit but it is related to increase degree of after-tax profit rate after merger. Merger profit will increase in case actual after-tax profit rate after merger increases by at least 12.5%. At this time, the possibility of enterprise merger has a negative correlation with the number of enterprises in this industry. In case the number of enterprises in this industry is less, the proportion of after-tax profit rate before merger 1-t’ and after-tax profit rate after merger 1-t will be less and the possibility of merger success will be higher; therefore, the possibility of merger will be higher; on the contrary, in case the number of enterprises in this industry is greater, requirements on after-tax profit after merger will be higher and it will be more difficult for merger to succeed.

In realistic economy, the number of enterprises in one industry is far more than 3 and the requirements for the proportion of after-tax profit rate before and after merger are much higher and it is more difficult for merger to succeed. To a certain degree, this conclusion proves that it is not easy to choose a company of tax nature for acquisition of tax payment to implement merger to succeed. In theoretical cycle, perspectives about influences of tax on selection of merger targets are not uniform. Especially, whether tax is an important factor influencing selection of merger target is a vexed topic. Alan J. Auerbach (1983) thinks that there is no enough convincing evidence proving tax property plays a decisive role in selection of merger target; Gilson and Ronald J. (1988) think that the theoretical base to regard tax as decisive factor in selection of merger target is relatively weak. Alan J. Auerbach and James M. Poterba (1987) also think that tax may not play a decisive role in determination of merger target but may have great influences in other aspects such as selection of merger opportunity.

(2) In case \( S \neq 0 \), the purpose of merger is to acquire synergy effects. At this time, conditions for merger are:

\[
g(n,m,s,t,t')=[\frac{\beta-C+S(n-m)}{n-m+1}\frac{2(1-t')}{(m+1)(1-\frac{m}{n+1})^2}]
\]

When other synergy effects \( S \) acquired from merger are not zero, the condition for merger to succeed is the proportion between after-tax profit rate after merger and after-tax profit rate before merger conforms to Equation (12). It is obvious that the right part of Equation (12) will be less in case \( S \) is greater and it is much easier to set the inequation hold; therefore, it is much easier for merger to succeed in case other synergy effects \( S \) acquired from merger are greater. Similarly, in case the number \( n \) of enterprises in this industry is greater, requirements on after-tax profit after merger will be higher and it will be more difficult for merger to succeed. Specially, when \( m=1 \), namely when two enterprises in one industry are merged, Equation (12) is as follows:

\[
1-t'>a(1-t)
\]

Enterprise merger area is \( t'>1-a+at \).

In case \( t'>1-a+at \)

The straight line represented by Equation (13) is the enterprise merger line. It is known from \( a=(m+1)I(1-\frac{m}{n+1})^2 \) that \( a<0 \) and three key points \( (0, 1-a), (1,1) and ((a-1)/a, 0) \) of straight line (13) can be acquired after respectively substituting \( t=0, t=1 \) and \( t=0 \) into Equation (13). And then we can draw the graph of straight line (13), which can be divided into the following two circumstances:

When \( 0<a<1 \) (as shown in Fig. 1), the dash area OABC refers to value range of inequation \( t'<1-a+at \) in interval of \([0, 1]\), namely merger area of enterprises; the dash area ABC refers to merger area of enterprises. It is obvious that the dash area is divided into two parts by the straight line OB \((t=t')\). In area of OBC, \( t'>t \), and the merger profit of enterprise at this time is greater than the sum of profits before merger. However, the actual tax rate \( t' \) after merger is greater than actual tax rate \( t \) before merger and the tax

\[
g(n,m,s,t,t')=[\frac{\beta-C+S(n-m)}{n-m+1}\frac{2(1-t')}{(m+1)(1-\frac{m}{n+1})^2}]
\]
synergy effects of merger shall be negative; in merger area of OAB, \( t' < t \), and enterprises have acquired positive tax synergy effects by means of merger.

When \( a \geq 1 \) (as shown in Fig. 2), the dash area ABC refers to enterprise merger area and it is just below the straight line OB and enterprises have acquired positive tax synergy effects.

When a \( \geq 1 \) (as shown in Fig. 2), the dash area ABC refers to enterprise merger area and it is just below the straight line OB and enterprises have acquired positive tax synergy effects.

\[
W = \frac{1}{2} \frac{(n-m)(\beta-C)+S}{n-m+1} t^2 + \left[ \frac{\beta-C + S(n-m)}{n-m+1} \right] 2(t-t') + (n-m-1)[(\beta-C-S)\beta-C-S]2(1-t')
\]

Changes in social welfare before and after merger:

\[
\Delta W = \frac{1}{2} \frac{(n-m)(\beta-C)+S}{n-m+1} t^2 + \left[ \frac{\beta-C + S(n-m)}{n-m+1} \right] 2(t-t') + (n-m-1)[(\beta-C-S)\beta-C-S]2(1-t') + (n-m-1)[(\beta-C-S)\beta-C-S]2(1-t') > 0
\]

