Non-Cooperation Game for Aircraft Pushback Slot Allocation Based on Dynamic Credibility Priority

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Abstract —Aircraft pushback management can make departure aircraft wait at the gate with engines off instead of waiting in queue before the runway with engines on, thus can reduce the fuel consumption. However, that can also cause delay the take-off time, which in turn reduces the capacity and passenger satisfaction. In view of this, this paper builds an non-cooperation game model for aircraft pushback slot allocation by considering the take-off time and fuel consumption, in which the delay and fuel utility of aircraft pushback management is quantified with modern game theory. The object function is the maximum delay and fuel utility of the system, the allocation strategy is first come first service (FCFS) and dynamic credibility priority, which the punishment of default is in consideration, then the hybrid decision algorithm based on dynamic credibility priority (HEAR) is proposed. Numerical results with real data of Xinzhou International airport demonstrate that the application of game theory can reflect the utility of participant, and HEAR provides better performance than FCFS, it can reduce the fuel consumption and the delay of departure aircraft.

Keywords- non-cooperation game; dynamic credibility priority; aircraft pushback; slot allocation; hybrid decision algorithm

I. INTRODUCTION

Nowadays most airports in China is in saturation, airport surface congestions have recently become a critical problem, their is long waiting queue before the runway in the peak hour of departure. Long queue leads to an increase in both fuel consumption and emissions. In fact, departure aircraft waiting at the gate can stop its engines and therefore save fuel. Besides, compared to arrival queues, departure queues can easily be controlled by allocating appropriate pushback time. Aircraft pushback management can make departure aircraft wait at the gate with engines off instead of waiting in queue before the runway with engines on, thus can reduce the fuel consumption. However, that can also cause delay the take-off time, which in turn reduces the capacity and passenger satisfaction. Therefore, aircraft pushback management strategy should be use with caution, and guarantee the negative effect is sufficiently small.

There are some researches about the pushback time. Jason A. D et al(2010)[1] discuss the relationship between slot compliance, equity and throughput in the research of TSAT allocation problem at London Heathrow. Atkin et al(2008)[2] propose a decision support system to improve the departure sequence at busy times at London Heathrow Airport, after that Atkin et al(2011)[3] further compare two methods for reducing take-off delay and consider the effect of TSAT allocation to aircraft sequence, the pushback time allocate is mentioned in the second method, although there is no detailed modeling and analysis, the potential benefits of aircraft pushback management has been shown through the combination with collaborative decision-making system.

After 2011, research about aircraft pushback started on the right track, there is a lot of outstanding theoretical achievements. Jason A. D et al(2013)[4] describes the problem of allocating pushback times as a problem can manage with cul-de-sac time rather than pushback time, analyzes the problem of pushback times allocation in detail with this two-stage approach, the separation of pushback sequencing and takeoff sequencing, and time-slot extensions. Ravizza et al(2013)[5] discuss the trade-off between taxi time and fuel consumption during taxing with a multi-objective model.

Simaiakis I(2013)[6] estimate the unimpeded taxi-out time distributions, develop a stochastic and dynamic queuing model of the departure runway, based on the analysis of D(t)/Ek(t)/1 queuing systems. The concept of Pushback Rate Control was presented by analyze the relations between departure throughput, arrival throughput in the next 15 minute and departure demand. The simulations of the Pushback Rate Control protocol at Philadelphia International Airport proves that the potential benefits, Simaiakis I also analyzed the implementation challenges of Pushback Rate Control. After that Simaiakis I et al(2014)[7] further determines a suggested rate to meter pushbacks and describes the field trials of a control strategy, this field tests at Boston Logan International Airport showed that the strategy can brings significant benefits in fuel saving. Sandberg et al(2014)[8] focus on the suggested rate, and support for its application, describes the field-testing of congestion control strategies at Boston Logan Airport. Fornés Martinez H(2015)[9] considers the pushback control policies which regulate departure pushback rates by holding aircraft at gates during congested periods at LaGuardia
Airport, this paper also analyzes the gate-holding limits in details.

At present, research about aircraft pushback both at home and abroad has no explicit punishment mechanism, the flight which in the event of default would lose the opportunities in this period, cause serious delay. For the effectiveness and fairness, the airline of the flight which in the event of default should be punished, the flight which in the event of default can also get slot resources.

In view of this, this paper focused on the problem of aircraft pushback slot allocation. The problem involves first proposing the theory of dynamic credibility priority to reflect the punishment of default, then establishing a non-cooperation game model for aircraft pushback slot allocation, the definitions and modules were explained extensively. The hybrid decision algorithm based on dynamic credibility priority was described, and the full details including basic idea and design processes of this algorithm were explained. The performance of this algorithm was verified by the numerical analysis on Xinzheng international airport.

