The Identification of Competitive Strategy based on Palepu Assumptions: A Mathematical Derivation Perspective

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Abstract — According to the prior research, we firstly establish an asymmetric Bowley utility function for the cost leadership firms as well as the differentiation firms. Secondly, we apply Bertrand model to deal with the Nash equilibrium for these firms pursuing cost leadership or differentiation strategies. Thirdly, subsequently utilizing the price elasticity of demand hypothesis and equilibrium performance condition of competitive strategies, we confirm that Palepu assumptions can be used as strategy identity indicators by mathematical verifications. Finally, we propose a summary of suggestions for future theoretical researches of Palepu assumptions.

Keywords — palepu assumptions; competitive strategy; bertrand equilibrium

I. INTRODUCTION

In the last two decades, there has been debating the epistemologies and methodologies applied in the strategic management both in academics and practice cycles [1-2]. Moreover, it is sometimes argued whether strategic management can be treated as a science—at least in the contexts of industrial organization. Besides, strategic management is equal parts mathematics and logic, empirical evidence and testing as with other sciences [3], from which indeed deploys a framework to infer principals, theoretical conclusions and somewhat empirical results [4-8].

The earlier literature mainly utilized cluster analysis to divide planning strategies into several groups based on subjective scales data [9-11]. In addition, some scholars applied cluster or factor analysis to group competitive strategies according to objective data mainly from PIMS databases [12-13]. These works only have simply grouped the competitive strategies, and the identified results differs among different data, which are quite difficult in forming general conclusions, sometime even “to identify and identify”, while ignoring the nature and intention of strategy identification.

In recent years, noting that Palepu and Healy [14] first creatively apply a modified Du Pont model based on financial statements analysis to competitive strategy identity as premise and methodology, producing a new multiple-discipline perspective of strategic management. Subsequently, Little [15] used the modified Du Pont model of financial ratio analysis to identify the drivers of financial success under alternative business strategies. Firms with high net operating income to sales and low operating asset turnover are assumed to be pursuing a differentiation strategy, and vice versa for cost leadership strategy. We call such rules for strategy identification as Palepu assumptions. Considering the availability and comparability of data, based on resource-based view, Tang [3] reconstruct Du Pont analysis to two indicators for identifying strategies: 1) net profit less adjusted tax to sales and 2) invested capital turnover. Similarly, other scholars [16] do such work on utilizing Du Pont model to competitive strategy identification. In general, it is still a new perspective for applying the financial analysis to strategic identity, which absolutely needs more scholars’ attention. As a matter of fact, there is a natural convergence between financial analysis and competitive strategy. That is, financial methodologies should not and won’t be absent in studying on strategic management.

As stated above, it is technically possible to utilize game model with pure mathematical derivations to identify competitive strategy, while indicators for strategy identity are not consistent and stable. At the meanwhile, financial analysis such as Palepu assumptions can provide fixed and significant indicators to categorize competitive strategies, while there is a need to be verified by somewhat deviations in theory. Thus, why not combine these two methodologies as a whole? That is the exact thing the paper would like to work on.

II. MATHEMATICAL DERIVATIONS ON STRATEGY IDENTIFICATION

In total, in this part of the paper, we would like to 1) provide three basic hypotheses based on Porter’s statements on strategy; 2) build an asymmetric Bowley utility function of two typical firms separately who is pursing cost leadership or differentiation strategies, to obtains the inverse demand function, demand function and objective function; 3) utilize Bertrand model to calculate the equilibrium solutions including equilibrium price, equilibrium output, and equilibrium profit in these two typical firms; 4) introduce price elasticity of demand constraint condition and equal performance constraint condition respectively to derive the conditions of these two pure strategies.
A. Basic Hypothesis

As a matter of fact, Porter [4] only defined differentiation strategy as: “seeks to develop and offer products or services that are perceived as being unique or superior to gain excessive premium in some way, such as design, brand image, technology etc.”. Besides, Porter [5] stated that differentiation strategy can make companies avoid competition by using the customers’ loyalty and the resulting decline in price elasticity, which implies that uniqueness can make differentiated products lower price elasticity of demand than cost leadership products. In a word, the stronger uniqueness of products, the lower price elasticity of demand. Thus, we assume two hypotheses as below:

H1: the differentiated product can hardly be replaced by cost leadership product, and the unit utilities of these two products are asymmetric.

H2: the price elasticity of demand for differentiation product is much lower than that of cost leadership product.