We can know

\[
1-t' > \frac{1}{2} (n2(\beta-C+S)\beta-C+S)j2 - \left\{ \frac{n-m(\beta-C+S)}{n-m+1} \right\} 2
\]

\[
1-t' > \frac{1}{2} (n2(\beta-C+S)\beta-C+S)j2 - \left\{ \frac{n-m(\beta-C+S)}{n-m+1} \right\} 2
\]

In case

\[
b = \frac{1}{2} (n2(\beta-C+S)\beta-C+S)j2 - \left\{ \frac{n-m(\beta-C+S)}{n-m+1} \right\} 2
\]

\[
b = \frac{1}{2} (n2(\beta-C+S)\beta-C+S)j2 - \left\{ \frac{n-m(\beta-C+S)}{n-m+1} \right\} 2
\]

\[
1-t' > b + d(1-t')
\]

and then the merger area permitted by the government is

\[ t' < 1-b-d+dt \]

In case

\[ t' = 1-b-d+dt \]
The straight line represented by Equation (14) is the merger line permitted by the government. After respectively substituting t=0, t=1 and t'=0 into Equation (14), we can know three key points (0, 1-b-d), (1, 1-b) and ((b+d-1)/d,0). Due to complex forms of b and d, it is impossible to determine their symbols and then the position of merger line permitted by the government will be discussed according to the following several circumstances.

Figure 3. \(d \geq 0, 0 < 1-b \leq 1, 1-b-d \geq 0\)

Figure 4. \(d \geq 0, 0 < 1-b \leq 1, 1-b-d < 0\)

1. \(d \geq 0, b \geq 0\). It can be known from \(d > 0\) that the straight line (14) has opposite symbols with intercept of horizontal axis \((b+d-1)/d\) and intercept of vertical axis \(1-b-d\); it can be know from \(b \geq 0\) that \(1-b \leq 1\) and the enterprise merger area permitted by the government is shown in OEDA in Fig. 3 and EDA cross grain in Fig. 4. Coordinate of point D is \((1, 1-b)\), and it is obvious that \(t' \geq 0\) is meaningless when \(b \geq 1\) namely \(1-b \leq 0\). Therefore, we only need to consider the circumstance of \(0 < 1-b \leq 1\) at this time.

Figure 5. \(d \geq 0, 1-b > 1, 1-b-d \geq 1\)

Figure 6. \(d \geq 0, 1-b > 1, 0 < 1-b-d < 1\)

Figure 7. \(d \geq 0, 1-b > 1, 1-b-d < 0\)

Figure 8. \(d \geq 0, 0 < 1-b < 1, 1-b-d = 0\)

2. \(d = 0, b < 0\). It can be known from \(b < 0\) that \(1-b > 1\). When \(1-b-d = 1\), the merger line permitted by the government is the position shown in Fig. 5 and the merger
area permitted by the government is the rectangle OABC; in case \(0<1-b-d<1\), the slash dash area OABCD shown in Fig. 6 is the merger area permitted by the government; when \(1-b-d \geq 0\), the slash dash area ABCD shown in Fig. 7 is the merger area permitted by the government.

(3) \(d<0,b \geq 0\). It can be known from \(b>0\) that \(1-b \leq 1\). In case y-axis intercept \(1-b-d \leq 0\) and \(t'<0\) is meaningless, we need to consider four circumstances when \(1-b-d>0\), which are slash dash area shown in Fig. 8, Fig. 9, Fig. 10 and Fig. 11.

(4) \(d<0,b<0\). It can be known from \(b<0\) that \(1-b>1\) and the merger area permitted by the government is the slash dash area shown in Fig. 12 and the enterprise merger in the whole rectangle OABC is good for social welfare maximization.

**E. Four Kinds of Merger Environment**

Next, we will analyze merger action of enterprises and regulation standards of government according to the plane figure. First of all, get to know the intersection point of two straight lines and then solve a system of equations according to Equation (15) and Equation (16)

\[
t' = 1 - a + at \\
t' = 1 - b - d + dt
\]

In this case, we can know the intersection point of two straight lines \(t = l - \frac{b}{a - d}, t' = l - \frac{ab}{a - d}\). It is obvious that the intersection point of two straight lines has no obvious characteristics. According to above-mentioned Fig. 1 to Fig. 12, we can reduce intersection of enterprise merger line and merger line permitted by the government into four types:

(1) The first kind of merger environment: the merger area permitted by the government is larger than enterprise merger area. For example, according to combination of Fig. 1 and Fig. 6 and as shown in Fig. 13 that the straight line (14) is located at the left side of straight line (13) and two straight lines intersect in interval \((-\infty, 0)\). Slash stripe area is the value range of inequation \(t'<1-b-d+dt\) in interval \([0,1]\), namely merger area permitted by the government. At this time, the government permits merger area is larger than enterprise merger area and the shade and slash overlapping area OABC is the optimal merger area which is permitted by the government and good for enterprises. In this kind of merger environment, merger action of enterprise is good for social welfare maximization such as energy saving, environmental protection, high and new technology and other industries. At this time, the tax policy for merger shall be “neutral”. At the same time, tax departments need to create more convenience for enterprises in tax declaration, approval, collection and other tax services so as to make it convenient for enterprises to accomplish merger.
(2) The second kind of merger environment: part of merger area permitted by the government intersects with enterprise merger area. According to combination of Fig. 1 and Fig. 3 and as shown in Fig. 14, two straight lines intersect in interval $[0,1]$. The slash stripe area OAFD is the merger area permitted by the government, gray shadow area OABC is the enterprise merger area and the overlapping area OAFEC of shadow and slash stripe is the merger area which is permitted by government and good for enterprises. The enterprise merger in shadow area in EFB will damage social welfare and it is the reflection of market failure and the government is required to interfere with enterprise merger and implement non-neutral tax policy especially in industry with serious excessive production capacity. The actual tax burden in enterprise merger area EFB, and therefore the government can prepare more preferential tax policy in OAFEC area to make profit of enterprises in EFB area after merger be less than after-merger profit in OAFEC area so as to guide merger action of enterprise to facilitating social welfare maximization.

(3) The third kind of merger environment: the merger area permitted by the government is less than enterprise merger area. According to combination of Fig. 1 and Fig. 4 and as shown in Fig. 15 that the public area with intersection of two straight lines is the shadow slash part consisting of the triangle ADE and enterprise merger in this area is good for social welfare maximization. However, the enterprise merger in dash area OCBDE is not good for social welfare maximization and serious market failure appears. This is the merger area where the government is required to interfere such as industries of equipment, backward technology and serious environmental pollution. Government can guide enterprise merger to ADE area by imposing sewage discharge fees and other heavy taxation policies.

(4) The fourth kind of merger environment: merger area permitted by the government has no intersection with enterprise merger area. According to combination of Fig. 2 and Fig. 11 and as shown in Fig. 16, two areas may have intersection or not. We have analyzed intersection circumstances and we will analyze the circumstances with no intersection. The straight line $t' = 1 - b - d + dt$ intersects with the straight line $t = 1 - a + at$ in the fourth quadrant and the merger area permitted by the government has no intersection with enterprise merger area in $[0, 1]$. Even in merger scope permitted by the government, enterprise merger can not realize increase in profit. Therefore, no enterprise is willing to carry out merger, which is also reflection of market failure. As a result, what government needs to do is to formulate preferential tax policy, reduce actual income tax rate of enterprises, make enterprise merger line move to the upper left and finally reduce actual income tax burden of enterprises and make the enterprise merger line intersect with merger line permitted by the government in the first quadrant and thus realize the guidance of tax policy on enterprise merger. As shown in Fig. 16, enterprise merger line moves upward to position of straight line $h$ and intersects with merger line permitted by the government at point F. The triangle area FGD is namely enterprise merger area caused...
by tax policy. In this area, enterprises increase profit through merger and this is also good for social welfare maximization.

IV. CONCLUSION

After introducing other synergy effect $S$ and tax factor $t$ into SSR mode, we can find condition for merger success depends on the proportion of actual after-tax profit rate after merger and actual after-tax profit rate before merger. It is not easy for merger implemented only for tax synergy effects to succeed and the difficulty level is inversely proportional to the number $n$ of enterprises in this industry; when acquired other synergy effect $S$ is not zero, it will lower the difficulty in merger caused by increase in number of enterprises in this industry and this is great possibility for merger to succeed.

By deduction of enterprise merger policy and change in social welfare after merger, we can know enterprise merger area and the merger area permitted by the government and thus knows four kinds of merger environment of the intersection of these two. Different tax policies are applicable to different merger environment. Characteristics of merger environment depend on position of merger line permitted by the government and enterprise merger line among which permitted merger line depends on $b$ and $d$ while enterprise merger line depends on $a$, $b$, and $d$ depend on total number $n$ of enterprises in this industry, number $m$ of merged enterprises, inverse market demand constant $\beta$, average marginal cost $C$ and other synergy effect $S$ that enterprise may acquire and these factors are collectively named factors of industry characteristics and they decide actual tax rate after enterprise merger. The government shall judge factors of industry characteristics, estimate enterprise merger line, judge merger environment of this industry and prepare different tax policies according to industry information so as to influence actual tax rate and actual profit rate after merger, guide enterprise merger decision to develop for the benefit of social welfare maximization and thus create merger environment which can facilitate optimization and upgrading of industrial structure. Even though preconditions of Cournot model are too simple when compared with reality, conclusions we have may be used for reference for government to prepare merger tax policy.

ACKNOWLEDGMENTS

Project supported by the National Social Science Foundation of China (15BJY172); the National Social Science Foundation of China (15BJY062), Research on tax structure optimization and operation path under the restriction of Stable tax in China(15CJY071).

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