A Mechanistic Model for Pressure Prediction in Deviated Wells During UBD Operations

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Abstract — Underbalanced drilling (UBD) offers a major advantage in increasing the rate of penetration and reducing lost circulation. In order to improve recovery, drilling of deviated and horizontal wells increased. However, little data is available regarding the effects of well deviation on the hydraulics of two phase flow in deviated and horizontal wellbores. Prediction of flow and pressure profiles while drilling underbalanced in such wells will help in designing and planning of the well. The main aim of this research is to develop a new mechanistic model to predict flow pattern and calculate flow behavior for each pattern in deviated wells. The proposed model was evaluated against field measurements and compared with empirical models. Simulation results show that the proposed model, which considering the effects of wellbore deviation, has an outstanding performance.

Keywords - mechanistic model; well deviation; flow pattern; pressure drop

I. INTRODUCTION

Under UBD conditions, two phase flow models are used to predict flow characteristic, such as pressure drop, flow pattern, liquid holdup and other parameters. The models can be divided into two categories: empirical correlations and mechanistic models. Although empirical models lead to acceptable results in certain wells, they fail or over predict for both vertical and horizontal operations sometimes[1]. Field applications show that mechanistic models, rather than empirical correlations, are being used with increasing frequency for design of multiphase production system because of the better accuracy[2]. Based on this trend of improvement, the application of mechanistic models to predict wellbore pressure and two phase flow parameters can increase the success of UBD operations. Taking into account the effects of well deviation, an improved mechanistic model for pressure prediction through a deviated well is presented in the paper.

II. DOMINANT UBD FLOW PATTERNS

The major flow patterns that exist in multiphase flow are dispersed bubble, bubble, churn and annular[3]. In UBD, due to well control safety and surface fluid handling consideration, choke pressure increase would drastically decrease superficial gas velocities and shift flow pattern from annular to churn or slug. In addition, pressure and temperature change along with the wellbore of a typical UBD well. Churn and annular flow may occur only at conditions close to the surface. Therefore, for the upward flow in the annulus, UBD operations deal mostly with dispersed bubble, bubble, and slug flow. For downward flow through the drill string, slug, bubble, and dispersed bubble flow are also the dominant flow patterns, depending on the combination of injected gas and liquid flow rates[4].

III. FLOW PATTERN PREDICTION MODELS

The following flow pattern models applied to both the drill string and the annulus with an inclination angle θ from horizontal.

A. Downward Flow through the Drill string Bubble to Slug Transition

Hasan proposed the following expression for transition boundary between bubble and slug flow [5].

\[ v_{\text{SL}} = C_0 v_\infty - v_\infty \sin \theta (1) \]

Harmathy correlation is used to calculate the terminal rise velocity (\(v_\infty\)) for upward flow in vertical and inclined channels as follows [6].

\[ v_\infty = 1.53 \left( \frac{\rho_L - \rho_G}{\rho_G} \right) g \sigma^{0.25} \]

Where, \(C_0\) is the velocity profile coefficient. When inclination is 10º-50º, \(C_0=1.05\); When inclination is 50º-60º, \(C_0=1.15\); When inclination is 60º-90º, \(C_0=1.25\). \(\rho_L\) is liquid density and gas density, respectively, kg/m³. \(g\) is the gravitational acceleration, m/s². \(\sigma\) is gas void fraction. \(v_{\text{SL}}\) is superficial liquid velocity, m/s. \(\sigma\) is liquid surface tension, N/m.

