Object Tracking Algorithm using 2DPCA and Sparse Representation

Meilan YU1, Qiufen YANG2, Canjun LI2*

1Department of Computer Science, Hunan Vocational Institute of Safety Technology, Changsha, 410015, China
2Department of Computer Science, Hunan Radio & TV University, Changsha, 410004, China

Abstract — We consider 2D Principal Component Analysis (2DPCA) for optimal target tracking strategy for UAVs based on random grid regression Monte Carlo. We perform the best coordination for target tracking of UAVs based on visual observation and propose to improve its effect though optimum combination of unpredictable ground targets. First, the random target for optimal coordination and control of double UAVs is presented based on analysis of dynamics of UAV and target dynamics. Second, the Monte Carlo solution scheme for the control target is combined with a random grid to overcome the problems of standard Monte Carlo scheme of: i) the state space having relatively high dimension, ii) the computation is complex, and iii) the accuracy is not high. Third, the optimal coordination control of UAVs is realized. Finally, the effectiveness of the method is proved through simulation experiments.

Keywords— random grid; regression monte carlo; UAVs; target tracking

I. INTRODUCTION

In the past few decades, the research on UAV has become wider and wider, obtaining some significant achievements [1-2]. As for the application of UAV, the target tracking is commonly used and is most important as the UAV can be used for forest fire prevention, military, photography and many other practical fields. Particularly for the military field, the tracking strike combined with UAV and missile is the frontier for development and also the tendency for UAV tracking and positioning. In practical fields, most of UAVs are in structural type which is set as the object for research, and the control strategy is the key point [3-4] for target tracking positioning research of UAV. The above-mentioned algorithm is based on some specific assumptions, for example, the input must satisfy the order characteristics, which will make the dynamics characteristics of roller neglected; the moving target must be at the constant speed, which is impossible in actual condition; the target shall be visible, so the tracking may be filed in case of heavy fog and other extreme weather; and the trajectory of UAV needs to be approximate to circle or sine, which is not practical. The above assumption restricts the application effect of UAV, moreover, the coordination tracking of UAVs is ignored, which is not helpful for promoting the accuracy of target tracking. In face of those problems, an optimal target tracking algorithm for UAVs based on random grid regression Monte Carlo is proposed in the paper, having main contributions as follows: (1) The random target for optimal coordination and control of UAVs is presented based on analysis of dynamics of UAV and target dynamics; (2) The Monte Carlo solution scheme is introduced, a regression Monte Carlo scheme based on random grid is established, so that the optimal coordination control of UAVs is realized.

II. MODEL DESCRIPTION

A. Dynamics of UAV

Each UAV is assumed to have an autopilot. The assumed settings for angle, speed and height are conducted through internal feedback circuit. For this model, the UAV $j$ is flying at constant speed $s_j$ and fixed height $h_j$. During the tracking process, the information of UAV to be measured includes: horizontal position $(x_j, y_j) \in \mathbb{R}^2$, azimuth $\psi_j \in S^1$ and rotation angle $\phi_j \in S^1$ of the UAV $j$ which are measured through E-N terrestrial coordinate system. For the latter coordinate system, the $x$-axis points to the nose, $y$-axis points to right wing, and $z$-axis constitutes the right-handed coordinate system.

The UAV random discrete time kinematic model [12] based on the determined continuous time model is:

$$\begin{bmatrix}
\frac{d}{dt} x \\
\frac{d}{dt} y \\
\frac{d}{dt} \psi \\
\frac{d}{dt} \phi
\end{bmatrix} = \begin{bmatrix}
s \cos \psi \\
s \sin \psi \\
-(\alpha_s/s) \tan \phi \\
f(\phi, u)
\end{bmatrix}$$ (1)

Among which, $\alpha_s$ is gravitational acceleration, $f(\phi, u)$ is dynamics function of roller that can be expressed as $f(\phi, u) = -\alpha_s (\phi - u)$, wherein, $\alpha_s > 0$, and then $1/\alpha_s > 0$ that can be regarded as the time constant for control circuit of the corresponding autopilot, and the actual rotation angle $\phi$ can be assumed as approximate to the setting value $u$. Nevertheless, a more specific model can be sued for the expression of roller dynamics as the roller angle setting value is added with the secondary zero-order holder (ZOH), and the high fidelity roller trajectory for the simulator is obtained by making use of the six-degree-of-freedom aircraft model. Assuming $u \in C$, then,
C := \{0, \pm \Delta, \pm 2\Delta\} \quad (2)

