Optimization Model for Separated Oil Production in Horizontal Well of Low-Permeability Reservoir with Edge Water

Shan LIU 1, Yu-liang SU 2, Ha LIU 3

1, 2 College of Science, China University of Petroleum, Qingdao 266580, China
3 Overseas Business Department of GRI, BGP, Zhuozhou, 072751

Abstract — We study separated oil production in horizontal well of low-permeability reservoir with edge water, and propose the separated completion method of changing blind-screen ratio and introduce the starting pressure gradient. On the basis of the potential superposition, reflection principle of mirror image and the idea of infinitesimal congruence of lines, the coupling models for the separated oil production in horizontal well of low-permeability reservoir with edge water under the ways of open-hole completion and perforation completion are respectively established. Taking the yield of horizontal well as the target function and the length and position of central point of production horizontal well segment as the optimal variables, the optimization models for two completion methods are respectively established. Finally, a genetic algorithm is utilized with numerical simulation software to carry out the optimization design for the separation scheme at the time of separated oil production with the horizontal well, while the optimal separation schemes under different segmentation modes are respectively given.

Keywords - low permeability; horizontal well; horizontal segment; separated oil production; genetic algorithm; oil reservoir/wellbore coupling model; optimization model

I. INTRODUCTION

Aiming at the low permeability reservoir, because the horizontal well itself has the advantages of large contact area, large drainage area and high output-input investment ratio, in the development and design of low permeability reservoir, the horizontal-well production technique [1-2] acquires the increasing extensive application. However, during the horizontal-well production, influenced by the bottom water and edge water, the containing water increases after a period of time for well production to form the water cone and thus influence the development effect of horizontal well. Sealing carried out at the position close to the water and the production scheme of separated oil production [3-5] can decrease the cost, put off the water breakthrough and enhance the oil recovery to reach the water control and oil stability. During the separated oil production in horizontal well of low permeability reservoir, the capacity of horizontal well is in close related to the segmentation number of horizontal wellbore and position and length of horizontal segment [6-7]. Aiming at the optimization problem of the separated oil production in horizontal well, most of the present research achievements all adopt the principle[8] that the production horizontal segments evenly distribute at the horizontal wellbore, to observe the rule of yield changing with the segmentation number of horizontal wellbore and length of production horizontal segment under the premise of number and length of fixed production horizontal segment, and do not start from the yield to reversely seek the optimization separation scheme of biggest yield. Aiming at the separated production in horizontal of low-permeability reservoir with edge water under the ways of the open hole completion and perforation completion, the writer starts from the yield, introduces the starting pressure gradient and adopts the separated completion method with blind pipe, in which the blind pipe and production horizontal segment in radial arrangement at the horizontal wellbore, to establish the optimization model of which the length and position of production horizontal segment is taken as the research object. Under the premise of biggest yield, the genetic algorithm is utilized to reversely seek the optimal separation scheme. Finally, through the designed open degree, segmentation number, starting pressure gradient and other parameters and combination of actual oil field data, the sensitivity analysis is carried out on the factors which influence the capacity of optimization model.

II. SEEPAGE MODEL FOR SEPARATED OIL PRODUCTION IN HORIZONTAL WELL OF LOW-PERMEABILITY RESERVOIR WITH EDGE WATER

Aiming at the non-linear seepage [9-10] existing in the low permeability reservoir, to simplify the problem, non-Darcy linear seepage [12-13] with starting pressure gradient [11] is adopted to research the problem of low permeability. The basic motion formula of non-Darcy seepage considering the starting pressure gradient is:

\[
\nu = \frac{k}{\mu} \left( \frac{dp}{dr} - G \right)
\]

(1)

Where: G is the starting pressure gradient.

define the fluid displacement pressure as \( \bar{p} = p - Gr \),
Where: $Gr$ is equivalent to the starting pressure of the point, namely, the displacement pressure of a point is equal to that formation pressure of the point subtracting the starting pressure of the point.

