A Study on the Two-Way Principal-Agent Relations and Models

Si-guang Dai*,1,3, Gui-hua Nie 2, Peng Zhang 2

1 School of Management, Wuhan University of Technology, Wuhan 430070, China
2 School of Economics, Wuhan University of Technology, Wuhan 430070, China
3 School of Business, Wuhan Huaxia University of Technology, Wuhan 430223, China

Abstract — This paper identifies the mutual principal-agent relation between two alliance members, then a two-way principal-agent model is constructed and is transferred into a single virtual principal dual entity agents’ principal-agent model. A sequential quadratic programming method combining interior point method is then proposed to solve it. Numerical examples are constructed to illustrate the idea of our model and the effectiveness of the designed algorithm. The examples show that the resources investment proportion and the profit distribution proportion of the alliance members constrained in the contract should be balanced to ensure that all contracts give the best value for the two players.

Keywords - mutual principal-agent relation; two-way principal-agent model; virtual principal; utility function; sequential quadratic programming

I. INTRODUCTION

The principal-agent theory developed in three stages: the basic model of the principal-agent theory, the multi-agent models and the dynamic models.

The basic model of the principal-agent theory with one principal and one agent is the basis of any other principal-agent models. It assumed principles and agents are all exclusively pursuing their material self-interest[1][2]. The research of it has focused on how principals design contract driving the agents to choose the action which is most beneficial to the principals[3]. Holmstrom and Milgrom proposed a multitasking single-agent principal-agent model[4] based on traditional linear principal-agent model[5], which initiated multitasking principal-agent research. Sohoni et al analyzed the effect of threshold contract on the automobile sales, and found that threshold contract is superior to the linear contract[6].

In multi-agent systems, the principal delegated the work to several agents and there are two cases in these models. One case is that all the agents work in a team with one observable output, which is first introduced by Holmstrom[7], in this case, it is impossible to avoid the “free-riding” opportunism[8], Zardkoooh et al pointed out that the opportunism is not only produced by agents but by also by principles, and they discussed three-dimensional opportunism: agents, principles and the confluence of agents and principles[9]. Steinle et al reduced the opportunism in the buyer-supplier relationships by testing whether both ex ante and ex post information asymmetries influence moral hazard[10]. Another case is that each agent in multi-agent systems produces his own output which is also introduced by Holmstrom[4], in this case, the “free-riding” opportunism problem can be avoided. Myerson established a generalized principal-agent model with multi-agent and illustrates that the equilibrium may not exist when the agents are non-cooperative[11].

The dynamic model is always in continuous time and divided into several periods, whether it should be governed by long-term or short-term contracts is a hot issue. Fudenberg et al identify four assumptions under which short-term contracts are not worse than long-term contracts [12]. Webb also proposed a model to a dynamic setting with two periods of investment and return [13], however, long-term contracts are needed when the some of the assumptions in short-term contracts are not satisfied, based on this, Her奥林匹 complicated the agent's action, renegotiation can improve welfare [14], Mirrlees discussed that the solution may not always be obtained by using the agent's first-order conditions when the agent’s information is unobservable[15], in recent years more and more scholars such as Páez-Pérez and Sánchez-Silva [16] focus on the research of dynamic models.

For those models above, they are assumed that the principal-agent relationship between the players is single and the players have only one status: principal status or agent status. But, in real world, the principal-agent relationships between the players are usually two-way or multi-way and the players all have two statuses: principal status and agent status. One example is the relationship between the suppliers and the distributors, On one hand, the suppliers commission the distributors to sell commodities, in this case, the suppliers act as principal and the distributors as agent; On the other hand, the distributors commission the suppliers to produce commodities, in this case, the suppliers act as principal and the distributors as agent. Thus, the suppliers and the distributors both have dual status: principal status or agent status, the principal-agent relationship between them are two-way. Few researches have been done in this field.

In this paper, we originally propose a two-way principal-agent model with two players and two-side constraints. This
paper is organized as follows. In Section II, the mutual principal-agent relation between the two alliance members is introduced, a two-way principal-agent model is constructed and is transferred into a single virtual principal dual entity agents’ principal-agent model, then, the sequential quadratic programming method combining interior point method is proposed to solve it in Section III. In Section IV, the analysis of differently investment shares, profit shares, and utility function values of alliance members are given to illustrate the idea of our model and the effectiveness of the designed algorithm. Finally, some conclusions are given in Section V.