Let $E_1$, $p_1^*$, $q_1^*$ respectively represent price elasticity of demand, equilibrium price and equilibrium outputs of firm 1 providing cost leadership products. Similarly, let $E_2$, $p_2^*$, $q_2^*$ respectively represent price elasticity of demand, equilibrium price and equilibrium outputs of firm 2 providing differentiated products.

$$E_1 = -\frac{\partial q_1}{\partial p_1} \frac{p_1^*}{q_1} > E_2 = -\frac{\partial q_2}{\partial p_2} \frac{p_2^*}{q_2}$$ (1)

As Porter [4] stated: “whatever strategy the firms choose, can be lead to success with specific competitive advantage of that exact strategy at the end”. Specifically, firms pursuing differentiation can gain premium price with their uniqueness in design, brand and technology advantages; In contrast, cost leadership firms can survive and develop by strict cost control and improve managerial ability to provide standard products or services, to realize scale economy. That means, it is possible to replace each other between these two pure strategies, moreover they are equal in financial performance in theory. Thus, we assume the third hypothesis as below:

H3: cost leadership and differentiation strategy can both lead to an equilibrium performance in theory.

Let firm 1 be the cost leadership, and firm 2 be the differentiation. Thus, if the return on asset (ROA) of firm 1 is $\rho_1$, then the return on asset (ROA) of is $\rho_2$. Per Hypothesis 3, there exists an equilibrium performance condition expressed as follows.

$$\rho_1^* = \rho_2^* = \frac{\pi_1^*}{A_1} = \frac{\pi_2^*}{A_2}$$ (2)

Where $\pi_1^*$ represents the equilibrium profit of firm 1, $\pi_2^*$ represents the equilibrium profit of firm 2. Similarly, let $A_1$ and $A_2$ respectively represent the asset of firm 1 and firm 2.

B. Modeling

Referring to Chen’s [16] approach, we try to improve the original symmetric Bowley quadratic utility function of two products into a general asymmetric model to distinguish and accommodate different strategies. Per hypothesis 1, let $\theta_1$ and $\theta_2$ be parameters for the substitutional factors instead of the substitution parameter in original symmetric utility function. Besides, the paper adds a relative utility coefficient, let parameter $b$ represent uniqueness factor of differentiation products, and $b > 1$.

Assume only two firms in a market, firm 1 is cost leadership, firm 2 is differentiation. Besides, whatever kind of firms only produce one product and sell out with the single price. Let $q_1$ and $q_2$ represent the output of cost leadership product and differentiated product respectively, let $m$ represent the utility of all other products. The new modified Bowley utility quadratic function is designed as follows.

$$u = (q_1 + q_2) - \frac{1}{2}(bq_1^2 + 2\theta_1 bq_1 q_2 + q_2^2) + m$$

$$= (q_1 + q_2) - \frac{1}{2}(bq_1^2 + 2\theta_2 q_1 q_2 + q_2^2) + m$$ (3)

where $\theta_1$ stands for the possibility of product in firm 2 (differentiation) substituting product in firm 1 (cost leadership), vice versa is $\theta_2$. Moreover, $\theta_1$ and $\theta_2$ are both restricted to the internal (0,1).

If set $b = 1$, $\theta_1 = \theta_2 = \theta$, equation (3) will degenerate to the original Bowley quadratic function as follows.

$$u = (q_1 + q_2) - \frac{1}{2}(q_1^2 + 2\theta q_1 q_2 + q_2^2) + m$$ (4)

Let $p_1$ and $p_2$ separately represent the price of cost leadership product in firm 1 and that of differentiation product in firm 2. According to the property of Bowley utility function, the first-order conditions of utility-maximization for cost leadership product and differentiation product are given as follows.

$$\frac{\partial}{\partial q_1} \left[ u(q_1, q_2) - \int_0^{q_1} p_1(q_1, q_2) dq_1 \right] = 0$$ (5)

And
$$\frac{\partial}{\partial q_2} \left[ u(q_1, q_2) - \int_{0}^{q_2} p_1(q_1, q'_2) dq'_2 \right] = 0 \quad (6)$$

Where $u(q_1, q_2)$ stands for the gross utility of products in these two firms, $\int_{0}^{q_2} p_1(q_1, q'_2) dq'_2$ stands for the market expenditure spent for the utility of cost leadership product, and $\int_{0}^{q_2} p_2(q_1, q'_2) dq'_2$ stands for the market expenditure spent for the utility of differentiation product. Solving equation (5) and (6), the inverse demand functions of these two products are calculated as below.