1) Bubble or Slug to Dispersed Bubble Transition

Caetano model is recommended for the bubble or slug to dispersed bubble flow transition [7-8], which is given by:

\[ \frac{2}{D_t} \left( \frac{v_{\text{SL}}}{D_t} \right)^{0.4} = \frac{1.6 \sigma g}{(\rho_L - \rho_G) g} \left( \frac{\rho_L}{\sigma} \right)^{0.3} \left( \frac{v_{\text{SL}}}{D_t} \right)^{0.6} \]

\[ = 0.725 + 4.15 \left( \frac{v_{\text{SL}}}{D_t} \right)^{0.5} \]

(3)
Fanning friction factor, \( f \), is calculated using the no-slip liquid holdup \((H_l)\) defined by:

\[
H_l = \frac{v_g}{v_M} (4)
\]

Where, \( D_{IT} \) is the inner pipe diameter, m. \( v_M \) is mixture velocity, m/s. \( v_g \) is superficial gas velocity, m/s.

B. Upward Flow through the Annuli Bubble to Slug Transition

During bubble flow, discrete bubbles rise with the occasional appearance of a Taylor bubble. The discrete bubble rise velocity in an inclined annulus, therefore, can be expressed as:

\[
v_{TB} = \left[0.345 + 0.1 \frac{D_{OT}}{D_{IC}}\right] \sin \theta (1 + \cos \theta)^{12} \left(\frac{g D_{IC} \rho_L - \rho_G}{\rho_L}\right)^{1/2} (5)
\]

Where, \( D_{OT} \) is the outside pipe diameter and \( D_{IC} \) is the inner casing diameter, m.

Hasan and Kabir stated that the presence of an inner tube does not appear to influence the bubble concentration profile \((C_0)\) and thus, the bubble-slug transition is defined by:

\[
v_{SL} = \frac{(4 - C_0) v_{SG}}{\sin \theta} - v_{TB} (6)
\]

1) Bubble or Slug to dispersed bubble transition

The hydraulic diameter \((D_h)\) is substituted for the pipe inside diameter \((D_{IC})\) in (3), the transition from bubble or slug to dispersed bubble flow is defined as follows:

\[
v_{SL} = 0.725 + 4.15 \left(\frac{v_{SG}}{v_M}\right)^{0.5} (7)
\]

The hydraulic diameter of the casing-tubing annulus is given by:

\[
D_h = D_{IC} - D_{OT} (8)
\]

2) Dispersed bubble to slug flow transition

Taitel et al. determined that the maximum allowable gas void fraction under bubble flow condition is 0.52 [10]. Higher values will convert the flow to slug, hence the transition boundary could be equated as follows:

\[
v_{SL} = 0.923v_{SG} (9)
\]

3) Slug to churn transition

Tengesdal et al. stated that the slug structure will be completely destroyed and churn flow will occur if the gas void fraction equals 0.78 [11]. Thus churn flow will occur. The transition from slug flow to churn flow can thus be represented by:

\[
v_{SL} = 0.0684v_{SG} - 0.292\sqrt{gD_{op}} (10)
\]

Where \( D_{op} \) is the equi-periphery diameter defined as follow.

\[
D_{op} = D_{IC} + D_{OT} (11)
\]

4) Churn to annular transition

Based on the minimum gas velocity required to prevent the entrained liquid droplets from falling back into the gas stream that would originate churn flow, Taitel et al proposed the following equation to predict the transition to annular flow [10]:

\[
v_{SG} = 3.1 \left(\frac{\rho_L - \rho_G}{\rho_G}\right)^{0.25} (12)
\]

IV. FLOW BEHAVIOR PREDICTION MODELS

For steady state flow, the total pressure gradient is composed of gravity, friction, and convective acceleration losses and is calculated as follows:

\[
\frac{dp}{dz} = \frac{dp}{dz}_{hy} + \frac{dp}{dz}_{fri} + \frac{dp}{dz}_{acc} (13)
\]

Where \( \frac{dp}{dz}_{hy} \) is the total pressure gradient, Pa/m; \( \frac{dp}{dz}_{fri} \) is the friction pressure gradient, Pa/m; and \( \frac{dp}{dz}_{acc} \) is the acceleration pressure gradient, Pa/m.