Then the motion equation (1) is turn into

$$v = \frac{k}{\mu} \frac{d\bar{p}}{dr} \tag{2}$$

In the low permeability reservoir, define the pseudo-potential:

$$\phi = \frac{k}{\mu} p = \frac{k}{\mu} (p - Gr) \tag{3}$$

$$v = \frac{d\phi}{dr}$$ Substitute the formula (3) into the formula (2) and get:

Substitute the defined displacement pressure and pseudo-potential function into the seepage formula of low permeability reservoir

$$\frac{d^2\bar{p}}{dr^2} + \frac{1}{r} \frac{d\bar{p}}{dr} = 0 \tag{4}$$

From this, it can be seen that, both the displacement pressure and pseudo-potential function which aim at the re-definition satisfy the linear seepage formula, namely: by introducing the concepts of displacement pressure and pseudo-potential function, turn the non-Darcy problem of low permeability into the Darcy linear problem. Therefore, the potential superposition, reflection principle of mirror image and others are still applicable in the low permeability reservoir.

Take a point convergence $(x_p, y_p, z_p)$ in the low permeability reservoir at will, then the pseudo-potential [14] created by the point convergence at any point $M(x, y, z)$ in the oil reservoir is:

$$\phi(x, y, z) = -\frac{Q}{4\pi r} + c \tag{5}$$

Where: $Q$ is the yield of point convergence; $r$ is the distance of point convergence to the point $M$.

Figure 1. Horizontal wells with separated production scheme

Assume that there is one horizontal well for separated oil production (as Fig.1), its effective length is $L$; horizontal wellbore is divided into $m$ segments; the length of $i$th segment is $L_i$ and the coordinate of middle point is $(x_i, y_i, z_i)$. Regarded it as a congruence of lines, then the pseudo-potential created by the $i$th horizontal segment on any point $M$ in the formation is equivalent to the integral of formula (4) at the $i$th horizontal segment, namely:

$$\phi_i(x, y, z) = \int_{L_{i-1}/2}^{L_{i}/2} \phi(x, y, z) dt$$

$$\phi_i(x, y, z) = \int_{L_{i-1}/2}^{L_{i}/2} -\frac{q_i}{4\pi L_i} \frac{1}{\sqrt{(x-t)^2 + (y-y_i)^2 + (z-z_i)^2}} dt + c$$

Where, $(t, y_i, z_i)$ is the coordinate of any point in the $i$ th segment and $q_i$ is the flow rate flowing from the formation into the $i$th segment of horizontal wellbore, m$^3$/d.

For the formula (5), get the integral:

$$\phi_i(x, y, z) = -\frac{q_i}{4\pi L_i} \ln \frac{r_1 + r_2 + L_i}{r_1 + r_2 - L_i} + c \tag{6}$$

Where:

$$r_1 = \sqrt{(x_i - L_i / 2 - x)^2 + (y_i - y)^2 + (z_i - z)^2}$$

Assume that the upper and bottom boundaries of edge-water reservoir are the closed outer boundaries and $y = a$ is the oil-water interface of edge water; the model schematic diagram is shown as the Fig.2.
Aiming at the edge-water reservoir, by utilizing the reflection principle of mirror image \[15\], the pseudo-potential created by the \(i\)th horizontal segment on any point \(M(x, y, z)\) in the oil reservoir is gotten:

\[
\psi_i(x, y, z) = -\frac{q_{ji}}{4\pi L_i} \sum_{n=0}^{\infty} \left\{ \xi_n(2nh + z_n, x, y, z) + \xi_n(-2nh + z_n, x, y, z) - \xi_n(-2nh - z_n, x, y, z) + \xi_n(z_n, x, y - 2a, z) - \xi_n(-z_n, x, y - 2a, z) \right\}
\]

\[
= -\frac{q_{ji}}{4\pi} \sum_{n=0}^{\infty} \left\{ \xi_n(2nh + z_n, x, y, z) + \xi_n(-2nh + z_n, x, y, z) - \xi_n(-2nh - z_n, x, y, z) + \xi_n(z_n, x, y - 2a, z) - \xi_n(-z_n, x, y - 2a, z) \right\} + c
\]

From the potential superposition\[16\], the pseudo-potentials created by each production segment of the whole horizontal well on any point \(M(x, y, z)\) in the oil reservoir is:

\[
\psi(x, y, z) = \sum_{i=1}^{m} \psi_i(x, y, z)
\]

\[
= \frac{1}{4\pi} \sum_{i=1}^{m} q_{ji} \phi_{ji}^E(x, y, z) + c
\]