II. THE TWO-WAY PRINCIPAL-AGENT MODEL

In this section, the problem description and notations used in the following section will be introduced first. The participation constraints of the two players and the utility function of the Virtual Principal who represents the alliance overall benefits are presented. Finally, a two-way principal-agent model will be proposed and can be proved to be a convex programming problem.

A. Problem Description and Notations

Let us consider a two-way principal-agent relationship with two players. The two players are risk-averse and there exists a mutual principal-agent relation between them.

To make it easier to understand the following exposition, all the notations that will be used hereafter are listed as follows:

\( x_1 \) and \( x_2 \): the effort level of the two players respectively;
\( \alpha \) and \( 1-\alpha \): the resources investment proportion of the two players respectively, where \( 0<\alpha<1 \);
\( \beta \) and \( 1-\beta \): the profit distribution proportion of the two players respectively, where \( 0<\beta<1 \);
\( \omega_1 \) and \( \omega_2 \): the fixed salary of the two players respectively;

\( F(x_1, x_2, \alpha) \) the utility function of the virtual principal who represents the overall benefits of the alliance, where \( F(x_1, x_2, \alpha) \) is a concave function, \( x_1 \) and \( x_2 \) are decision variables;

\( F_1(x_1, x_2, \alpha, \beta, \omega_1, \omega_2) \): the utility function of the first player, where \( F_1(x_1, x_2, \alpha, \beta, \omega_1, \omega_2) \) is a concave function, \( x_1 \) and \( x_2 \) are decision variables;

\( F_2(x_1, x_2, \alpha, \beta, \omega_1, \omega_2) \): the utility function of the second player, where \( F_2(x_1, x_2, \alpha, \beta, \omega_1, \omega_2) \) is a concave function, \( x_1 \) and \( x_2 \) are decision variables;

\( U_1 \) and \( U_2 \): the reservation utility of the two players respectively;

\( Y \): the overall benefits of the alliance, which is also called the benefit of the virtual principal;

\( y_1 \) and \( y_2 \): the benefit of the two players respectively, which could be the linear contract or non-linear contract;

\( C_1(x_1) \) and \( C_2(x_2) \): the effort cost of the two players respectively.

B. The Two-Way Principal-Agent Model

There are two problems of the alliance which consists of two players: firstly, whether the alliance could be established? Namely whether the participation constraints of the two players can be satisfied; secondly, how to design a contract to make the two players to choose the behavior that is most beneficial to the alliance, that is also the benefit or risk allocation problems in the alliance. Hence, it faces the issues about how to meet the participation constraints of the two players and how to maximize the utility of the alliance. Here, a virtual player was introduced as the principal (hereinafter referred to as “Virtual Principal”) representing the profit of the alliance, the original two players reserve their agent status (hereinafter referred to as “Entity Agents”), and their delegated power will be transferred to the virtual principal for centralized management. Thus, the two-way principal-agent model will be equally transferred into a single virtual principal dual entity agents’ principal-agent model. Two-way principal-agent problem aims to maximize the utility function of the virtual principal subject to the participation constraints of the two entity agents.

Based on the above analysis, the two-way principal-agent model after introducing the virtual principal could be showed as follows:

\[
\begin{align*}
\text{Max} & \quad F(x_1, x_2, \alpha) \\
\text{st} & \quad F_1(x_1, x_2, \alpha, \beta, \omega_1, \omega_2) \geq U_1 \quad (a) \\
& \quad F_2(x_1, x_2, \alpha, \beta, \omega_1, \omega_2) \geq U_2 \quad (b) \\
& \quad x_1, x_2 \in K \quad (c)
\end{align*}
\]

Where constraint (a) and constraint (b) denote the participation constraint of the two entity agents respectively, which means that if the expected benefits of them exceed their reservation profit levels respectively, they would participate in the alliance, otherwise the establishment of the alliance wouldn’t be realized. The constraint (c) states the threshold constraint of the two entity agents’ effort levels. So the economic meaning of the model (1) is how to design a contract to maximize the benefit of the alliance or the virtual agent’s utility under the participation constraints of the two entity agents.

