An Application of Layered Grey Correlation Analysis to Senior Personnel Selection

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Abstract — As project manager is the central figure in the process of project management, so the optimization of project manager has a decisive influence on the success of a project. Based on the actual demand and characteristics of engineering projects, reasonable quantitative indicators have been added, the selection index system of project manager has been established, and Analytic Hierarchy Process, AHP, method was used to calculate scaling factors of each criteria. On this basis, the system evaluation mathematical model was set up using the grey correlation method, correlation contrast curve was formed using the plot function of Matlab. Case analysis has shown that the mechanism of project manager selection is scientific and effective, which has a broad application prospect.

Keywords - project manager; AHP; grey correlation method; Matlab

I. INTRODUCTION

Project manager responsibility system is an EPC (The full name is Engineering Procurement Construction) project management target responsibility system. Project manager as the main responsibility of this system, whose management level affects the realization of the goal of engineering project directly [1]. The selection of a good project manager can not only improve the project quality, save costs and increase profits, but also can reduce the accident rate to a certain extent, and help enterprise to maintain a good business reputation and competitiveness. At this stage, the domestic analysis on project manager selection mostly focus only on the macro-analysis of qualitative indicators, whose over-reliance on the reviewer's subjective consciousness and lacking of effective qualitative indicators assessment mechanism, limiting objective reasonableness of the selection results. From reality, this paper established a project manager selection system including a set of qualitative and quantitative indicators, made assessment method for every index and made quantitative analysis for qualitative index. Then, combined with the advantages of AHP and gray correlation method, this paper proposed a project manager selection mechanism.

II. ESTABLISHING THE EVALUATION INDEX SYSTEM FOR PROJECT MANAGER

An investigation on Tianyuan Group Co., Ltd., Qingdao Construction Group Co., Ltd., Shandong Laiwu Steel Construction Co., Ltd. and eight Construction Group Co., Ltd., Shanxi was made, those construction units provided many data for this paper. Based on information obtained, the main factors influencing the project manager selection were summed.
III. DETERMINATION OF INDEX WEIGHTS BASED ON THE ANALYTIC HIERARCHY PROCESS

Analytic Hierarchy Process (AHP) is a multi-objective decision-making system analysis method including qualitative and quantitative analysis, which was put forward by the US operations researcher TL.Saaty [2-3]. On the basis of hierarchical structure model, judgment matrix was structured. Then, the weight vector can be calculated and consistency will be tested. The index system in this paper includes too many indicators to ensure the accuracy of this research. In order to reduce error in judgment, this paper selected “three scaling method” for solving [4].

A. Establishment of Indirect Judgment Matrix

Firstly, using three scale method to calculate the comparison matrix:

\[ B = \left( b_{ij} \right)_{n \times n} \]  

(1)

where:

\[ b_{ij} = \begin{cases} 
2 : & \text{The i-th element is more important than the j-th element} \\
1 : & \text{The i-th element and the j-th element are equally important} \\
0 : & \text{The j-th element is more important than the i-th element}
\end{cases} \]

Then, calculate the sum value \( r_j = \sum_{i=1}^{n} b_{ij} \) of each row. Elect the maximum \( r_{\text{max}} \) and the minimum \( r_{\text{min}} \), so comparative basis scaling \( d_a \) can be calculated. Finally, indirect judgment matrix \( D = \left( d_{ij} \right)_{n \times n} \) will be obtained.

Where,

\[ d_{ij} = \begin{cases} 
\frac{(r_i - r_j)}{(r_{\text{max}} - r_{\text{min}})} (d_{ij} - 1) + 1 & r_i - r_j > 0 \\
1 & r_i = r_j \\
\frac{(r_j - r_i)}{(r_{\text{max}} - r_{\text{min}})} (d_{ij} - 1) + 1 & r_i - r_j < 0
\end{cases} \]  

(2)

B. Calculation of Weight Vector

\[ \omega = \sqrt[M]{M} \]  

(3)

\[ M = \prod_{j=1}^{n} d_j \ (i = 1, 2, 3, \cdots, n) \]  

(4)

\[ \overline{W} = \left[ \frac{\omega_1, \omega_2, \cdots, \omega_n}{} \right]^T \]  

