Transmission Lines Modeling Based on Vector Fitting Algorithm and RLC Active/Passive Filter Design

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Abstract — In the modeling of the transmission line, the electric energy produced at generating stations is transported over high-voltage transmission lines to utilization points, and the trend toward higher voltages is motivated by the increased line capacity while reducing line losses per unit of power transmitted. An electric transmission line is modeled using series resistance, series inductance, shunt capacitance, and shunt conductance. For some studies, it is possible to omit the shunt capacitance and conductance and thus simplify the equivalent circuit considerably. Frequency Response Analysis (FRA) on the transmission line application is utilized for behavior prediction and fault diagnosis, but a need for more investigation is important for the bases on which the diagnosis determined. The utilities of the measured FRA data points need to be enhanced with suitable or developed modeling category to facilitate the modeling and analysis process. This research proposes a new method for modeling the transmission line based on a rational approximation function which can be extracted through the Vector Fitting (VF) method, which attempts on the frequency response measured data points. A set of steps needs to be implemented to achieve this by setting up the extracted partial fraction approximation, which results from a least square RMS error via VF, in such a way that would construct as real numbers, first and second order parts as well as the gain constant. Active and passive filter design circuits are attempting to construct the model of that rational function of the transmission line. RLC design representation has been implemented for modeling physically the system with MATLAB, Simulink for verifying the results.

Keywords - Transmission lines modeling, Vector fitting method.

I. INTRODUCTION

The electrical power system mainly consists of three standard divisions; Generating Stations, Transmission System and Distribution System. The transmission line (TL) is the most important energy corridor in a power system [1]. The performance of a power system is mainly based on the performance of the transmission lines in the system. In general, transmission lines have three parameters; resistance, inductance, and capacitance (RLC). All these parameters are distributed over the length of the line.

The examination of the transmission lines can be divided into three categories:

- A short transmission line has a length not more than 80 km.
- Medium transmission line length is between 80 km and 250 km.
- A long transmission line has a length beyond 250 km.

The classification of transmission lines depends upon the accuracy of the model and its simplicity. While the short transmission line model is quite simple, the longer line model is somewhat complex and the medium transmission line is in-between the two. In the field of transmission lines modeling, the author presented a technique to model transmission lines using time domain synthesis approach [2]. The parameters of each transmission line are determined by using a constraint, nonlinear, least squares optimization technique performed in the time domain. The proposed modeling technique has several applications in microwave measurements, material characterization and microwave device modeling. While in another work, the authors presented two highly accurate transmission line models [3]. The first model is particularly suitable for overhead lines and in the second model, is used for propagation a modal decomposition with a constant transformation matrix and an optional phase domain correction term. Both models are computationally highly efficient due to their time domain realizations based on vector fitting.

II. VECTOR FITTING METHOD

Vector Fitting (VF) method is an approximation of a measured transmission line frequency response (FR) that recently adopted in this application which is based on complex rational function models. Frequency dependent transmission line models have been used in the simulation of power system since the 1970s [4][5]. The vector fitting method used in transmission line transient mentioned in [6] proposed by the author to be fast and robust method for rational fitting of frequency domain responses, well suited
for both scalar and vector transfer functions (TF). Application of the new method results in increased computational efficiency for transmission line models using model decomposition with frequency dependent transformation matrices. This is due to the fact that the method allows the fitted elements of each eigenvector to share the same set of poles, and that accurate fitting can be achieved with a relatively low number of poles. In [7] the time delay for identification for transmission lines modeling was studied. It shows that to simply use the time delay of lossless propagation can lead to significant loss of accuracy of the rational approximation. A procedure is shown which optimizes the time delay together with the poles and residues of the rational approximation. This is achieved by combining Brent’s method with Vector Fitting.

In the above literature, the related studies either addressed the modeling of transmission line in different categories or they used Vector Fitting Algorithm (VFA) with power transformer only. In this work, the fundamental method for transmission lines modeling from measured data points based on RLC modeling of the Rational Approximation Function (RAF) obtained from the Vector Fitting method was proposed. The RLC modeling in this research uses the concept of the passive and active filter representation of the partial fractions of the system transfer function.