For simplicity, this paper makes the assumptions as follows:

1) The effort costs of the two players are decided by their effort levels which are expressed as \( C_i(x_i) = \frac{1}{2}x_i^2 \) and \( C_2(x_2) = \frac{1}{2}x_2^2 \).
2) The alliances profit is expressed by the Cobb–Douglas production function as \( Y = x_1^{a}x_2^{1-a} + \varepsilon \), where, \( \varepsilon \) is a random variable in normal distribution;

3) The distribution function of the alliances profit between two entity agents can be obtained by Holmstrom and Milgrom’s research[5], the specific definition is as follows:

\[
y_1 = \alpha_1 + \beta(Y - \alpha_1 - \alpha_2) \\
y_2 = \alpha_2 + (1-\beta)(Y - \alpha_1 - \alpha_2)
\]

Considering the assumptions listed above, the utility function of the virtual principal can be formulated as:

\[
F(x_1, x_2, \alpha) = E(Y) - C_1(x_1) - C_2(x_2) = x_1^{a}x_2^{1-a} - \frac{1}{2}x_1^{2} - \frac{1}{2}x_2^{2}
\]

The two entity agents’ utility function \( F_1(x_1, x_2, \alpha, \beta, \alpha_1, \alpha_2) \) and \( F_2(x_1, x_2, \alpha, \beta, \alpha_1, \alpha_2) \) can be respectively expressed as follows:

\[
F_1(x_1, x_2, \alpha, \beta, \alpha_1, \alpha_2) = y_1 - C_1(x_1) = \alpha_1 + \beta(Y - \alpha_1 - \alpha_2) - \frac{1}{2}x_1^{2}
\]

\[
F_2(x_1, x_2, \alpha, \beta, \alpha_1, \alpha_2) = y_2 - C_2(x_2) = \alpha_2 + (1-\beta)(Y - \alpha_1 - \alpha_2) - \frac{1}{2}x_2^{2}
\]

Then, the two-way principal-agent model (1) can be turned into as follows:

\[
\begin{align*}
\min & \quad \frac{1}{2}x_1^{2} + \frac{1}{2}x_2^{2} - x_1^{a}x_2^{1-a} \\
\text{s.t.} & \quad \alpha_1 + \beta(x_1^{a}x_2^{1-a} - \alpha_1 - \alpha_2) - \frac{1}{2}x_1^{2} \geq U_1 \quad (a) \\
& \quad \alpha_2 + (1-\beta)(x_1^{a}x_2^{1-a} - \alpha_1 - \alpha_2) - \frac{1}{2}x_2^{2} \geq U_2 \quad (b) \\
& \quad 0 \leq x_1 \leq 1 \quad (c) \\
& \quad 0 \leq x_2 \leq 1 \quad (d)
\end{align*}
\]

Where constraint (a) denotes the participation constraint of the first entity agent; constraint (b) indicates the participation constraint of the second entity agent; constraint (c) states the threshold constraint of \( x_1 \); constraint (d) states the threshold constraint of \( x_2 \).

**Theorem 1.** Let \( 0 < \alpha < 1 \) and \( 0 < \beta < 1 \), then the model (2) is a convex programming problem.

**Proof.**

1) Let the objective function of the model (2) be \( f(x_1, x_2) \), where \( f(x_1, x_2) = \frac{1}{2}x_1^{2} + \frac{1}{2}x_2^{2} - x_1^{a}x_2^{1-a} \), so \( f(x_1, x_2) \) is a convex function.

Let:

\[
\begin{align*}
f_1(x_1, x_2) &= \frac{1}{2}x_1^{2} + \frac{1}{2}x_2^{2} \\
f_2(x_1, x_2) &= -x_1^{a}x_2^{1-a}
\end{align*}
\]

Because the Hesse matrix of \( f_1(x_1, x_2) \) is \( \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \), which is a positive definite matrix, so \( f_1(x_1, x_2) \) is a convex function.

Similarly, Let the Hesse matrix of \( f_2(x_1, x_2) \) be \( H \), \( H \) can be expressed as

\[
\begin{bmatrix}
\alpha(1-\alpha)(\frac{x_1}{x_2})^{a} & \alpha(\alpha-1)(\frac{x_1}{x_2})^{a} \\
\alpha(\alpha-1)(\frac{x_1}{x_2})^{a} & \alpha(1-\alpha)(\frac{x_1}{x_2})^{a}
\end{bmatrix}
\]

Because \( |h_{11}| = \alpha(1-\alpha)(\frac{x_1}{x_2})^{a} \frac{x_2}{x_1} \geq 0 \) and \( |H| = 0 \), so \( H \) is a positive semi-definite matrix, thus \( f_2(x_1, x_2) \) is a convex function.