(5)

After normalization of \( \overline{W} \) following this formula:

\[ \omega_i = \frac{\omega_i}{\sum_{i=1}^{n} \omega_i} \ (i = 1, 2, \cdots, n) \]  

(6)

We get:

\[ W = \left[ \omega_1, \omega_2, \cdots, \omega_n \right]^T \]  

(7)

C. Consistency Test

Consistency index:

\[ CI = \frac{\lambda_{\text{max}} - n}{n - 1} \]  

(8)

where,

\[ \lambda_{\text{max}} = \sum_{i=1}^{n} \left( \frac{DW}{n\omega_i} \right) \]  

If \( CI = 0 \), it shows the judgment matrix is consistent completely, or consistency ratio \( \frac{CR}{RI} \) should be calculated. If \( CR < 0.1 \), the result is acceptable.

IV. ESTABLISHING OF GREY RELATIONAL MODEL

Grey correlation is a multi-objective decision analysis method, which is not limited by the number of samples and the dimensionless index difference constraint. After the establishment of index feature matrix and virtual "ideal scheme", gray correlation degrees between alternatives and
"ideal scheme" will be calculated. The size of the correlation reflects the degree of approximation between general scheme and ideal scheme [5].

A. Determination of the Index Characteristic Values

Assuming the number of alternatives is “m”, the number of evaluation indexes is “n”. Then, the index characteristic amount matrix can be calculated:

\[
X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\
x_{21} & x_{22} & \cdots & x_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}
\]

\[(9)\]

1) Determine the eigenvalues of qualitative indicators

Moral qualities: Making fuzzy evaluation of the evaluation objects mainly from the law-abiding, integrity, honesty and other aspects.

Physical fitness: Hiring units organize medical work, coherent evaluation response according to the examination results.

Psychological qualities: Hiring units organize psychological test (in the form of self-made), evaluation object will get a test score.

Education: The value set [3, 2, 1, 0.5] corresponds to degree set [Doctor, Master, Undergraduate, Specialist].

Knowledge of building law, technology, economy and management: Hiring units organize an examination, each evaluated object will get a set of assessment scores.

Innovation ability, communication skills, leadership skills: Hiring units organize an interview, each evaluated object will get an interview score.

- Index values of moral qualities and physical quality: Refer to Table 2.

TABLE II. REFERENCE TABLE OF QUALITATIVE INDICATORS EIGENVALUES

<table>
<thead>
<tr>
<th>Index</th>
<th>Very good</th>
<th>Good</th>
<th>General</th>
<th>Poor</th>
<th>Very poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalues</td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

2) Determine the eigenvalues of quantitative indicators

Quantity of similar project you participated, working life, quantity of published papers and the number of disciplinary actions: According to the real number or situation.

B. Normalization of Feature Matrices

To eliminate the adverse effects different dimensions caused, the characteristic matrix “X” will be normalize to standard matrix “Y” according to formula (11) and (12).

\[
Y = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\
y_{21} & y_{22} & \cdots & y_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
y_{m1} & y_{m2} & \cdots & y_{mn} \end{pmatrix}
\]

\[(10)\]

where,

\[
y_{ij} = \frac{x_{ij} - \bar{x}_j}{\sigma_j}
\]

\[(11)\]

Benefit index : \[ \bar{y}_j = \frac{\sum_{i=1}^{m} y_{ij}}{m} \]

Cost index : \[ y_{ij} = \frac{x_{ij} - \bar{x}_j}{\sigma_j} \]

C. Calculation of Correlation Coefficient

Assuming “ideal scheme” is:

\[
Y' = \begin{pmatrix} y_{1}' & y_{2}' & \cdots & y_{n}' \end{pmatrix}
\]

Where,

\[
y_{i}' = \max \{ y_{ij} \} \quad (i = 1, 2, \cdots, n)
\]

The jth “ideal solution” is \[ y_{ij}^* \]. The decision matrix \[ Y = [y_{ij}] \] corresponds to ith alternative. Contrast \[ Y \] with \[ Y' \], we can get correlation coefficient of each index: \[ r_{ij} [6-7] \].