II. GAME ANALYSIS OF AIRCRAFT PUSHBACK SLOT ALLOCATION AND DYNAMIC CREDIBILITY PRIORITY

The process of aircraft pushback includes close the door, apply for pushback (airline), allocate pushback slot (airport), off-block, aircraft pushback, reach the entrance of taxiway, start the engine.

Aircraft pushback means the process from off-block to the tractor out of the plane, pushback slot allocation namely the airport specify a time period for each flight, and the flight should completed the process of pushback during this period of time. For single flight, the earlier of pushback time starting time, the earlier of flight get into the take-off queuing, and less of delay.

Conventional pushback slot allocation follows "first come first service" (FCFS), the airport select and allocate pushback slot to the first prepared flight. FCFS algorithm is simple, but there were some drawbacks.

1) FCFS does not refer to take-off time, untimely pushback will lead to long waiting time in the entrance of taxiway.
2) If late pushbacked flight has an early take-off time, the queuing behind those late take-off flights will result in serious delay.
3) The punishment to default was not considered.

A. Game Analysis

Aircraft pushback operation can be regarded as a separate process. As interest subjects, the airline is selfish, its goal is to pushback flights as soon as possible, so as to reduce the loss of this airline delays (If the actually pushback time is later than the latest pushback time, delay will be occurred), so a non-cooperative game competition for limited resources is formed between airlines.

Untimely pushback will lead to long queue in the entrance of taxiway, aggravate congestion on airport surface. As the manager and implementer of pushback management, the airport’s target is to minimize the airport surface congestion, this is about to limit the pushback time. Therefore, the airport needs to intervene the rationality of the pushback slot resources allocation in the process of pushback decision-making, thus another non-cooperative game is also formed between the airport and airlines.

B. Theory of Credibility Priority

According to the modern game theory, the way to reduce default behaviour is the implementation of the "tit-for-tat" strategy[10], in which one party who don't compliance the agreement will receive credible, inevitable, hard punishment, to guarantee it dare not to default. In order to future utility, airlines have to sacrifice immediate benefits for the time being. In a standard market with perfect credit management mechanism, faithless airlines will suffer faithless punishment, and faithless costs are much higher than the faithless utility, only the creditworth airlines can get long-term development.

In view of this, this study defines credibility priority as high credibility airline has the priority in the process of competing for pushback slot resources. In this theory, should a selected airline does not complete pushback within the scheduled time period, this would constitute an act of default, and it should be punished by lowering its credibility priority in later allocations.

III. NON-COOPERATION GAME MODEL FOR AIRCRAFT PUSHBACK SLOT ALLOCATION

A. Stackelberg Non-cooperation Game Model

Aircraft pushback decision-making system is a master-slave hierarchical decision-making system[11] which is composed of airport (leader) and airlines (followers), namely the stackelberg decision:

< P, A >

Definition 1. Airlines A = < Li, { s i }, priorityL i >

1) L i : Set of the airline, Li = {L1, L2, ..., Ln}, L i represents the airline, Li =0 or 1, a is the request of pushback, b is the application for pushback. If a=0, L i does not participate in slot allocation; if a=1 and b=1, L i participates in slot allocation.
2) { s i } : The strategy set of all airlines, S = (s1, s2, ..., sn)
3) PriorityL i : The priority of airline i. In this research, the primary credibility priority of each airline is calculated by its departures’ punctuality rate.
4) PriorityF : The priority set of all airlines, PriorityF= (priorityf1, ..., priorityf...)

Definition 2. The strategy set of airline i : si = {Ti1, Ti2, Ti3, Ti4}

1) T i1 : The optimal strategy of airline i. Ti1 is the ideal pushback time period which enables flight take-off on time.
2) T i2 : The suboptimal strategy of airline i. Ti2 is the pushback time period which result in slight delay.

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41.2
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3) $T_3$: The poor strategy of airline $i$. $T_3$ is the pushback time period which result in acceptable delay.

4) $T_a$: The worst strategy of airline $i$. $T_a$ is the pushback time period which result in serious delay.

This paper focus on the rationality of pushback slot resource allocation in time, utility value of different strategies: $T_{i3}=3$, $T_{i2}=2$, $T_{i1}=1$, $T_{ia}=1$.

Taking airline $i$ as an example, the time distribution of pushback strategy $S_i$={$T_{i1}$, $T_{i2}$, $T_{i3}$, $T_{ia}$} is shown in Figure 1.