A. Downward Flow through the Drill string Bubble Flow Model for Drill string

The drift flux approach is used to calculate liquid holdup considering the slippage between the phases and non-homogenous distribution of bubbles. The slip velocity using the drift flux approach can be expressed as follows [12]:

\[
v_s = \frac{v_{SG}}{1 - v_{SL} v_M} - C_0 v_M (14)
\]

With an inclination angle \( \theta \) the proposed model as shown below:

\[
v_s = v_{SG} H_h sin \theta (15)
\]

Combining (14) and (15) we get the following expression:

\[
v_{SG} = \frac{v_s}{1 - H_h sin \theta} - 1.2v_M (16)
\]

Newton-Raphson method was used to solve for liquid holdup \( H_h \) from (16).

The gravity component is given by:

\[
\frac{dp}{dz}_{hy} = \rho_g g sin \theta (17)
\]

Where, \( \rho_g = \rho_L H_h + \rho_G (1 - H_h) \) (18)

The frictional pressure loss is given by:

\[
\frac{dp}{dz}_{fri} = \frac{f_M \rho_M v_s^2}{2D_{IT}} (19)
\]

Where \( f_M \) is the Moody friction factor and is calculated using Reynolds number \( N_Re \).
Moody friction factor is four times the Fanning friction factor and it is calculated using the Colebrook function [13].

\[
\frac{1}{\sqrt{f_M}} = -4 \log \left( \frac{0.269 \varepsilon}{D_{ir}} + \frac{1.255}{N_{re} \sqrt{f_M}} \right) \tag{20}
\]

Where, \( \varepsilon \) is tube roughness, mm.

The acceleration pressure gradient components is calculated using Beggs and Brill approach as follow [14].

\[
\left( \frac{dp}{dL} \right)_f = \frac{\rho_M v_M v_{SG} dp}{p} \tag{21}
\]

The acceleration term \( (E_k) \) is defined as follow.

\[
E_k = \frac{\rho_M v_M v_{SG}}{p} \tag{22}
\]

Then the total pressure drop is calculated by (23):

\[
\left( \frac{dp}{dL} \right)_T = \frac{\left( \frac{dp}{dL} \right)_f - \left( \frac{dp}{dL} \right)_e}{1 - E_k} \tag{23}
\]

1) **Dispersed bubble flow model for drill string**

Since nearly a uniform bubble distribution in the liquid, the flow can be treated as homogenous flow. Thus, the liquid holdup is very close to the no-slip holdup \( H_{LS} \). The pressure gradient components are calculated as those in bubble flow.

2) **Slug flow model for drill string**

Assuming that the liquid and gas phases in the liquid slug behave analogously to fully developed bubble flow and that the bubble swarm effect in downward flow is negligible \( (n=0) \) [15], the liquid holdup in the liquid slug can be calculated by:

\[
H_{LS} = 1 - \frac{v_{SG}}{C_0 v_M - v_c} \tag{24}
\]

The liquid holdup in the Taylor bubble in downward flow may be calculated by:

\[
H_{TB} = 1 - \frac{v_{SG}}{C_0 v_M - v_c} \tag{25}
\]

After extensive validations, Hasan recommended using \( C_0=1.2 \) and \( C_1=1.12 \).

Considering a slug unit formed by a Taylor bubble and a liquid slug regions, the liquid holdup in the slug unit may be approximated to:

\[
H_{SU} = 1 - \left[ \frac{H_{TB}}{L_{SU}} \left( 1 - H_{TB} \right) + \frac{L_{LS}}{L_{SU}} \left( 1 - H_{LS} \right) \right] \tag{26}
\]

The slug unit length can be calculated by the following expression based on the superficial gas velocity.

\[
L_{SU} = \frac{160 D_{ir} v_{SG}}{C_0 v_M - v_c} \quad \text{for} \quad v_{SG} > 0.4 \text{m/s} \tag{27}
\]

\[
L_{SU} = \frac{64 D_{ir} v_{SG}}{C_0 v_M - v_c} \quad \text{for} \quad v_{SG} \leq 0.4 \text{m/s} \tag{28}
\]