Where: the superscript \(E\) represents the edge-water reservoir,

\[
\phi_{ji}^E(x, y, z) = \frac{1}{L_i} \left\{ \xi_n(z_n, x, y, z) + \xi_n(-z_n, x, y, z) \right\}
\]

Suppose the pseudo-potential at the middle point \((x_j, y_j, z_j)\) of the \(j\)th segment of horizontal wellbore is \(\psi_{w_j}\) and its wellbore flowing pressure is \(p_{w_j}\); then, from the formula (7), get:

\[
\psi_{w_j} = \frac{1}{4\pi} \sum_{i=1}^{m} q_{ji} \phi_{ji}^E(x_j, y_j, z_j) + c
\]
\[ \psi_{ij}(x_j,y_j,z_j) = -\frac{1}{4\pi} \sum_{i=1}^{m} q_i \phi_i^E(x_j,y_j,z_j) \Delta - \frac{1}{4\pi} \sum_{i=1}^{m} q_i \phi_i^E \]  \tag{8}

Take a point at the supply boundary of edge water; set the boundary pressure in correspondence with the point is and its boundary pseudo-potential is; then, from the formula (8), get:

\[ \psi_c(L/2,a,z_w) = -\frac{1}{4\pi} \sum_{i=1}^{m} q_i \phi_i^E \left( \frac{L}{2}, r_e, z_w \right) \Delta - \frac{1}{4\pi} \sum_{i=1}^{m} q_i \phi_i^E \]  \tag{9}

The formula (8) minus the formula (9) and get:

\[ \psi_{ij} - \psi_c = -\frac{1}{4\pi} \sum_{i=1}^{m} q_i (\phi_i^E - \phi_i^F) \]  \tag{10}

From the definition of pseudo-potential in formula (4), also can get:

\[ \psi_{ij} = \frac{k}{\mu} (p_{ij} - Gr_w) \quad \psi_c = \frac{k}{\mu} (p_c - Gr_e) \]  \tag{11}

Substitute the formula (11) into the formula (10) and can trim to get the infiltration model for the top and bottom closed boundary of low permeability reservoir which is:

\[
\left[
\phi_1^E - \phi_1^F \quad \phi_2^E - \phi_2^F \quad \cdots \quad \phi_{m-1}^E - \phi_{m-1}^F \quad \phi_m^E - \phi_m^F
\right] q_i \\
\vdots \\
\phi_1^F - \phi_1^E \quad \phi_2^F - \phi_2^E \quad \cdots \quad \phi_{m-1}^F - \phi_{m-1}^E \quad \phi_m^F - \phi_m^E
\]  \tag{12}

\[
\begin{align*}
\frac{4\pi k}{\mu} & \begin{pmatrix}
p_c + G(r_e - r_c) - p_{w_1} \\
p_c + G(r_e - r_c) - p_{w_2} \\
\vdots \\
p_c + G(r_e - r_c) - p_{w_{m-1}} \\
p_c + G(r_e - r_c) - p_{w_m}
\end{pmatrix} \\
= & \frac{4\pi k}{\mu} \begin{pmatrix}
p_e + G(r_e - r_c) - p_{w_1} \\
p_e + G(r_e - r_c) - p_{w_2} \\
\vdots \\
p_e + G(r_e - r_c) - p_{w_{m-1}} \\
p_e + G(r_e - r_c) - p_{w_m}
\end{pmatrix}
\end{align*}
\]

Namely:

\[ \sum_{i=1}^{m} q_i (\phi_i^E - \phi_i^F) \]  \tag{12}

\[ = \frac{4\pi k}{\mu} [p_e + G(r_e - r_c) - p_{w_j}] \quad j = 1 \cdots m \]

Where: \( p_{w_j} \) is the flowing pressure at the middle point of \( j \) th segment of horizontal wellbore, MPa; \( p_e \) is the primary pressure at oil drainage boundary, MPa; \( G \) is the starting pressure gradient, MPa/m; \( r_w \) is the oil drainage radius, m; \( r_c \) is the wellbore radius, m; \( q_i \) is the flow rate flowing from the formation into the \( j \) th segment of horizontal wellbore, m\(^3\)/d; \( \mu \) is the fluid viscosity, mPa·s; \( k \) is the effective permeability;

\[ \phi_i^E \neq \phi_i^F(x_j,y_j,z_j) \quad, \quad \phi_i^E \neq \phi_i^F \left( \frac{L}{2}, r_e, z_w \right) \]