![Figure 1. The time distribution of pushback strategy](image)

- $f_{pa}$: The initial time of optimal pushback time period which enables flight take-off on time, $f_{pa}=t_{take-off} - t_{push - t_{taxi}} - t_{wait}$.
- $t_{take-off}$: The initial time of take-off, min;
- $t_{push}$: The pushback time, min;
- $t_{taxi}$: The taxiing time, min;
- $t_{wait}$: The waiting time before the runway, min;
- $f_{pa}$: The initial time of suboptimal pushback time period which result in slight delay, $f_{pa}=f_{pa} + l$ where $l$ is the number of aircrafts apply for pushback;
- $f_{pa}$: The initial time of worst pushback time period, which result in serious delay for particularly late pushback, $f_{pa}=f_{pa} + l$.

D: The time that aircraft stays in the airport for a personal reason, such as default.

Definition 3. Airport: $P=\{S_1, U_P\}$

1) $S_i$: The strategy set of $P$, $S_i=\{S_{i1}, S_{i2}, \ldots, S_{in}\}$;
2) $k$: The number of decision makings, $k=1, 2, \ldots, m$.
3) $U_P$: Total utility value, which obtained by the formula below:

$$U_P=\sum_{i=1}^{n} ax+by+cz+do$$

Where $u_i$ is utility value of each decision, $n$ is the number of airlines, $m$ is the number of slot, $x$, $y$, $z$, $o$ represents the number of aircraft which adopt different strategies, $a$, $b$, $c$, $d$ represents the utility value of different strategies in table 1.

Definition 4. Equilibrium solution $S_{eq}$

$S_{eq}$ belongs to $S_i$ is a plan of pushback slot resources allocation. For the formula $U_P^{*}(S_{eq}/S_{eq}) <= U_P^{*}(S_{eq}/S_{eq})$, if their is at least one $i$ make the equals sign can not hold, $S_{eq}$ is a equilibrium solution of the game model (1)\(^2\).

The ultimate goal of this game model for aircraft pushback is to optimize the plan of pushback slot resources allocation $S_{eq}$, optimize the proportion of $x$, $y$, $z$, $o$. That is to maximum the value of $U_P$ while considering the utility of individual airline, more ideal pushback, little slight delay, avoid acceptable time, and try to eliminate the serious delay, finally to minimize the delay and fuel consumption of all aircrafts.

B. Behavior Rules

Behavior rules are as follows:

1) IF $L_i=0 \Rightarrow \phi$; Wait for the next cycle;
2) IF $L_i= l=\geq m\Rightarrow \{T_{i1}, T_{i2}, T_{i3}, T_{ia}\}$;
3) $\forall L_i=1, 2 \Rightarrow Priority^{F}$;
4) IF $w=v$, then new $S=S$, go to Step 7; (For each allocation, $w$ is the maximum pushback slots number provided by the airport, $v$ is the number of aircrafts apply for pushback.)
5) IF $w<v \wedge priority^{F} < priority^{m}$ Then delete $s_i$, delete the strategy of flight with low priority;
6) IF $w<v \wedge priority^{F} > priority^{m}$ Then retain $s_i$, select the strategy of flight with high priority;
7) $\forall S_i \in S_l \Rightarrow U_P \geq \max i(S_i)$
8) $P$ allocate pushback slots to $L_i$.

IV. THE HYBRID DECISION ALGORITHM BASED ON DYNAMIC CREDIBILITY PRIORITY

A. Basic Idea of HEAR Algorithm

According to the procedure of pushback, the aircraft pushback process was regarded as a game between the airport and airlines, thus the non-cooperation game model of aircraft pushback decision was established. In the process of allocation, the theory of “credibility priority” is proposed to punish the behavior of default, then the hybrid decision algorithm based on dynamic credibility (HEAR) is proposed.

This also is a game problem: In the competition for slot resources, the airport follows the principle of credibility priority and FCFS, airlines will have priority to slot(under the condition of the same credibility priority, follow the principle of FCFS), if a selected airline with high credibility does not complete aircraft pushback within the scheduled period, this would be a default, and it should be punished by lowering its credibility priority and be obviously in inferior position in later competition.

In order to give small airlines an fair competition, further regulate the big airline’s private behavior, the credibility priority of each airline is obtained only based on its departures’ punctuality rate in the statistical period. There is no consideration of its scale and strength, qualification, product brand.

B. The Design of HEAR Algorithm

1) Determine $L_i$, $L_i=\{L_1, L_2, \ldots, L_n\}$
2) Calculate priority $L_i$ of each $L_i$.