III. PASSIVE FILTER DESIGN

Low pass, high pass, band pass and band reject (notch) filters are the basic filter types which are shown in Fig. 1 [8]. A passive filter is consists of resistors, inductors, and capacitors. The number of capacitors and inductors is equal to the highest power of frequency, and the order depends on the order of the filter, in the cases of Butterworth and Chebyshev. In the Elliptic filter case, the number of capacitors indicates the order of the filter. The lumped elements values for each filter type for the normalized frequency have been computed and tabulated [9], as \( \omega = \frac{1}{\text{Sec}} \) and source and load impedances are \( Z_{\text{dc}} = Z_{\text{Load}} \) (\( \Omega \)).

In this work, only the passive filter circuits are used and described in some details.

A. Voltage Divider Circuit

The voltage divider circuit consists of two series resistors to simulate the attenuation of the input signal is shown in Fig. 2.

\[ \frac{v_o}{v_i} = \frac{R_2}{R_1 + R_2} \]

(1)

B. Low Pass Filter Circuit

Low Pass Filter circuit (1st order transfer function) is shown in Fig. 3.

\[ \frac{v_o}{v_i} = \frac{1}{S + \frac{1}{RC}} \]

(2)

C. RLC 2nd order Low Pass Filter Circuit

Low pass filter circuit (2nd order transfer function) is shown in Fig. 4.

\[ \frac{v_o}{v_i} = \frac{1}{S + \frac{1}{RC} + \frac{1}{S^2LC + SRC}} = \frac{1}{S + \frac{1}{RC} + \frac{1}{S^2 + \frac{R}{L} + \frac{1}{LC}}} \]

(3)

D. Band Pass Filter Circuit

Band Pass Filter circuit (2nd order transfer function) is shown in Fig. 5.

\[ \frac{v_o}{v_i} = \frac{R}{S + \frac{1}{SC} + \frac{SRC}{S^2LC + 1 + RSC}} = \frac{SRC}{S^2 + \frac{R}{L} + \frac{1}{LC}} \]

(4)
As mentioned above, this research addressed some filters circuit that is used and satisfied the modeling verification.

IV. METHODOLOGY

The algorithm of the vector fitting is described in different aspects in [10][11]. In this research, MATLAB environment has been used to implements the procedure because MATLAB has suitable commands to obtain the parameters and the rational approximation function which is the key equations to acquire.

After the transmission line rational approximation function is obtained which is based on the frequency response analysis data points by utilizing Vector Fitting method, a set of steps need to implement such as follows. Firstly, setting up the extracted partial fraction approximation, which results have less RMS error due to VF method and it is expressed as in eq. (5)

\[
H(S) = \frac{r_1}{S-p_1} + \frac{r_2}{S-p_2} + \ldots + \frac{r_s}{S-p_s} + K
\]  

Eq. (5) consists of three parts as follows:-

• Real numbers, first order parts \( \frac{r}{S+p} \)
• Second order parts results from the product of two complex conjugate poles parts \( \frac{as+b}{S^2+cs+d} \)
• Constants K.

Secondly, active and passive filter design circuits are conducted in order to construct the model that represents the transfer function of the transmission line by:-

• Implementing the required calculations to find out the values of the circuits components.
• Using MATLAB Simulink to verify the results.

Based on the concept that mentioned in [12], the flowchart which describes the algorithm of the research is shown in Fig. 6, where the measured data points are presented as an input to the program. These data are changed to Cartesian form to generate a row of complex matrix for n dimension, and the process is described clearly in the flowchart.

Referring to the flowchart of Fig. 6, the resultant rational approximation function (RAF) needs to be processed by some numerical filters to ignore the very low or the ineffective imaginary part from each element with respect to its combined real part. As an example, if one complex number has a value like \( pn= a*10^6+b*10^{-7}j \), then \( pn= a*10^6 \), then, rearranging eq. 1, all the rational partial parts of the system function again to sum each two pair that has complex conjugate numbers together to get second order partial function elements without imaginary part for all system RAF, so, as an example, the eq. 5. could be changed to the following form.

\[
H(S) = \frac{r_1}{S-p_1} + \frac{r_2}{S-p_2} + \frac{as+b}{S^2+cs+d} + \ldots + \frac{r_s}{S-p_s} + K \quad (6)
\]

where \( \frac{as+b}{S^2+cs+d} \) represents the result of summing \( \frac{r_1}{S-p_1} + \frac{r_2}{S-p_2} \) and \( r_3 \) and \( r_4 \) has a(=real+J imaginary)

Therefore, the resultant RAF, of eq. 6, was not a complex numbers and can be passed to the next step which is the RLC modeling for H(s) of eq. 6 by comparing each partial part with its equivalent circuits (Fig. 3, Fig. 4. and Fig. 5).
V. APPLICATION AND RESULTS

In order to validate the proposed method, a model mentioned in [13] as shown in Fig. 7 has been adopted to analyze the frequency response analysis (FRA). The modeling of the transmission line is opted as a data to represents the TL behavior and has been selected to be modeled with this proposed method such as shown in Fig. 8 together with the curve fitting of the modeling procedure.