The liquid slug length is given by:

\[
L_{LS} = 16 D_s \tag{29}
\]

Perez-Tellez showed that, for a fully developed Taylor bubble, the total hydrostatic and frictional pressure losses can be calculated by:

\[
\left( \frac{dp}{dL} \right)_{T_{fH}} = \left[ (1 - \beta) \rho_{M,s} + \beta \rho_{M,g} \right] g \tag{30}
\]

\[
\left( \frac{dp}{dL} \right)_{T_{fF}} = \frac{2 f_{f_r} \rho_{M,s} v_M^2}{D_{ir}} (1 - \beta) \tag{31}
\]

Where \( \rho_{M,s} \) is the mixture density in the liquid slug zone and the friction factor is calculated with Colebrook function using Reynolds number, kg/m\(^3\); \( \beta \) is the relative bubble length parameter, 1; \( \rho_{M,g} \) is the mixture density in the Taylor bubble zone, kg/m\(^3\); and \( v_{TB} \) is the in-situ liquid velocity in the Taylor bubble zone, which are function of the slug flow conditions, m/s.

For fully developed Taylor bubble slug flow:

\[
\beta = \frac{L_{TB}}{L_{SU}} \quad \text{and} \quad \rho_{M,s} = \rho_G \tag{32}
\]

and for developing Taylor bubble slug flow:

\[
\beta = \frac{L_{TB}}{L_{SU}}, \quad \rho_{M,s} = \rho_L H_{TB} + \rho_G \left( 1 - H_{TB} \right) \tag{33}
\]

Since in UBD, the most common flow patterns in downward flow are dispersed bubble and bubble, the acceleration component in drillstring geometries is relatively small and may be either neglected or calculated using the approach suggested for bubble flow, (23).

B. **Upward Flow through the Annulus**

1) **Bubble Flow Model for Annular Geometries**

For a bubbly flow the holdup is calculated as reported by Hasan and Kabir as follows [9].

\[
H_b = 1 - \frac{v_{SG}}{v_a - C_0 v_M} \tag{34}
\]

The gravity pressure gradient is calculated using (17). For the frictional pressure loss is calculated from (19). Caetano suggested the use of the calculation developed by Gunn and Darling for a turbulent flow as follow:

\[
\left[ f \left( \frac{F_p}{F_{CA}} \right)^{0.45 \exp \left[ -(N_{re} - 3000) \times 10^{-6} \right]} \right]^{1/0.5} = \tau \tag{35}
\]

\[
4 \log \left( N_{re} \left( \frac{f \left( \frac{F_p}{F_{CA}} \right)^{0.45 \exp \left[ -(N_{re} - 3000) \times 10^{-6} \right]} }{ \left( F_{CA} \right)^{0.45 \exp \left[ -(N_{re} - 3000) \times 10^{-6} \right]} } \right) \right) = -0.4 \tag{36}
\]

Where \( F_p \) and \( F_{CA} \) are geometry parameters defined by (36) and (37).

\[
F_p = 16 / N_{re,M} \tag{36}
\]

\[
F_{CA} = \frac{16 (1 - K) \sqrt{1 - K^4} - 1 - K^2}{1 - K - \ln (1/K)} \tag{37}
\]

The acceleration component is calculated using Beggs and Brill approach using (23).

2) **Dispersed Bubble Flow Model**

The dispersed bubble holdup is assumed equal to the no-slip holdup \( H_L \). The same equations as in the bubble
flow are used to calculate the total pressure gradient.

3) Slug Flow Model

The same model used by Perez-Tellez for the case of downward flow inside the drill string is used. The hydraulic diameter is used instead of the inner tubing diameter in (30) for calculating Reynolds number.