(1) For the primary allocation, the credibility priority of each airline is obtained based on its departures’ punctuality rate in the statistical period. For subsequent allocations, the priority is obtained based on dynamic credibility priority algorithm;

(2) If their are four or more airlines participate in the competition, in order to ensure the fairness of competition,
the initial value of credibility priority will be diminished in groups of 0.1.
3) Select $L_i$
Select and allocate pushback slots to $1-m$ aircrafts with high priority. For $k$th allocation, if airlines which apply for pushback have the same credibility priority, follow the principle of FCFS.
4) Determine $S_k$
(1) Determine available plans of $S_k$.
(2) Select $S_k$: calculate $U_{p}^{k}$ of all available plans, then select the $S_k$ which can maximize the value of $U_{p}^{k}$ as the final plan of $k$th allocation.
5) Update priority $L_i$ of each $L_i$

$$priorityL_{i}^{k} = priorityL_{i}^{k-1} + \Delta priorityL_{i}^{k-1} \quad (3)$$

Where $k$ is the number of decision makings(1, 2, ..., $k$, ..., $m$), $\Delta$ priority $L_{i}^{k-1}$ is the change to airline $i$’s priority after the ($k-1$)th allocation.

The updating principles of priority $L_i$:
1) $L_i=0 \Rightarrow priorityL_{i}^{k} = priorityL_{i}^{k-1}$;
2) $L_i = 1 \land priorityfj < priorityfm \Rightarrow priorityL_{i}^{k} = priorityL_{i}^{k-1}+1$;
3) $L_i = 1 \land priorityfj > priorityfm \land performance \Rightarrow priorityL_{i}^{k} = priorityL_{i}^{k-1}-1$;
4) $L_i = 1 \land priorityfj > priorityfm \land defaults \Rightarrow priorityL_{i}^{k} = priorityL_{i}^{k-1}-2$.
For airline which not involved in resource application, the value of priority is kept unchanged. For those which applied for pushback but did not get resource for low priority, adds one to the old value. For those selected and pushback on schedule, minus one to the old value. For those selected but defaulting, minus two to the old value.
5) Return to 3), continue again and again until the end.

V. NUMERICAL ANALYSIS
Numerical analysis was conducted to determine the following:
1) The effectiveness of the developed non-cooperation game model in terms of its ability to decrease the delay of take-off time and reduce the fuel consumption.
2) Whether the punishment to default have been considered.
3) Whether the proportion of strategy have been optimized.
Numerical analysis was performed using real data of Xinzheng International Airport, China (10:00-11:00, Dec.9, 2015).

A. Data Preparation
The scheduled departure time, flight number, airline and actual departure time were shown in Table 1.

<table>
<thead>
<tr>
<th>NO.</th>
<th>Scheduled</th>
<th>Flight number</th>
<th>Airline</th>
<th>Actual</th>
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<td>10:00</td>
<td>GS7843</td>
<td>GS</td>
<td>10:14</td>
</tr>
<tr>
<td>2</td>
<td>10:10</td>
<td>MF8205</td>
<td>MF</td>
<td>10:13</td>
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<td>3</td>
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<td>CZ</td>
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<td>4</td>
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<td>3U</td>
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<td>3U</td>
<td>11:50</td>
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<td>9</td>
<td>10:30</td>
<td>NS3309</td>
<td>NC</td>
<td>10:49</td>
</tr>
</tbody>
</table>

Table 2. The primary credibility priority of each airline

<table>
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<th>NO.</th>
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<th>Punctuality rate</th>
<th>Priority</th>
<th>NO.</th>
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<th>Punctuality rate</th>
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<td>77.86</td>
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<td>3</td>
<td>3U</td>
<td>85.07</td>
<td>99.8</td>
<td>9</td>
<td>JR</td>
<td>77.61</td>
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<td>HU</td>
<td>84.76</td>
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<td>99.5</td>
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</table>

The primary credibility priority of each airline is calculated by its departures’ punctuality rate in October, 2015 (Table 2).

The supplied data including parameters were shown in Table 3.

<table>
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<tr>
<th>Parameter</th>
<th>Time(min)</th>
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<th>Time(min)</th>
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<td>$t_{d1}$</td>
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<td>$t_{d2}$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3. The supplied data
All the parameters and initial conditions were taken to the non-cooperation game model. The hybrid decision algorithm based on dynamic credibility priority (HEAR) was adopted by use of MATLAB to calculate the mathematical model.