As the anisotropy is considered [17], suppose that the permeability in horizontal direction is \( k_x = k_y = k_h \), and the permeability in vertical direction is \( k_z = k_v \); then, the effective permeability of the formation is \( k = \sqrt{k_h k_v} \) and the anisotropy coefficient is \( \beta = \frac{k_h}{k_v} \). As the influence of anisotropy is considered, amend \( \phi_h^E \) and \( \phi_v^E \) in the formula (12), while \( z_w \) is amended as \( \beta z_w \) and \( h \) is amended as \( \beta h \).

III. PRESSURE-DROP CALCULATION MODEL OF HORIZONTAL WELLBORE

A. Open hole Completion

Because the fluid constantly flows from the formation into the wellbore along with the horizontal wellbore, the flowing fluid in horizontal wellbore is the variable mass flow [18]. Under the open-hole completion method, adopt the separated completion method of changing blind-screen ratio and take an infinitesimal with length of \( \Delta x \) in the horizontal wellbore.

From the principle of mass conservation and theorem of momentum [19], get:

\[ \Delta p_w = \frac{8f \rho Q^2 \Delta x}{\pi^2 D^2} + \frac{32 \rho Q q}{\pi^2 D^3} + \frac{16 \rho q^2}{\pi^2 D^4} \]  \tag{13}

Where: \( Q \) is the average flow rate at the upstream end of main stream of the segment in the wellbore, m\(^3\)/d; \( f \) is the friction factor on the wall of perforation horizontal wellbore; \( q \) is the total flow rate of all holes at the segment from the oil reservoir into the wellbore, m\(^3\)/d; \( \rho \) is the fluid density; \( D \) is the diameter of horizontal wellbore, m.

The above formula is the pressure drop model for the well completion of the whole well segment and the separated oil production is considered. Suppose that the horizontal wellbore with length of \( l \) is divided into \( m \) segments, then
the pressure drop of \( j \)th horizontal wellbore of open hole completion is:

\[
\Delta p_{w j} = \frac{8 f_j \rho Q_j^2 l_j}{\pi^2 D^5} + \frac{32 \rho Q_j q_j}{\pi^3 D^3} + \frac{16 \rho q_j^2}{\pi^4 D^4} \quad (14)
\]

Where: \( f_j \) is the friction factor of \( i \)th segment; \( Q_j \) is the flow rate at the terminal of \( i \)th segment, m\(^3\)/d; \( \Delta p_{w j} \) is the pressure drop in the \( i \)th segment of horizontal wellbore, MPa; \( L_j \) is the length of \( i \)th well segment, m; \( q_j \) is the flow rate flowing from the formation into the \( i \)th segment of horizontal wellbore, m\(^3\)/d; \( D \) is the diameter of horizontal wellbore, m.

Because the blind pipe segment and production horizontal segment alternatively appear, the pressure drop calculation model of the wellbore of open hole completion for the separated oil production in horizontal well is:

\[
\Delta p_{w j} = \begin{cases} 
\frac{8 f_j \rho Q_j^2 l_j}{\pi^2 D^5} + \frac{32 \rho Q_j q_j}{\pi^3 D^3} + \frac{16 \rho q_j^2}{\pi^4 D^4} & q_j \neq 0 \\
8 f_j \rho Q_j l_j & q_j = 0 
\end{cases} \quad j = 1, 2, \ldots, m
\]

\[
\Delta p_{w j} = \begin{cases} 
\frac{8 f_j \rho Q_j^2 l_j}{\pi^2 D^5} + \frac{32 \rho Q_j q_j}{\pi^3 D^3} \left[ 1 + \frac{1}{6n^2} \left( \frac{q_j}{Q_j} \right)^2 \right] & q_j \neq 0 \\
\frac{32 \rho Q_j q_j}{\pi^3 D^4} \left( 1 + \frac{q_j}{2Q_j} \right) & q_j = 0 
\end{cases} \quad (j = 1, 2, \ldots, m) \quad (18)
\]

\[
D p_{w j} = p_{wf j - 1} + 0.5(\Delta p_{w j - 1} + \Delta p_{w j}) \quad (j = 1, 2, \ldots, m) \quad (19)
\]