According to the properties of the convex function, when \( f_1(x_1, x_2) \) and \( f_2(x_1, x_2) \) are convex functions, \( f_1(x_1, x_2) + f_2(x_1, x_2) \) is a convex function, too. So we could reach a conclusion that the objective function of the model (2) \( f(x_1, x_2) = \frac{1}{2}x_1^{2} + \frac{1}{2}x_2^{2} - x_1^{a}x_2^{1-a} \) is a convex function.

2) The feasible region of the model (2) is a convex set.

Let:

\[
\begin{align*}
f_1(x_1, x_2) &= \alpha_1 + \beta(x_1^{a}x_2^{1-a} - \alpha_1 - \alpha_2) - \frac{1}{2}x_1^{2} - U_1 \\
f_2(x_1, x_2) &= \alpha_2 + (1-\beta)(x_1^{a}x_2^{1-a} - \alpha_1 - \alpha_2) - \frac{1}{2}x_2^{2} - U_2
\end{align*}
\]

Hence, \( f_1(x_1, x_2) \) can be turned into:

\[
\begin{align*}
f_3(x_1, x_2) &= \alpha_1 - \beta(\alpha_1 + \alpha_2) - U_1 - \frac{1}{2}x_1^{2} + \beta x_1^{a}x_2^{1-a}
\end{align*}
\]

Where, \( \beta = \beta(\alpha_1 + \alpha_2) - U_1 - \frac{1}{2}x_1^{2} \) is a convex function, in addition, because \( 0 < \beta < 1 \) and \( -x_1^{a}x_2^{1-a} \) is a convex function, so \( \beta x_1^{a}x_2^{1-a} \) is a convex function and so that

\[
\begin{align*}
f_4(x_1, x_2) &= \alpha_1 + (1-\beta)(x_1^{a}x_2^{1-a} - \alpha_1 - \alpha_2) - \frac{1}{2}x_2^{2} - U_2
\end{align*}
\]

is a convex function too.
Because $f_3(x_1, x_2)$ and $f_4(x_1, x_2)$ are concave functions, Meanwhile $0 \leq x_1 \leq 1$ and $0 \leq x_2 \leq 1$ are linear constraint functions, so we could get the conclusion that the feasible region of the model (2) is a convex set.

Because the objective function of the model (2) is a convex function and its feasible region is a convex set, so model (2) is a convex programming problem.

**Which ends the proof.**

### III. **SOLVING OF THE TWO-WAY PRINCIPAL-AGENT MODEL**

In this section, a sequential quadratic programming (SQP) method [17] will be proposed to solve the Model (2) which is a convex planning problem with non-linear constraints.

Let

$$f_1(x^k) = f_3(x_1^k, x_2^k) + \sum_{i=1}^{k} \left[ \left(1 - \frac{1}{k_i^k} \right) \partial f_3(x_1^k, x_2^k) \right]$$

Then the first-order Taylor expansion of $f_1(x)$ in $x^k$ is as follows:

$$f_1(x^k) = f_3(x_1^k, x_2^k) + (x_1 - x_1^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_1} + (x_2 - x_2^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_2}$$

Where,

$$f_3(x_1, x_2) = \frac{1}{2} \left[ (x_1 - x_1^k)^2 + (x_2 - x_2^k)^2 \right]$$

Thus, the second-order Taylor expansion of $f_3(x)$ in $x^k$ can be obtained as follows:

$$f_3(x^k, x_2^k) + \sum_{i=1}^{k} \left[ \left(1 - \frac{1}{k_i^k} \right) \partial f_3(x_1^k, x_2^k) \right]$$

Let

$$f_2(x) = f_3(x_1, x_2) = \alpha_1 + \beta (x_1^a x_2^{1-a} - \alpha_1 - \alpha_2) - \frac{1}{2} x_1^2 - U_1$$

Then the first-order Taylor expansion of $f_2(x)$ in $x^k$ is as follows:

$$f_2(x^k) = f_3(x_1^k, x_2^k) + (x_1 - x_1^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_1} + (x_2 - x_2^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_2}$$