\[
r_{ij} = \frac{\min \min |y_{ij} - y_{ij}'| + \xi \max \max |y_{ij} - y_{ij}'|}{|y_{ij} - \bar{y}_j| + \xi \max \max |y_{ij} - y_j'|} (i = 1, 2, \cdots, n)
\]

\[(15)\]

Where,

\[
\xi = \text{resolution coefficient}, \quad \xi \in [0, 1]. \quad \text{Generally, take} \quad \xi = 0.5.
\]

Then, we have correlation coefficient matrix:

\[
R = \left[ \begin{array}{cccc} r_{11} & r_{12} & \cdots & r_{1m} \\
r_{21} & r_{22} & \cdots & r_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
r_{m1} & r_{m2} & \cdots & r_{mm} \end{array} \right]
\]

\[(16)\]

D. The Calculation of Correlation Degree

Ideal degree of the ith scheme is positively related to the association degree \[ z_i, \quad i = 1, 2, \cdots, n \]. Correlation calculated by the following formula:

\[
Z = RW = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\
r_{21} & r_{22} & \cdots & r_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
r_{m1} & r_{m2} & \cdots & r_{mm} \end{pmatrix} \begin{pmatrix} \omega_1 \\
\omega_2 \\
\vdots \\
\omega_n \end{pmatrix} = \begin{pmatrix} z_1 & z_2 & \cdots & z_n \end{pmatrix}^T
\]

\[(17)\]
The alternative which is corresponding to \( \max \{ z_i \} \) is the optimal scheme.

### E. Structure Weight Coefficient of Correlation Contrast Curve

Formula (16) shows that the ith scheme’s correlation coefficient vector is \( \mathbf{R}_i = [r_{i1}, r_{i2}, \ldots, r_{in}] \). Each index correlation coefficient multiplied by the corresponding weight value, the weight coefficient of correlation vector corresponding will be obtained: \( \mathbf{R}_i = [\omega_1 r_{i1}, \omega_2 r_{i2}, \ldots, \omega_n r_{in}] \). Similarly, the weight vector of correlation coefficient of the ideal scheme can be calculated: \( \mathbf{R}_I = [\omega_1, \omega_2, \ldots, \omega_n] \).

Using the plot function of matlab, those known date can be transformed into a weight coefficient of correlation contrast curve, which is consisted of \( m+1 \) lines and will show the characteristics of each filing program.

### V. CASE ANALYSIS

Combined with Table1,7 experts from different construction companies were invited to calculate index weights. To judge the matrix \( A - A \) as an example, the calculation process is shown in Table 3.

**TABLE III. CALCULATION PROCESS OF FIRST GRADE INDEX WEIGHT**

<table>
<thead>
<tr>
<th>( A )</th>
<th>( A_i )</th>
<th>( A_j )</th>
<th>( A_k )</th>
<th>( m = \sum l_i q_i )</th>
<th>( m - \frac{m}{\sum q_i} )</th>
<th>( \lambda_{max} )</th>
<th>( CI )</th>
<th>( CR )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>1/4</td>
<td>1/7</td>
<td>1/4</td>
<td>0.0089</td>
<td>0.3071</td>
<td>0.0552</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_2 )</td>
<td>1/4</td>
<td>1/7</td>
<td>1/4</td>
<td>1.0</td>
<td>0.1798</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_3 )</td>
<td>4</td>
<td>1</td>
<td>1/4</td>
<td>112.0</td>
<td>3.2532</td>
<td>0.5851</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_4 )</td>
<td>4</td>
<td>1/4</td>
<td>1</td>
<td>1.0</td>
<td>0.1798</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\lambda_{max} = \frac{\sum (DW)}{m} = 4.0863, \ CI = \frac{\lambda_{max} - n}{n-1} = 0.0288, \\
CR = \frac{CI}{RI} = 0.032 < 0.1 \text{(A satisfactory consistency)}
\]

Similarly, other judgment matrix weight sets can be calculated, the results are shown in Table 4.