Where: \( p_{wf} \), \( \Delta p_{w0} = 0 \)

The total yield \( Q = q_1 + q_2 + \cdots + q_m \)
SHAN LIU et al: OPTIMIZATION MODEL FOR SEPARATED OIL PRODUCTION IN HORIZONTAL WELL OF …

Where: $m$ is the segmentation number of horizontal well; $p_{w,j}$ is the pressure drop at the middle point of the $j$th segment wellbore, MPa; $\Delta p_{w,j}$ is the pressure drop of the $j$th segment wellbore, MPa; $B_o$ is the oil volume factor.

Wellbore/oil reservoir coupling model of low permeability reservoir with edge water is:

Model (Ⅰ): the formula (12), (15) and (19) form the coupling model for separated oil production in horizontal well of low permeability reservoir with edge water of open hole completion.

Model (Ⅱ): the formula (12), (18) and (19) form the coupling model for separated oil production in horizontal well of low permeability reservoir with edge water of perforation completion.

V. OPTIMIZATION MODEL FOR SEPARATED OIL PRODUCTION IN HORIZONTAL WELL OF LOW PERMEABILITY RESERVOIR

Taking the yield of horizontal well as the target function and the length of each production horizontal segment and the position of middle point as the optimization variables, establish the optimization model for separated oil production in horizontal well of low permeability reservoir.

$$Q = q_1 + q_2 + \ldots + q_m$$

Variable to be optimized: $L_i$, $x_i$ ($i=1 \cdots m$)

Optimization model for separated oil production in horizontal well of low permeability reservoir with edge water of open hole completion: Model Ⅰ + Model Ⅲ.

Optimization model for separated oil production in horizontal well of low permeability reservoir with edge water of perforation completion: Model Ⅱ + Model Ⅲ.

VI. ANALYSIS OF SENSITIVITY FACTORS

Aiming at the coupling model and optimization model of above two kinds of oil reservoirs, the genetic algorithm [23] is applied to work out the numerical simulation software; the instance data is adopted to carry out the optimization solution and the analysis of sensitivity factors is carried out with aiming at the model. The relevant parameters are: in the genetic algorithm, the selective probability is 0.8; the mutation probability is 0.07; the starting pressure gradient is 0.001MPa/m; the supply pressure is 30MPa; the flowing pressure at well bottom is 15MPa; the horizontal permeability is 0.01 $\mu m^2$ and the vertical permeability is 0.005 $\mu m^2$; the length of horizontal well is 1000m; the thickness of formation is 20m; the wellbore radius is 0.1m; the supply radius is 1000m; the density is 840kg/m³ and the oil volume factor is 1.05.

Take the 60% of horizontal wellbore and separated oil production in horizontal well of low permeability reservoir with edge water, respectively give the optimal segmentation schemes under the different segmentation modes.

(1) The optimal segmentation scheme when there are two segments $m=2$

<table>
<thead>
<tr>
<th>$L_i$</th>
<th>$X_i$</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>292.815458</td>
<td>204.800000</td>
<td>52.048661</td>
</tr>
<tr>
<td>307.184542</td>
<td>754.788826</td>
<td>52.048661</td>
</tr>
</tbody>
</table>

Figure 3. The distribution of horizontal interval when $m=2$

There into: is the production horizontal well segment and the rests are the blind pipe segment.

(2) The optimal segmentation scheme when there are three segments $m=3$

<table>
<thead>
<tr>
<th>$L_i$</th>
<th>$X_i$</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>228.351474</td>
<td>194.141480</td>
<td>52.617101</td>
</tr>
<tr>
<td>112.800000</td>
<td>566.524724</td>
<td>52.617101</td>
</tr>
<tr>
<td>258.848526</td>
<td>848.153210</td>
<td>52.617101</td>
</tr>
</tbody>
</table>

Figure 4. distribution diagram of $m=2$ production level

There into: is the production horizontal well segment and the rests are the blind pipe segment.