Where,

$$f_3(x_1, x_2) = \frac{1}{2} \left[ (x_1 - x_1^k)^2 + (x_2 - x_2^k)^2 \right]$$

Thus, the first-order Taylor expansion of $f_3(x)$ in $x^k$ can be obtained as follows:

$$f_3(x^k) = f_3(x_1^k, x_2^k) + (x_1 - x_1^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_1} + (x_2 - x_2^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_2}$$

Let

$$f_4(x) = f_3(x_1, x_2) = \alpha_1 + \beta (x_1^a x_2^{1-a} - \alpha_1 - \alpha_2) - \frac{1}{2} x_1^2 - U_1$$

Then the first-order Taylor expansion of $f_4(x)$ in $x^k$ is as follows:

$$f_4(x) = f_3(x_1^k, x_2^k) + (x_1 - x_1^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_1} + (x_2 - x_2^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_2}$$

Where,

$$f_3(x_1, x_2) = \frac{1}{2} \left[ (x_1 - x_1^k)^2 + (x_2 - x_2^k)^2 \right]$$

Thus, the first-order Taylor expansion of $f_3(x)$ in $x^k$ can be obtained as follows:

$$f_3(x^k) = f_3(x_1^k, x_2^k) + (x_1 - x_1^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_1} + (x_2 - x_2^k) \frac{\partial f_3(x_1^k, x_2^k)}{\partial x_2}$$
Then the first-order Taylor expansion of \( f_4(x) \) in \( x^k \) is as follows:

\[
 f_4(x^k, x^k) + (x_1 - x^k_1) \frac{\partial f_4(x^k, x^k)}{\partial x_1} + (x_2 - x^k_2) \frac{\partial f_4(x^k, x^k)}{\partial x_2}
\]

Where,

\[
 f_4(x^k, x^k) = \omega_2 + (1 - \beta)((x^0_1)^(\alpha)(x^0_2)^(\beta)) - \omega_1 - \omega_2
\]

\[
 \frac{1}{2}(x^k_2)^2 - U_2
\]

\[
 \frac{\partial f_4(x^k, x^k)}{\partial x_1} = \alpha(1 - \beta)(x^k_1)^(\alpha)
\]

\[
 \frac{\partial f_4(x^k, x^k)}{\partial x_2} = -x^k_2 + (1 - \alpha)(1 - \beta)(x^k_2)^(\beta)
\]

Thus, the first-order Taylor expansion of \( f_4(x) \) in \( x^k \) can be obtained as follows:

\[
 f_4(x^k, x^k) + (x_1 - x^k_1) \frac{\partial f_4(x^k, x^k)}{\partial x_1} + (x_2 - x^k_2) \frac{\partial f_4(x^k, x^k)}{\partial x_2}
\]

\[
 = \alpha(1 - \beta)(x^k_1)^(\alpha)(x_1) + (-x^k_2)^2 + (1 - \alpha)(1 - \beta)(x^k_2)^(\beta)(x_2) + \omega_1 + \omega_2 - U_2
\]

\[
 1 \geq \epsilon
\]

Thus, the objective function value of the model (4) can be solved by solving model (3) respectively be \( x^0 \) and \( F(x^0) \), then the sub-problem of the model (2) can be turned into as follows:

\[
 \begin{align*}
 \min & \ A(x^k, x^k) + A_1(x^k, x^k) + B(x^k, x^k, x, x) \\
 \text{s.t.} & \quad M(x^k, x^k) + D(x^k) \geq 0 \\
 & \quad N(x^k, x^k) \leq 0
\end{align*}
\]

Step 1. Let:

\[
 F(x^k) = A_1(x^k, x^k) + B(x^k, x^k, x, x) + C_1(x^k, x, x) + C_2(x^k, x, x)
\]

at the same time, let the initial feasible solution and the objective function value of the model (3) respectively be \( x^0 \) and \( F(x^0) \), then the sequential quadratic sub-programming problem can be described as follows:

\[
 \begin{align*}
 \min & \ A(x^k, x^k) + A_1(x^k, x^k) + B(x^k, x^k, x, x) \\
 \text{s.t.} & \quad M(x^k, x^k) + D(x^k) \geq 0 \\
 & \quad N(x^k, x^k) \leq 0 \quad (4)
\end{align*}
\]

Then \( x^k \) is the optimal solution of the model (4) using an interior point method[18]. If \( \left| F(x^k) - F(x^0) \right| < \epsilon \quad \epsilon \leq 10^{-6} \), then \( x^k = x^* \), \( F(x^*) = F(x^k) \), where \( x^* \) and \( F(x^*) \) are the optimal solution and the optimal objective function value of the model (4). Otherwise, turn Step 2.