### B. Simulation Result

According to the non-cooperation game model and HEAR, the detailed result of pushback slot allocation was listed in Table 4.

#### Table 4. Result of pushback slot allocation

<table>
<thead>
<tr>
<th>NO.</th>
<th>Flight</th>
<th>Airline</th>
<th>Pushback time</th>
<th>NO.</th>
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<th>Airline</th>
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#### Table 5. Aircraft pushback utility of original and optimized

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<th>Optimized</th>
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<td>MU</td>
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<td>2</td>
</tr>
</tbody>
</table>

#### Table 6. Airline average pushback utility of original and optimized

<table>
<thead>
<tr>
<th>Airline</th>
<th>CA</th>
<th>SC</th>
<th>3U</th>
<th>HU</th>
<th>CZ</th>
<th>MU</th>
<th>MF</th>
<th>GS</th>
<th>JR</th>
<th>8L</th>
<th>NC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>3.0</td>
<td>0.5</td>
<td>0.0</td>
<td>1.0</td>
<td>0.5</td>
<td>1.3</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Optimized</td>
<td>3.0</td>
<td>1.0</td>
<td>1.7</td>
<td>1.0</td>
<td>1.0</td>
<td>1.7</td>
<td>2.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**1) Utility Test:**

1) Utility of $P$ the aircraft pushback utility of original and optimized was shown in Table 5.

Utility here can directly reflect the delay and fuel consumption, the larger the numerical value of utility, the smaller the degree of delay and fuel consumption. Table 5 shows that the utility increases from 21 to 28 after optimizing, it is said that the game model and HEAR can effectively decrease the delay and fuel consumption.

2) Utility of $L$

the airline average pushback utility of original and optimized was shown in Table 6, Figure 2.

For FCFS, the maximum airline average pushback utility value is 3(CA) while minimum is 0(3U), their is a big utility gap between CA and 3U which indicates the unbalanced resources allocation of pushback slot, it’s unfair to 3U.

For HEAR, the maximum is 3(CA) while minimum is 1(SC, HU, CZ, GS, JR, 8L, NC), the more balanced resources allocation makes the curve much smoother.

Taking aircraft 8, 11, 15 as examples, when in the event of default, FCFS will send it to the next period, that lead to serious delay, HEAR considers the penalties by lowering its credibility priority so as to avoid serious delay.

**2) Default Test:**

For FCFS, aircraft’s act of default has no effect on its airline, the priority of the airline to participate in competition, it is unfair to good-faith airline. HEAR algorithm punished the behavior of default by lowering its credibility priority. Taking airline 3U and CZ as examples, they both have high initial priority (table 2), each of them has 3, 2 aircrafts. The average utility should be large, however there are two flights with the act of default in 3U, they were punished by lowering their credibility priority so as to restrict the airline's behavior(Figure 2).
(3) The pushback strategy Test: 1) Total proportion of all aircrafts

The total proportion of optimal, suboptimal, poor and worst strategy have been changed, the contrast shown in Figure 3. Compared with FCFS, the proportion of optimal strategy have been increased from 11.1% to 22%, the proportion of worst strategy have been decreased from 16.7% to 0%.

Total proportion of optimal, suboptimal, poor and worst strategy have been optimized.

2) The proportion of each airline

Taking MU and 3U (both have 3 aircrafts) as examples, the proportion of optimal, suboptimal, poor and worst strategy of each airline was shown in Figure 4, Figure 5.

MU’s proportion of suboptimal strategy have been increased from 33.3% to 66.7%, and poor strategy have been decreased from 66.7% to 33.3%. 3U’s proportion of optimal strategy have been increased from 0% to 33.3%, and worst strategy have been decreased from 66.7% to 0%.

It is also said that the game model and HEAR can effectively optimize the proportion of airline.

VI. CONCLUSION

To reduce the delay and fuel consumption of departure flight, this paper focused on the problem of aircraft pushback slot allocation and hybrid decision algorithm based on dynamic credibility priority (HEAR) was considered.

In this research, the theory of “credibility priority” is proposed to punish the behavior of default. In order to avoid the delay of take-off time caused by aircraft pushback management, the delay and fuel utility of different pushback slot strategy is quantified with modern game theory. This paper builds an non-cooperation game model for aircraft pushback slot allocation, its object function is the maximum of delay and fuel consumption utility. The hybrid decision algorithm based on dynamic credibility priority was developed, it combines two principles- dynamic credibility priority and FCFS. The numerical example on Xinzheng international airport proves its ability to reduce the delay and fuel consumption, reflect the the punishment to default, optimize the pushback strategy.

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REFERENCES