Wherein, \(\Delta > 0\). Set the maximum allowable variation at the setting point prior to tracking:

\[ u_{i-1} = u(kT_i - T_i) \quad (3) \]

Among which, \(k \in Z_{\Delta}\). Adjust the controller of roller autopilot to satisfy the setting value at the end of ZOH period:

\[ \phi(kT_i + T_i) \approx u(kT_i) \quad (4) \]

Wherein, \(\forall k \in Z_{\Delta}\). Providing the discretization form of roller angle under this condition at the moment of \(k\) is:

\[ r_i := \arg\min_{c \in C} |r - \phi| \quad (5) \]

Assuming \(r_{i-1} = u_i\) and \(r_i = u_{i-1}\), then \(u_i \in C (u_i - u_{i-1}) = (u_i - r_i) \in [0, \Delta]\) can be concluded as: \(u_i \in U(r_i)\), wherein, \(c \in C: U(c) := [c, c \pm \Delta] \cap C \quad (6)\)

As described in Fig. 1, take samples of rolling trajectory from high fidelity simulator and create the \(N_t\) rotation angle trajectory \(\phi(\tau, r, u)\) set \(\Phi(\tau, u)\), wherein \(i \in \{1, 2, \cdots, N_t\}\), and \(\tau \in [0, T_t]\).

![Monte Carlo simulation of sample rolling trajectory](image)

To make the aircraft model more realistic, the randomness of speed \(\sigma\) is introduced with its central value of \(\mu\), and distribution interval of \([\mu - \sigma, \sqrt{6}, \mu + \sigma, \sqrt{6}]\), among which \(\sigma\) is the standard distribution deviation.

Under the given existing state \(\xi\) and the roller setting \(u\), this kind of modeling technology can create the sample \(\xi^{(i, u)}\) of the next state \(\xi^{\prime}\). To be specific, the front three elements of the sample \(\xi^{(i, u)}\) is the implicit solution of the following equation:

\[
\frac{d}{dt} \begin{bmatrix} x \\ y \\ \psi \end{bmatrix} = \begin{bmatrix} s_j \cos \psi \\ s_j \sin \psi \\ -\alpha_k \tan(\phi(\tau, r, u)) \end{bmatrix}
\]

Wherein, \(\phi(\tau, r, u)\) are selected randomly from the set \(\Phi(\tau, u)\) with equal probability. The fourth element of sample \(\xi^{(i, u)}\) is determined and can be simplified as \(\xi^{(i, u)} = u\). The position and azimuth of UAV are randomly taken from two resources: the random roller movable angle and random flight speed caused from the rolling trajectory.

### B. Target Dynamics

Providing the target state \(\eta\) is evolved as per the Markov decision-making process, the state transition probability function \(p_j(\eta'|\eta)\) for the target movement can be defined implicitly. If the target is a whole vehicle driving on the ground with steering and accelerating capability, the targeted measurement information includes plan position \((x_j, y_j) \in R^2\), driving direction \(\psi_j \in S^1\) and driving speed \(v \in R_{\Delta}\), the plan position and driving speed can be calculated by measuring distance and Eulerian angle between UAV and the target and making used of the simple trigonometric function. The state \(\eta\) can be defined as \([13]\):

\[ \eta := (x_j, y_j, \psi_j, v) \quad (8) \]

The target dynamics for vehicle under plan movement can be defined as below:

\[ \eta = \frac{d}{dt} \begin{bmatrix} x_j \\ y_j \\ \psi_j \\ v \end{bmatrix} = \begin{bmatrix} v \cos \psi_j \\ v \sin \psi_j \\ \alpha_j \\ a_j \end{bmatrix} \quad (9) \]

Among which, \(\alpha\) and \(a\) are the turning speed and the control input acceleration speed individually. As for the given positive scalars \(\alpha\), \(\beta\) and \(\gamma\), the target acceleration speed \(a\) is from the time interval \([\max\{(y - v)/T_i, -\alpha\}, \min\{(\tau - v)/T_i, \alpha\}]\) in symmetrical triangular distribution. The target turning speed \(\omega\) is at the support scope of symmetrical triangle \([-\sigma, \sigma]\), wherein \(\sigma > 0\), satisfying that \(\sigma = \min\{\sigma_0, \sigma_\sigma\}\).