Step 2. When \( k = m \quad (m \geq 1 \quad m \in \mathbb{Z}^+) \), let the optimal solution and the optimal objective function value of the \( m \)th iteration sub-problem of the model (4) respectively be \( x^m \) and \( F(x^m) \), the procedures of solving SQP can be described as follows:
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\( (\mathcal{Q}_P) \min \{ 4(x'^2) + 4(x''(x')^2) + B(x') + C(x')(x') + C(x)(x') \}

\[
\begin{align*}
M(x') + M(x'') + D(x') & \geq 0 \\
N(x') + N(x'') + D(x') & \geq 0 \\
0 & \leq x' \leq 1 \\
0 & \leq x' \leq 1
\end{align*}
\]

(5)

Then \( x^{m+1} \) (the optimal solution) and \( F(x^{m+1}) \) (the optimal objective function value) of the Model (5) can be obtained by solving Model (5) using an interior point method.

Step 3. If \( |F(x^{m+1}) - F(x^{m})| \leq \varepsilon, \varepsilon \leq 10^{-6} \), then the optimal solution \( x^{*} \) and the optimal objective function value \( F(x^{*}) \) of the model (4) can be obtained as follows: \( x^{*} = x^{m+1} \), \( F(x^{*}) = F(x^{m+1}) \). Otherwise, let \( k = m + 1 \) and turn Step 2.

IV. Numerical Example

In this section, a numerical example is given to express the idea of the proposed model. Assume that there are two players in the alliance, the one is A, the other one is B, thus A and B are the two entity agents mentioned above. Suppose that the fixed salary of A and B take the same value; the reservation utility of A and B take the same value. Here, we discuss the effect of the investment shares and profit shares on the utility function value of the alliance. In this example, we use the interior point method and the SQP algorithm to solve the problem. The results are as follows.

When the investment share of the player A (\( \alpha \)) is gotten \( 0.05, 0.1, ..., 0.6 \) between the interval values of \([0.05, 0.6]\), the profit share of the player A (\( \beta \)) is gotten \( 0.2, 0.25, ..., 0.8 \) between the interval values of \([0.2, 0.8]\), the utility function values of the alliance (\( F \)) can be obtained as Table I by solving the Model (2). Accordingly, the effect of the investment share and profit share of the player A on \( F \) can be showed in Figure 1.

Table I and Figure 1 show that: 1) when \( \alpha = 0.1, \beta = 0.2 \), the investment share and profit share of A is low which mean low risk and low reward, on the contrary, the investment share and profit share of B are very high, which are 0.9 and 0.8 respectively, thus, the player B will work hard to create more wealth for the alliance and lower the risks, which result in that the utility function value of the alliance reaches the maximum value 0.3031; 2) when \( \alpha \) obtains a certain value, \( F \) decreases firstly with increasing \( \beta \), and then increases. For example, when \( \alpha = 0.1, F \) decreases firstly from 0.3031 to 0.2500 with increasing \( \beta \), and then increases from 0.2500 to 0.2602, it is because that when \( \alpha = 0.1 \), the risk A bears is steady and low but the return of A is increasing with increasing \( \beta \), the risk B bears is steady and high but the return of B is decreasing with increasing \( \beta \); which results that A and B will not to raise their effort level, thus the utility value of the alliance decrease firstly. When \( \beta \) increases to a certain value (0.5), the high return share of A promote him to work harder to get more returns with increasing \( \beta \), thus ,the utility value of the alliance started to increase.
In this paper, we discuss the two-way principal-agent relationship between two players, in which the two players both have dual status: the principal status and the agent status. We introduce a virtual principal to represent the overall benefits of the alliance, then propose a two-player two-way principal-agent model with the participation constraints of the two players. Since the proposed model is a convex planning problem with non-linear constraint, we use the sequential quadratic programming method and the interior point method to obtain the optimal strategy. Then a numerical example is given to illustrate the application of the proposed model and demonstrate the effectiveness of the design algorithm for solving our model. Numerical results show that it is necessary to balance the investment and return of the alliance members to maximize the total utility of the alliance and achieve real incentive for them by determining the appropriate investment share and profit share.

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REFERENCES