(3) The optimal segmentation scheme when there are four segments $m=4$

<table>
<thead>
<tr>
<th>$L_i$</th>
<th>$X_i$</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>152.497257</td>
<td>105.375836</td>
<td>53.174419</td>
</tr>
<tr>
<td>196.587579</td>
<td>378.528829</td>
<td>53.174419</td>
</tr>
<tr>
<td>70.000000</td>
<td>710.176655</td>
<td>53.174419</td>
</tr>
</tbody>
</table>

Figure 5. The distribution of horizontal interval when $m=4$

There into: is the production horizontal well segment and the rests are the blind pipe segment.

A. Analysis of Sensitivity Factors for the Coupling Model for Separated Oil Production in Horizontal Well of Low Permeability Reservoir with Edge Water under the Open-hole Completion Method

From Fig. 6, it can be seen that: the yield of horizontal well of low permeability reservoir with edge water of open hole completion increases gradually with the increasing of segmentation number of horizontal wellbore; compared to the open length of horizontal wellbore, the segmentation number of the horizontal wellbore has little influence on the yield and the increasing amplitude is little.
From the Fig.7, it can be seen that: following the moving of oil-water interface of the edge water, the yield of horizontal well of low permeability reservoir with edge water of open hole completion gradually decreases. With the fixed oil-water interface of edge water, the more segments there are, the more yields there will be.

From the Fig.8, it can be seen that: for the open hole completion, following the increasing of starting pressure gradient, the yield of separated completion of low permeability reservoir with edge water gradually decreases. For the same one starting pressure gradient, the bigger the well open ratio is, the higher the yield is.

From Fig.9, it can be seen that: for the same thickness of formation, the bidder the well open ratio is, the bigger the yield is; Following the increasing of thickness of formation, the yield of horizontal well of low permeability reservoir with edge water gradually increases, but the increasing amplitude tend to be gentle, which shows that the capacity of improving the yield by utilizing the development of horizontal well is limited under the thick formation of low permeability; therefore, for the thick formation of low permeability reservoir with edge water, it is not suggested to consider about using the horizontal well for carrying out the development and production.

B. Analysis of Sensitivity Factors for the Coupling Model for Separated Oil Production in Horizontal Well of Low-permeability Reservoir with Edge Water under the Perforation Completion Method
Following the increasing of thickness of formation, the yield of horizontal well low permeability reservoir with edge water gradually increases, but the increasing amplitude tends to be gentle, which shows that the capacity of improving the yield by utilizing the development of horizontal well is limited under the thick formation of low permeability.

(3) Taking the yield of horizontal well as the target function and the length of central point of production horizontal well segment and the position of central point as the optimal variables, the optimization models for the separated oil production in horizontal well of low-permeability reservoir with edge water under two completion methods are respectively established. The numerical simulation software of genetic algorithm is utilized to carry out the optimization solution for it, while the optimal separation schemes under the different segmentation modes are respectively given.

(4) The sensitivity analysis is carried out for the capacity influence factors of optimization model by utilizing the instance data. The research shows that: as for the low permeability reservoir with edge water, the more the number of production horizontal segments are, the bigger the yield is. However, with the increasing of number of the production horizontal segments, the increasing speed of yield tends to be gentle; therefore, in consideration of the economic benefits, the segments shall not be too many.

VII. CONCLUSION

(1) With introduction of the starting pressure gradient and on the basis of idea of infinitesimal congruence of lines, by utilizing the potential superposition principle and reflection principle of mirror image, the coupling models for the separated oil production in horizontal well of low-permeability reservoir with edge water under the ways of open-hole completion and perforation completion are respectively established.

(2) The separated completion method with blind pipe is adopted, of which the blind pipes and production horizontal segments are in radial arrangement, and the calculation model of wellbore pressure drop for separated oil production in horizontal well is established. By utilizing the coupling connection formula, the oil reservoir/wellbore coupling models for separated oil production in horizontal well of low permeability reservoir with edge water under 2 kind of completion methods are established; the sensibility analysis is carried out for the capacity influence factors of coupling models by utilizing the instance data. The research shows that: under the open-hole completion method, the yields of horizontal wells of low permeability reservoir with edge water all gradually decrease with the increasing of the starting pressure gradient. As the starting pressure gradient is fixed, the bigger the well open ratio is, the bigger the yield is.

REFERENCES