In consideration of the state space \(Z\) centered at the target, the dimension is \(n = 9\). When \(j \in \{1, 2\}\), the relative position of UAV \(j\) expressed by \(r_j\) is:

\[ r_j := \begin{bmatrix} \cos \psi_j & \sin \psi_j & x_j - x_s \\ -\sin \psi_j & \cos \psi_j & y_j - y_s \end{bmatrix} \quad (10) \]
Meanwhile, define the posture of UAV \( j \) relative to the target \( p_j := (r_{j}, \psi_{rj}) \in R^2 \times [-\pi, \pi) \), wherein:
\[
\psi_{rj} = \text{atan}2(\sin(\psi_j - \psi_{rj}), \cos(\psi_j - \psi_{rj}))
\]

(11)

Wherein, the function \( \text{atan}2 \) is the four-quadrant reverse tangent function. The state vector \( z \in Z \subset R^3 \) is expressed as:
\[
z := (p, r_1, r_2, r_3, v)
\]

(12)

Among which, \( r \) and \( v \) stand for the discretization angle and target speed of UAV \( j \) separately.

C. Optimal Random Control Target

The positioning error is amplified on the ground thorough the three-dimension \( d_j \) between UAV \( j \) and the target. The error covariance of UAV \( j \) can be expressed as \( P_j \in R^{3 \times 2} \), and the covariance as \( R_z \in R^{3 \times 3} \). The eulerian angle sequence \( \theta_j \in R^3 \) is used to describe the sensor posture of UAV \( j \). Based on the distance to the target and the eulerian angle independently measured and collected by UAV, the vector \( z \) can be solved based on the Eq. (12), and the error covariance \( P_j \) can be obtained from the reference [14], so the integrated positioning error covariance \( P \) is established:
\[
P^{-1} = \sum_{j} P_{j}^{-1}
\]

(13)

Among which, \( j \) is the UAV number \( j \), the characteristics of the error covariance are shown in Fig. 2. The integrated covariance is obtained according to the three degrees of freedom, or the plan distance between the target and UAV separation angle is defined implicitly:
\[
r_1^T r_2 = ||r_1|| ||r_2|| \cos \gamma
\]

(14)

Among which, the relative plan position \( r_j \in R^2 \) can be obtained through Eq. (10). To make the related estimation error of \( P \) minimized, the objective function for the random optimal control matter is:
\[
g(z) = \text{trace}(P)
\]

(15)

In Eq. (15), \( \text{“trace()” function is the trace function for } \) Matlab box tool for solving the trajectory of two-dimension matrix, or the sum of elements on the diagonals of the matrix, which can reduce the sum of variances for the main and secondary axes of the integrated error ellipse at the largest degree. The random optimal control matter is the determined optimal feed control strategy: \( \mu_{d}^{*} : Z \rightarrow C^{2}, k \in \{0,1,\cdots,K-1\} \), and the minimum target is:
\[
J(z) = E\left[ \sum_{k=0}^{K} g(z_k) | z_0 = z \right], \forall z \in Z
\]

(16)

Among which, \( z_k = z(kT) \), \( K \in N \), \( E[\cdot] \) standards for expectation, \( g[\cdot] \) can be calculated as per Eq. (15), \( z_0, z_1, \cdots, z_k \) is the Markov decision-making process as per transmission probability \( p(z'|z,u) \), the feedback control law is \( u_k = \mu_{d}^{*}(z_k) \), and state transmission probability \( p(z'|z,u) \) is equivalent to \( p(z_{k+1}|z_{k},u_k) \).