$$p_1 = 1 - bq_1 - b\theta_q q_2 \quad (7)$$

And

$$p_2 = 1 - q_2 - \theta_q q_1 \quad (8)$$

Considering the long-term nature of strategy, the paper utilizes Bertrand equilibrium as a methodology for further solutions. Due to the characteristic of Bertrand model derivation on price, we need to apply equation (7) and (8) as a combined equation set to solve the demand functions of these two products.

$$q_1 = -\frac{(p_1 - 1) - b\theta_q (p_2 - 1)}{b(1 - \theta_1 \theta_2)} \quad (9)$$

And

$$q_2 = -\frac{b(p_2 - 1) - \theta_q (p_1 - 1)}{b(1 - \theta_1 \theta_2)} \quad (10)$$

Without considering the fixed cost of these two firms, let $c_1$ and $c_2$ represent unit cost of non-differentiated product and differentiation product individually. Thus, the profit before tax of firm 1 and firm 2 can be separately computed by equation (11) and (12) as follows.

$$\pi_1 = (p_1 - c_1) q_1 \quad (11)$$

And

$$\pi_2 = (p_2 - c_2) q_2 \quad (12)$$

To be mentioned, the demand functions and profit before tax of these two products above are solved step by step based on the improved asymmetric utility function.

C. Bertrand Equilibrium

As assumed there only two firms in a duopoly market, the original Bertrand model is given by equation (11) as below.

$$\begin{bmatrix} \frac{\partial \pi_1}{\partial p_1} \\ \frac{\partial \pi_2}{\partial p_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (13)$$

Where $\pi_i (i=1,2)$ stands for the profit before tax of the firm $i$, considering the research purpose of this paper, we use the return on asset (ROA) instead of profit before tax, which can be expressed as $\rho_i = \pi_i / A_i \quad (i=1,2)$, where $A_i$ stands for the total asset of firm $i$.

Adding the return on asset (ROA) as objective function into equation (13), a modified first-order condition of Bertrand equilibrium is given as follow.

$$\begin{bmatrix} \frac{\partial \rho_1}{\partial p_1} \\ \frac{\partial \rho_2}{\partial p_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial (\pi_i / A_i)}{\partial p_i} \\ \frac{\partial \pi_i}{\partial p_i} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} \frac{\partial \rho_1}{\partial p_1} \\ \frac{\partial \rho_2}{\partial p_2} \end{bmatrix} \quad (14)$$

In essence, it is quite easy to find that equation (13) is equal as equation (14). Using equation (14), we can solve the equilibrium price and equilibrium output of these two products.

$$p_1^* = \frac{b\theta_1 (c_2 - 1) - (2 - \theta_1 \theta_2) (c_1 - 1)}{4 - \theta_1 \theta_2} + c_1 \quad (15)$$

And

$$p_2^* = \frac{\theta_2 (c_1 - 1) - b (2 - \theta_1 \theta_2) (c_1 - 1)}{4 - \theta_1 \theta_2} + c_2 \quad (16)$$

Without considering the fixed cost of these two firms, let $c_1$ and $c_2$ represent unit cost of non-differentiated product and differentiation product individually. Thus, the profit before tax of firm 1 and firm 2 can be separately computed by equation (11) and (12) as follows.

$$q_1^* = \frac{b\theta_1 (c_2 - 1) - (2 - \theta_1 \theta_2) (c_1 - 1)}{b(1 - \theta_1 \theta_2)(4 - \theta_1 \theta_2)} \quad (17)$$

And

$$q_2^* = \frac{\theta_2 (c_1 - 1) - b (2 - \theta_1 \theta_2) (c_1 - 1)}{b(1 - \theta_1 \theta_2)(4 - \theta_1 \theta_2)} \quad (18)$$

We can easily see that, whatever non-differentiation firm or differentiation firm, the equilibrium price of product will
come up following the firm’s increasing unit cost, which happens to be contrary to the equilibrium outputs in both firms. 