Referring to eq. 5 and the proposed algorithm of [12], the resultant Rational Approximation Function parameters can be listed as follows:

\[ r = -21.0633 + 0.0000i \]
\[ 1.0510 + 0.0000i \]
\[ -0.4939 - 0.0546i \]
\[ -0.4939 + 0.0546i \]
\[ P = -4.8095 + 0.0000i \]
\[ -1.3091 + 0.0000i \]
\[ 0.0593 + 1.1941i \]
\[ 0.0593 - 1.1941i \]
\[ K = 4 \]

So the partial fraction expansion for the above transfer functions are as follows:-

\[ TF_1 = \frac{-21}{S + 4.809} \]
\[ TF_2 = \frac{1.051}{S + 1.309} \]
\[ TF_3 = \frac{-0.4939 + 0.0546i}{S - (0.0593 + 1.1941i)} \]
\[ TF_4 = \frac{-0.4939 - 0.0546i}{S - (0.0593 - 1.1941i)} \]
After all transfer functions are obtained, the equation is rearranged to eq (5):

$$H(S) = \frac{-21}{S + 4.809} + \frac{1.051}{S + 1.309} + \frac{-0.4939 + 0.0546}{S - (0.0593 + 1.194)} + \frac{-0.4939 - 0.0546}{S - (0.0593 - 1.194)} + 4$$

Based on eq. (5), each of two complex conjugate poles must be added to each other such as follows to obtain equation such as in eq. (6):

$$H(S) = \frac{-21}{S + 4.809} + \frac{1.051}{S + 1.309} + \frac{-0.9878S + 0.189}{S^2 - 0.1186S + 1.429} + 4$$

This procedure is based on the RLC passive filter design. The block diagrams described in Fig. 9 represent the procedure.

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### TABLE 1. ELEMENTS OF EQUATION 6

<table>
<thead>
<tr>
<th>2nd order partial fractions</th>
<th>1st order partial fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{-0.9878S + 0.189}{S^2 - 0.1186S + 1.429}$</td>
<td>$\frac{-21}{S + 4.809}$</td>
</tr>
<tr>
<td>$\frac{1.051}{S + 1.309}$</td>
<td>$\frac{1.051}{S + 1.309}$</td>
</tr>
</tbody>
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From Fig. 10, the transfer functions are converted to RLC passive filter circuit using eq. 2, 3 and 4. The new transmission lines modeling is shown in Fig. 11.

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VI. DISCUSSION AND CONCLUSION

In this research, two controversial issues are investigated and concluded. Firstly, it is essential for any electromagnetic system, such as the transmission line to be modeled in terms of the basic electrical elements i.e. RLC,
or a combination of mathematical modeling equation together with some RLC modeling circuits.

This proposed method shows a common response to electromagnetic interaction between the transmission line elements and that for geometrical changes for the transmission line active part, different TF’s types of the extracted RAF have different sensitivity towards various frequency response behaviour of the transmission line. This method provides an uncomplicated way to divide not only the frequency range, but the overlap and effects of each first or second order partial fraction of the attained RAF onto FRA chart to explore more facilities for diagnosis process to be estimated from the end-to-end transfer function.

The vector fitting algorithm has been utilized here to provide a suitable solution to find analytical expressions for measured transfer functions. The necessary accuracy of the approximation can be controlled by the user according to the needs of the application. The main advantage of a mathematical explanation of RAF of the numerical data is that subsequent algorithms and assessment rules can be implemented for more analysis.

This paper suggests deriving an electrical equivalent model from measured frequency response data points. The model is subdivided into two groups of partial fractions 1st and 2nd order transfer functions. For the 1st order, two parts of circuits are needed which are voltage divider and RC low pass filter, equivalent resonance circuits with physical meaning can be identified from measured data points. For the 2nd order transfer functions group, three parts of circuits need to be used and an electrically equivalent circuit can be identified using the analytical expression gained by RAF of vector fitting.

The proposed model can be used to create precise electrical representation for transmission line and help to know the range that the FRA may succeed to be utilized for physically modeling.

REFERENCES


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