**TABLE IV. CALCULATION RESULTS OF SECOND GRADE INDEX WEIGHT**

<table>
<thead>
<tr>
<th>Judgment matrix</th>
<th>Weight set ( W_i )</th>
<th>( \lambda_{max} )</th>
<th>( CI )</th>
<th>( CR )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A - A_j )</td>
<td>([0.7143,0.1429,0.1429] )</td>
<td>3.0002</td>
<td>0.0001</td>
<td>0.0007</td>
</tr>
<tr>
<td>( A - A_{ij} )</td>
<td>([0.4557,0.1988,0.0734,0.0734,0.1988] )</td>
<td>5.2277</td>
<td>0.0569</td>
<td>0.0508</td>
</tr>
<tr>
<td>( A - A_{ij} )</td>
<td>([0.1047,0.2583,0.6370] )</td>
<td>3.0385</td>
<td>0.0193</td>
<td>0.0333</td>
</tr>
<tr>
<td>( A - A_{ij} )</td>
<td>([0.3750,0.3750,0.1250,0.1250] )</td>
<td>4.0000</td>
<td>0.0000</td>
<td>0</td>
</tr>
</tbody>
</table>

\( CR < 0.1 \text{(A satisfactory consistency)} \)

On the basis of Table 3 and Table 4 available, the comprehensive index weight set of evaluation system can be obtained:

\[
W = [\omega_1, \omega_2, \ldots, \omega_n]^T = [0.0394,0.0079,0.0079,0.0819,0.0357,0.0132,0.0132,0.0357,0.0613,0.1511,0.3727,0.0674,0.0674,0.0225,0.0225]^T
\]

(18)

A construction company intends to pick a project manager from 4 candidates, evaluations of 4 indicators are shown in Table 5.

**TABLE V. EVALUATION OF CANDIDATE INDICATORS**

<table>
<thead>
<tr>
<th>Candidates</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{11} )</td>
<td>Good</td>
<td>Very good</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>( A_{12} )</td>
<td>Very good</td>
<td>General</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>( A_{13} )</td>
<td>Specialist</td>
<td>Undergraduate</td>
<td>Undergraduate</td>
<td>Undergraduate</td>
</tr>
<tr>
<td>( A_{14} )</td>
<td>Specialist</td>
<td>Undergraduate</td>
<td>Undergraduate</td>
<td>Undergraduate</td>
</tr>
<tr>
<td>( A_{21} )</td>
<td>85</td>
<td>83</td>
<td>90</td>
<td>92</td>
</tr>
<tr>
<td>( A_{23} )</td>
<td>70</td>
<td>73</td>
<td>87</td>
<td>72</td>
</tr>
<tr>
<td>( A_{24} )</td>
<td>92</td>
<td>83</td>
<td>80</td>
<td>85</td>
</tr>
<tr>
<td>( A_{31} )</td>
<td>70</td>
<td>75</td>
<td>85</td>
<td>72</td>
</tr>
<tr>
<td>( A_{32} )</td>
<td>85</td>
<td>80</td>
<td>90</td>
<td>85</td>
</tr>
<tr>
<td>( A_{33} )</td>
<td>79</td>
<td>88</td>
<td>80</td>
<td>75</td>
</tr>
<tr>
<td>( A_{34} )</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>( A_{41} )</td>
<td>10years</td>
<td>7years</td>
<td>5years</td>
<td>5years</td>
</tr>
<tr>
<td>( A_{43} )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( A_{44} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**A. Determination of the Index Feature Matrix**

Quantify all indicators in Table 5, characteristic index matrix will be obtained:

\[
X = \begin{bmatrix}
0.8 & 1.0 & 87 & 0.5 & 85 & 88 & 70 & 92 & 70 & 85 & 79 & 3 & 10 & 1 & 0 \\
1.0 & 0.6 & 83 & 1.0 & 83 & 80 & 73 & 83 & 75 & 80 & 88 & 2 & 7 & 0 & 0 \\
0.8 & 0.8 & 90 & 1.0 & 82 & 81 & 87 & 80 & 85 & 90 & 80 & 4 & 5 & 1 & 1 \\
0.8 & 0.8 & 92 & 2.0 & 90 & 76 & 72 & 85 & 72 & 85 & 75 & 2 & 5 & 2 & 0
\end{bmatrix}
\]