In addition, the pressure drop due to acceleration across the mixing zone at the front of the liquid slug by:

\[
\left( \frac{dp}{dZ} \right)_{ac} = \frac{H_{l_{is}} \rho_L}{L_{SLu}} \left( \nu_{g_{is}} + \nu_{i_{is}} \right) \left( \nu_T - \nu_{g_{is}} \right) (38)
\]

The average holdup over the entire slug unit \( H_{i_{su}} \) for either developed of fully developing Taylor bubble can be calculated by:

\[
H_{i_{su}} = 1 - \frac{\nu_{g_{is}} + \left(1 - H_{i_{is}}\right) \left(\nu_T - \nu_{g_{is}}\right)}{\nu_{g_T}} (39)
\]

Where \( \nu_{g_{is}} \) is in-situ gas velocity in the liquid slug, m/s.

4) Annular flow model

As explained above, in common UBD operations, the window of occurrence of annular flow is quite limited and when it occurs, it takes place in the annulus at a few meters close to the surface. The simplified annular flow model proposed by Taitel and Barnea was implemented only to avoid convergence problems during the computations [16].

\[
\left( \frac{dp}{dL} \right)_{\tau} = \frac{4\tau}{D_e} - \left[ \rho_L H_L + \rho_v \left(1 - H_L\right) \right] g \sin \theta (40)
\]

The annular film thickness \( \delta \) can be defined as follow:

\[
\delta = 0.115 \left( \frac{\mu_L^2}{g \left( \rho_v - \rho_L \right) \rho_L} \right)^{1/3} \left( \frac{\rho_L \nu_{g_{SL}} D_e}{\mu_L} \right)^{0.6} (41)
\]

\( D_e \) is the equivalent pipe diameter and is calculated by:

\[
D_e = \sqrt{D_{in} - D_{in}^2} (42)
\]

The interfacial shear stress (\( \tau_i \)) is defined by:

\[
\tau_i = 0.5 f_s \rho_L \nu_{g_{SL}}^2 (43)
\]

The interfacial shear friction factor is calculated as suggested by Alves et al as follows [17].

\[
f_s = f_{sc} I (44)
\]

Where \( f_{sc} \) is the superficial core friction factor (gas phase) and is calculated based on the core superficial velocity, density and viscosity. The interfacial correction parameter \( I \) is used to take into account the roughness of the interface. The parameter \( I \) is an average between the horizontal angle and the vertical angle and is calculated based on an inclination \( \theta \).

\[
I = I_h \cos^2 \theta + I_v \sin^2 \theta (45)
\]

The horizontal correction parameter is given by Henstock and Hanratty[18].

\[
I_{h} = 1 + \frac{800 F_i}{(46)
\]

Where,

\[
F_i = \left[ 0.707 N_{RE,SL}^{2} \right]^{2.5} + \left[ 0.0379 N_{RE,SL}^{9} \right]^{2.5} \left( \frac{\rho_L}{\rho_G} \right) \left( \frac{\rho_G}{\rho_L} \right)^{0.5}
\]

Where \( \rho_{RE,SL} \) and \( \rho_{RE,SG} \) are the superficial liquid and gas Reynolds number respectively. Both are calculated below:

\[
N_{RE,SL} = \frac{\rho_L \nu_{g_{SL}} D_e}{\mu_L} (48)
\]

\[
N_{RE,SG} = \frac{\rho_G \nu_{g_{SG}} D_e}{\mu_G} (49)
\]

The vertical correction parameter is given by Wallis as follow [19].

\[
I_v = 1 + 300 \left( \frac{\delta}{D_e} \right) (50)
\]

Considering that the liquid film thickness \( \delta \) is constant, the liquid holdup can be estimated by:

\[
H_L = 4 \left( \frac{\delta}{D_e} \right) \left( \frac{\delta}{D_e} \right)^2 (51)
\]

V. BIT MODEL

Using the mechanical energy balance along with the gas weighting fraction and neglecting frictional pressure drop, Perez-Tellez formulated the following expression for calculating the pressure drop across the bit nozzles.

\[
v_T^2 + \frac{1 - w_G}{\rho_L} \left( P_{bh} - P_{ap} \right) + \frac{w_G}{\rho_G} R T \ln \frac{P_{bh}}{P_{ap}} = 0 (52)
\]

Where \( v_T \) is the nozzle velocity, m/s; \( w_G \) is the gas weighting factor; \( P_{bh} \) is the bottom hole pressure, Pa; \( P_{ap} \) is the upstream pressure, Pa; \( M_G \) is the gas molecular weight, kg.