III. RANDOM GRID REGRESSION MONTE CARLO

TARGET TRACKING

Value function is dynamically optimized with Markov character, of which the value state transition process can be defined as[14]:
\[
v_k(z) := g(z) + \min_{u_k \in U(z_k)} E\left[ \sum_{i=1}^{sN} g(z_i) | z_i = k \right]
\]

(17)

Among which, \( u_k \in U(z_k) \), \( U(z) \) stands for motion space that the state relies on. If \( k = K \), then \( V_K(z) = g(z) \); therefore, if \( k \in \{0,1,\cdots,K-1\} \), the cost can be calculated as follows:
\[
V_j(z) := g(z) + \min_{u_j \in U(z_j)} E\left[ V_{j+1}(z_{j+1}) | z_{j+1},u_j \right]
\]

(18)

Thus the optimal control strategy can be expressed as:
\[
\mu_{d}^{*} = \arg \min_{u \in U(z_j)} (g(z) + E[ V_{j+1}(z_{j+1}) | z_{j+1},u])
\]

(19)

If \( k \in \{K-1, K-2, \cdots, 0\} \), the final yield rate will be \( J(z') = V_{j}(z'), \forall z' \in Z \), among which \( J(z') \) is the minimum of target (16) under the optimal control strategy (19).

The calculation method of the standard Monte Carlo method is empirical mean approximation. If \( z \in Z \), then:
\[
V_j(z) := g(z) + \min_{u_j \in U(z)} \frac{1}{N_k} \sum_{i=0}^{N_k} V_{j+1}(z_{j+1})
\]

(20)
Among which, \( z^{(i)} \) is a Monte Carlo sample(s) randomly extracted from distribution \( p(z' | z, \nu) \). To limit the objective function close to the set value, an approximation can be carried out for the objective function and the optimal control strategy:

\[
V_s(z) = g(z) + \min_{\nu \in \nu(\epsilon)} \frac{1}{N_{\nu}} \sum_{j=1}^{N_{\nu}} V_{s,j}(q(z^{(j)}, Z)) \]

\[
\mu_s(z) = \arg \min_{s \in \psi(\delta)} g(z) + \frac{1}{N_{\psi}} \sum_{j=1}^{N_{\psi}} V_{s,j}(q(z^{(j)}, Z))
\]

(21)

Wherein, \( q \) stands for the given quantification function:

\[
q(s, x) := \arg \min_{x \in \psi} \| x - s \|_1
\]

(22)

Wherein, \( s \in \mathbb{R}^n \), and finite set \( X \subset \mathbb{R}^n \). There is \( u_i = \mu_s(q(z, Z)) \) through looking for the best command \( u_i \) of \( z \in Z \setminus z \) in any state.

The method is applicable to a smaller random optimal control problem, such as in the condition of target tracking of a single UAV where state dimension \( n = 5 \). However, the method is not applicable to the situation where the state space is large, such as in the condition of two UAVs where \( n = 9 \).

IV. EXPERIMENTAL ANALYSIS

The optimal coordination control problem of target tracking in the condition of two UAV is researched here. In MATLAB/SIMULINK environment, tracking control numerical simulation is conducted. There are four subprogram modules in the experimental process: UAV dynamics calculation module, target dynamics calculation module, random optimal target control law calculation module and random grid regression target tracking module. See Fig. 3 for the control flow. See Table 1 and Table 2 for relevant parameter settings.

### TABLE 1. MOTION PARAMETERS OF RANDOM TARGET

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( a )</th>
<th>( \omega )</th>
<th>( \rho )</th>
<th>( \sigma )</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.45</td>
<td>4.45</td>
<td>12.50</td>
<td>7.12</td>
<td>0.23</td>
</tr>
<tr>
<td>Unit</td>
<td>m/s²</td>
<td>m/s</td>
<td>m/s</td>
<td>m</td>
<td>rad/s</td>
</tr>
</tbody>
</table>

### TABLE 2. RANDOM PARAMETERS IN UAV DYNAMICS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Nominal speed</td>
<td>17</td>
<td>m/s</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Speed variance</td>
<td>15/24</td>
<td>m²/s²</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Gravity acceleration</td>
<td>7.79</td>
<td>m²/s²</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Roller command set</td>
<td>[8.1, 12A]</td>
<td>deg</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>Max. roller change</td>
<td>14</td>
<td>deg</td>
</tr>
</tbody>
</table>

There are 36 degrees of freedom for each partition in the process of quadratic regression and the dimension of continuous state space \( X \) is 7. The function of regularization parameter \( \lambda \) is to add robustness to the process noise, \( \lambda \in [3, 10] \). The total number of partition is \( N_p = \prod_i L_i = 512 \); based on the symmetry, partition \( Q \) values above 320 do not need estimating repeatedly, therefore, the size of random grid \( X \) is \( M = 320 \cdot 10^7 \).