**B. Normalization of Feature Matrices**

The matrix \( X \) is transformed to standardized matrix \( Y \) according to the equation (11) and (12):
C. Calculation of Correlation Coefficient

On the basis of matrix $Y$, matrix of ideal solution can be calculated: $Y^*=[1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$, where,

$$\min_{j} \min_{i} |y^*_i-y_j|=0, \max_{i} \max_{j} |y^*_i-y_j|=1.$$  

Take $\xi=0.5$, correlation coefficient can be calculated according to formula (15):

$$r_{ij} = \frac{0.5}{|y^*_i-y_j|+0.5} \quad (i=1,2,\ldots,m; \ j=1,2,\ldots,n)$$

Then, we get the matrix of correlation coefficient:

$$R = \begin{bmatrix} 0.3333 & 1 & 0.3333 & 0.3333 \\ 1 & 0.3333 & 0.5 & 0.5 \\ 0.4737 & 0.3333 & 0.6923 & 1 \\ 0.3333 & 0.4286 & 0.4286 & 1 \\ 0.4444 & 0.3636 & 0.3333 & 1 \\ 1 & 0.4286 & 0.4616 & 1 \\ 0.3333 & 0.3778 & 1 & 0.3617 \\ 1 & 0.4 & 0.3333 & 0.4616 \\ 0.3333 & 0.4286 & 1 & 0.3658 \\ 0.5 & 0.3333 & 1 & 0.5 \\ 0.4194 & 1 & 0.4483 & 0.3333 \\ 0.5 & 0.3333 & 1 & 0.5 \\ 1 & 0.4545 & 0.3333 & 0.3333 \\ 0.5 & 0.3333 & 0.5 & 1 \\ 1 & 0.3333 & 1 & 1 \end{bmatrix}$$

D. Calculation of Correlation Degree

The matrix of correlation degree $Z = RW^T = [0.5084, 0.6501, 0.5888, 0.5004]^T$

Sort the four correlation degrees of each candidate: $z_4 > z_3 > z_2 > z_1$. It shows that the second candidate is closest to the ideal scheme, so the second candidate should be chosen to the project manager.

E. Structure Weight Coefficient of Correlation Contrast Curve

The weight of correlation coefficient vector is calculated based on the correlation coefficient matrix $R$ and index weight set $W$:

$$W = \begin{bmatrix} 0.01313, 0.0079, 0.0074, 0.0273, 0.0187, 0.0132, 0.0044, \\ 0.0357, 0.0043, 0.007556, 0.1563, 0.0137, 0.0074, 0.0125, 0.00225 \end{bmatrix}$$

The weight of correlation coefficient vector is calculated based on the correlation coefficient matrix $R$ and index weight set $W$:

$$W = \begin{bmatrix} 0.0094, 0.00263, 0.00263, 0.051, 0.051, 0.051, 0.0066, 0.00499, \\ 0.0428, 0.0267, 0.0267, 0.0267, 0.0267, 0.0267, 0.0267, 0.0075, 0.00225 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.00131, 0.00095, 0.00847, 0.00851, 0.00819, 0.00069, 0.00132, \\ 0.0019, 0.00613, 0.01511, 0.01606, 0.0074, 0.00224, 0.0125, 0.0075 \end{bmatrix}$$

There are 15 sub-indexes in this system, take $x=[1:1:15]$ in the horizontal ordinate. Then, use the plot function of matlab, those known date can be transformed into a weight coefficient of correlation contrast curve (Fig.1). Fig.1 shows that the second candidate is closest to the ideal scheme, this result is consistent with the result of correlation rank.

![Fig.1. The contrast curve of weight of the correlation coefficient](image)

VI. CONCLUSIONS

Through site visits, telephone counseling, mailed questionnaires, online surveys and other ways, relevant information about project manager selection was obtained from multiple construction company. Then, evaluation system which includes quantitative and qualitative indexes was established. Mathematical model was established by
using Grey Incidence Analysis, comprehensive index weight set was calculated by using AHP method. After a fair scheduling of all the correlation degree, the best scheme will be found. The mathematical model used in this paper can provide useful reference basis for other selection activities in engineering project.

REFERENCES