Using the continuity equation for the gas liquid mixture the following expression is reached to express the conservation of mass

\[
\rho_G v_{SL} A_t = q_L \rho_L + q_G \rho_G = \text{constant} (53)
\]

And the nozzle velocity is calculated by:

\[
v_T = \frac{q_L \rho_L + q_G \rho_G}{A_t} (54)
\]

The above three equations are solved numerically to obtain the bit nozzle upstream pressure given the bottom hole pressure.

VI. MODEL VALIDATION

In order to demonstrate the validity of the model, a field case was simulated and results compared with measurements.

Injection and bottom hole pressure at depth 2308m and 2328m was measured by a pressure recording tool, which is obtained from Table I. Drill string and operating parameters for the above two depths are shown in Table II,III.

<table>
<thead>
<tr>
<th>Depth</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2308</td>
<td>Injection Pressure ( P_{bh} / \text{MPa} )</td>
<td>11.7</td>
</tr>
</tbody>
</table>

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Inclination angles are read from survey file, which are shown in Table IV. In order to find the angle at any depth, linear interpolation is used.

TABLE IV MEASURED INCLINATION ANGLE

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Inc. Angle (°)</th>
<th>Depth (m)</th>
<th>Inc. Angle (°)</th>
<th>Depth (m)</th>
<th>Inc. Angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2169.46</td>
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<td>89.09</td>
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<td>2190.52</td>
<td>89.74</td>
</tr>
</tbody>
</table>

Hasan and Kabir, Beggs and Brill models were also tested in compare to the result of the new model. The error of the developed model’s predictions, the empirical models' results with filed measurements are shown in Table V. The average absolute error $E_a$ is given by:

$$E_a = \left| \frac{P_{\text{calc}} - P_{\text{meas}}}{P_{\text{meas}}} \right| \times 100\% \quad (55)$$

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Run 1#</th>
<th>Run 2#</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calc. Pres /MPa</td>
<td>$E_a$</td>
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<tr>
<td>Developed</td>
<td>$P_{in}$</td>
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<td>Model</td>
<td>$P_{in}$</td>
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</tr>
<tr>
<td>Beggs &amp; Brill</td>
<td>$P_{in}$</td>
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</tr>
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<td>Hasan &amp; Kabir</td>
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<td>16.3</td>
</tr>
<tr>
<td></td>
<td>$P_{in}$</td>
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<tr>
<td></td>
<td>$P_{in}$</td>
<td>18.6</td>
</tr>
</tbody>
</table>

Table V shows that the results of the developed model have a good match with the measured value where at the average absolute error $E_a$ has an average value of less than 10%. The Beggs & Brill, Hasan & Kabir models are based on empirical correlations, the results shown in Table V indicate that the empirical models predict the bottom hole pressure reasonably, however the mechanistic model outperform both.

VII. CONCLUSIONS

A model is developed for the prediction of flow pattern and flow behavior in deviated wellbores and pipelines under UBD conditions using mechanistic steady state model. In the annulus, dispersed bubble, bubble, slug, churn, annular flow are considered. And three flow patterns (dispersed bubble, bubble, slug) are taken into account in the drill string.

The developed model has been validated against field data and existing empirical models. The model have a good match with the measured data, which has an average absolute error of less than 10%. The model comparison shows that the developed model performs better than empirical models when trying to design UBD operation within a pressure window.
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