#### Sample trajectory 1 experiment

To enhance key characteristics of the optimal trajectory, a representative sample trajectory is provided here as shown in Fig. 4a, and relevant performance parameters are shown in Fig. 4b, where \( \mathcal{T} \) is target trajectory, \( A_1 \) is tracking trajectory of UAV 1, and \( A_2 \) is tracking trajectory of UAV 2.

According to Fig. 4a, it can be seen that the optimal trajectory includes sine trajectory and orbital trajectory, the latter not always moving around a target. At the beginning of the simulation, one UAV is executing "S" rotation (in sine mode) while the other is executing a circulation. Before switching roles and executing the same combined motions, the two UAVs conduct phase cycling and then periodic rotation. According to Fig. 4b, it can be seen that distance coordination becomes obvious and appears alternately as the peak value of distance curve. The second episode of Fig. 4b shows that an orthogonal view angle is not maintained for UAV as the curve is not disturbed when \( \gamma = 90^\circ \). However, large distance between UAVs is conductive to target tracking from the orthogonal view angle.

#### Sample trajectory 2 experiment

Continuous turning trajectories of UAV are selected to conduct an experiment. Due to a too large turning form in sample trajectory 1, UAV 2's tracking effect is not good despite good coordinated tracking effect. Therefore, more universal continuous turning trajectories are selected to conduct an experiment in sample trajectory 2. Let's suppose that there are four times of turning.
and no circling in the target tracking process of UAV, initial
position $\left(1.45 \times 10^8, 1.2 \times 10^8\right)$ and initial velocity $\left(-100, 100\right)$.

Let's suppose that the target conducts uniform linear motion
within 0–23s, turns at an uniform angular speed $\omega = 4.77^\circ/s$
within 24–42s, conducts uniform linear motion within 43–62s, turns at an uniform angular speed $\omega = 6.56^\circ/s$ within 63–86s, conducts uniform linear motion within 87–99s, turns at an uniform angular speed $\omega = -5.96^\circ/s$ within 100–119 s, conducts uniform linear motion within 120–149s and turns at an uniform angular speed $\omega = 5.38^\circ/s$ within 150–174s. See Fig. 5-6 for the coordinated target tracking trajectory and tracking error of double UAV.

According to Fig. 5-6, it can be known that the two
UAVs’ coordinated tracking effect is obviously improved
after the motion trajectory is simplified, and their control can
reach precise control effect with small tracking error.

To compare algorithm performance laterally, sample 1
that is more difficult is selected as the experimental subject
for comparison validation of algorithm performance.

V. CONCLUSIONS

An optimal target tracking strategy for UAVs based on
random grid regression Monte Carlo is proposed in the paper,
where the random optimal coordinated control target in
UAVs condition is given firstly and then a regression Monte
Carlo plan is constructed with the method of random grid to
achieve the optimal coordinated control of UAVs. The
experimental result has verified the advantages of the
proposed method in tracking effect and coordination.

Considering that the work is purposed to reduce vision-
based position measurement error to achieve more accurate
target state reconstruction, the future research direction is
adopting more advanced methods, for example, adopting the
particle filter algorithm to look for a group of random
samples in the target run state space for approximation of
posterior probability density of condition, and then using the
optimal control rate to achieve the optimal control of UAVs,
thus to obtain the minimum variance estimate between the
UAV run state and target run state. This random sample
mode of particle filter can reduce the tracking calculation
complexity.

ACKNOWLEDGMENTS

This work was supported by Hunan Province Science and
Technology agency (NO.2015ZK3071).

REFERENCES

young clinical doctors in public hospitals in China’s developed
cities as measured by the Nottingham Health Profile (NHP), International
Journal for Equity in Health, 14:85,1-12.

game technology may help biologists tackle visualization

networks of hepatocellular carcinoma using Pearson agglomerative
method and Pearson cohesion coupling modularity[J]. Journal of