Incorporating these expressions given from (15) to (18) into equation (11) and (12), the equilibrium profit of these two firms can be separately solve as follows.

$$\pi_1^* = \left( p_1^* - c_1 \right) q_1^* = \frac{\left( p_1^* - c_1 \right)^2}{b(1-\theta_1\theta_2)} \quad (19)$$

And

$$\pi_2^* = \left( p_2^* - c_1 \right) q_2^* = \frac{\left( p_2^* - c_2 \right)^2}{(1-\theta_1\theta_2)} \quad (20)$$

D. Introducing Constraint Conditions

Utilizing equilibrium solutions above, with the help of hypothesis 2 and 3, we introduce constraint condition of price elasticity of demand as well as constraint condition of equal performance in succession, to export the identification condition of Porter’s two pure strategies.

Firstly, using hypothesis 2, we can easily confirm the comparison result of price elasticity of demand between these two firms from equation (1) as below.

$$E_1^* > E_2^* \quad (21)$$

Where $E_1^*$, $E_2^*$ stands for equilibrium price elasticity of demand for cost leadership product and differentiated product respectively, incorporating equation (9) -(10), and equation (15)-(18) into equation (21), we can compute as follows.

$$E_1^* = -\frac{\partial q_1^*}{\partial p_1} \cdot \frac{p_1^*}{q_1^*} \cdot \frac{1}{(p_1^* - c_1)} \quad (22)$$

And

$$E_2^* = -\frac{\partial q_2^*}{\partial p_2} \cdot \frac{p_2^*}{q_2^*} \cdot \frac{1}{(p_2^* - c_2)} \quad (23)$$

Secondly, using hypothesis 3 from equation (2), we can easily confirm that there exists an equal equilibrium performance under Bertrand condition.

$$\rho_1^* = \frac{\left( p_1^* - c_1 \right) q_1^*}{p_1 q_1^*} \cdot \frac{p_1^* q_1^*}{A_i} = \pi_1^* \quad (24)$$

Where $\left( p_i^* - c_i \right) q_i^*/p_i q_i^*$ represents net income to sales ratio of firm $i$, and $\left( p_i^* q_i^* \right)/A_i$ represents asset turnover ratio of firm $i$, let $A_i$ stand for asset of firms $i$.

Let $\omega_1^*$, $\omega_2^*$ respectively stands for net income to sales ratio of firm 1 and firm 2, shown as follows.

$$\omega_1^* = \frac{\left( p_1^* - c_1 \right) q_1^*}{p_1 q_1^*} = \frac{p_1^* - c_1}{p_1} \quad (25)$$

And

$$\omega_2^* = \frac{\left( p_2^* - c_2 \right) q_2^*}{p_2 q_2^*} = \frac{p_2^* - c_2}{p_2} \quad (26)$$

E. Identification Conditions of Two Pure Strategies

Compared with equation (22) and (25), we can easily find the relation between them as below.

$$\omega_1^* = \frac{1}{E_1} \quad (27)$$

Similarly, we can easily find the relation between equation (23) and (25) as below.

$$\omega_2^* = \frac{1}{E_2} \quad (28)$$

From equation (21), $E_1^* > E_2^*$, we can make a conclusion as below.

$$\omega_1^* < \omega_2^* \quad (29)$$

Per hypothesis 3, if the first term $\left( p_i^* - c_i \right) q_i^*/\left( p_i q_i^* \right)$, to be equal ROI in these two firms, the second term $\left( p_i^* q_i^* \right)/A_i$ in firm 1 must be greater than $\left( p_i^* q_i^* \right)/A_i$ in firm 2.

With the result of equation (29), we can surely confirm that another latent conclusion that $\left( p_i^* q_i^* \right)/A_i$ in firm 1 is greater than $\left( p_i^* q_i^* \right)/A_i$ in firm 2 can be drawn automatically based on the equilibrium performance condition of these two firms.

To the end, Palepu assumptions on competitive strategy identification are fully supported by mathematical
derivations based on game theory. Finally, we obtain the identification conditions of Porter’s generic strategy as follows.

**First Identification Condition**: the net income to sales ratio of pure differentiation firms is higher than that of pure cost leadership firms.

**Second Identification Condition**: the asset turnover of pure cost leadership firms is higher than that of pure differentiation firms.

III. CONCLUSIONS

Taking the process a step further, we can bring Palepu assumptions as a basic method for strategy identity in empirical studies on strategic management. Noted that, considering the environment gaps of industries and countries, as well as the availability of financial data access, Palepu assumptions need necessarily to be adjusted and improved to match the major circumstances. The possibilities certainly merit further investigation, and form some outstanding conclusions for future research.

ACKNOWLEDGMENTS

This research is financially supported by the National Social Science Council of the Republic of China (11AGL001).

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