A Study on Financing and Pricing Strategies for Supply Chains with Capital Constraints on Manufacturers

Yongwei Zhou*1,2, Dayong Wu1, Hehua Fan1

1. College of Science, Zhengzhou University of Aeronautics, Zhengzhou, Henan, China, 450015
2. Collaborative innovation center for aviation economic development of Henan province, Zhengzhou, Henan, China, 450015

Abstract — We consider the case of a supply chain system: i) composed of one risk neutral manufacturer and ii) one risk averse retailer, iii) selling a single product, and iv) the manufacturer is faced with the problem of insufficient initial fund. We study the supply chain financing and pricing strategy under the manufacturer’s different initial funds by setting up the related profit model. The research results show that: i) as long as the financing rate provided by the financing organization is less than a certain threshold value, ii) the financed manufacturer can increase its expected revenue, iii) the retailer’s certainty equivalent revenue is also increased; iv) the retailer’s optimal retail price and v) manufacturer’s optimal wholesale price are all below the optimal pricing of unfinanced supply chain. The numerical experiment has shown the effectiveness of the conclusions.

Keywords - Capital constraint; Supply chain; Financing; Pricing strategy

I. INTRODUCTION

The supply chain is developed from “the economy chain” proposed by Peter F. Drucker, the supply chain management emphasized the effective integration and coordination of logistics, information flow and capital flow. The existing supply chain management references mostly focused on the coordination and management of logistics and information flow, but they neglected the capital constraint problem [1-8], the study of supply chain will have greater practical significance combing with capital constraint. In the supply chain with capital constraint, on account of the capital constraint, the supply chain node enterprises may not possible realize the optimal decision, this will affect the enterprise’s performance of supply chain, even the whole supply chain’s performance, if this kind enterprises can obtain the financing service, and realize the supply chain’s optimal decision by financing, then the financing service creates the new value for the capital constraint supply chain.

In recent years, there have some scholars devoted themselves to study the problems, such as the influence of capital constraint on supply chain, the related operations management, the financing service coordination and so on. Zhao Ai-mei [9] studied the capital constraint supply chain’s financing problem under the condition of non-competition, Chen Xiao-xu et al [10] studied the supply chain’s coordination problem when the retailer’s fund is insufficient, the researches showed that manufacturer’s guarantee contract can coordinate the supply chain, Zhang Xiao-juan, Wang Yong [10] mainly studied the dual-channel supply chain’s pricing strategy under the condition of retailer’s fund is insufficient, the studies showed that the retailer can obviously improve the manufacturer and retailer’s profit by its financing behaviour. Form the angle of financing institution’s profit maximization, Raghavan and Mishra [12] analysed the problem of financing institution offer financing to the both sides of supply chain under the centralized and decentralized conditions. Chang and Rhee [13] assumed that the both sides of supply chain all have capital restriction and the inventory financing cost is large than 0, and analysed the optimal reaction behaviour between financing institution and supply chain. Lai and Debo [14] considered how to share the inventory risk of supply chain in the premise of capital restriction. But the above mentioned researches often studied under the condition of certainty demand, and the manufacturer and retailer are all risk neutral, in fact, with the rapid changes in the market, the demand faced by enterprises is highly random, and the manufacturer and retailer’s attitudes to risk are very different. Different to above mentioned, the article take the supply chain system composed of one single manufacturer and one single retailer with sufficient initial fund as research objects, the manufacturer and retailer’s attitudes to risk are different, the manufacturer is risk neutral, the retailer is risk averse, and the retailer faces with random demand, the article mainly discusses the pricing and financing strategy of supply chain, and provides theoretical support for related enterprises’ decision-making.

II. MODEL DESCRIPTION

The article considers the supply chain system which are composed of one risk neutral manufacturer and one risk averse retailer, and sells a single product, the manufacturer’s unit product cost is $c$, the manufacturer’s unit product wholesale price to retailer is $w$, the retailer’s unit product retail price is $p$, the manufacturer’s initial fund is $B$, if the manufacturer’s initial fund is insufficient, to satisfied the
market demand, he/she can financing to the outer financing institution. It assumed that the market demand is \( Q \), and 
\[
Q = a - g \cdot p + \varepsilon 
\]  
(1)

Where \( a \) is the market’s total demand, \( g \) is the price elastic coefficient, \( \varepsilon \) is an exogenesis random variable, it refers to the uncertainty demand, such as the change of customers preference, the market environment change and so on, it assumed that \( \varepsilon \sim N(0, \sigma^2) \), obviously, \( a - gc \geq 0 \), \( a - gw \geq 0 \). In order to discuss conveniently, the article supposes that stock out is not allowed, and the residual products value is 0, that is to say, product recovery problem is not exist.

According to the above assumption, the manufacturer and retailer’s profit functions are determined by

\[
\pi_m = (w - c)Q = (w - c)(a - g \cdot p + \varepsilon) 
\]  
(2)

\[
\pi_r = (p - w)Q = (p - w)(a - g \cdot p + \varepsilon) 
\]  
(3)

respectively.

When the retailer is risk averse, according to reference [15], we described the retailer’s risk aversion degree by absolutely risk aversion degree, and the retailer’s utility function is the classical constant risk aversion function \( \mu(\pi_r) = e^{-\rho \pi_r} \), where \( \rho \) is the risk aversion measurement, when \( \rho < 0 \), it indicated that the retailer is risk preference, when \( \rho = 0 \), it indicated that the retailer is risk neutral, when \( \rho > 0 \), it indicated that the retailer is risk aversion, we suppose that the retailer’s profit \( \pi_r \sim N(\mu, \sigma^2) \), then the retailer’s expected utility

\[
E[\mu(\pi_r)] = \int_{-\infty}^{\infty} e^{-\rho x} f(x)dx = \int_{-\infty}^{\infty} e^{-\rho \pi_r} \frac{1}{\sqrt{2\pi \sigma_r}} dx = e^{-(\frac{1}{2} \rho^2 + \frac{1}{2} \frac{\rho^2}{\sigma^2})}. 
\]  
(4)

Where

\[
\mu = E(\pi_r) = E[(p - w)Q] = (p - w)(a - gp) \]  
(5)

\[
\sigma_r^2 = D(\pi_r) = D[(p - w)Q] = (p - w)^2 \sigma^2 . 
\]  
(6)

The retailer maximize its expected utility function \( E[\mu(\pi_r)] \) is equivalent to its certainty equivalent profit [16], namely

\[
\pi_c = \mu - \frac{\rho}{2} \sigma^2 = (p - w)(a - gp) - \frac{\rho}{2} (p - w)^2 \sigma^2 . 
\]  
(7)

III. THE MANUFACTURE AND RETAILER’S PRICING STRATEGY BEFORE FINANCING

In the decentralized supply chain system, the manufacturer and retailer maximize their own profits independently, this paper models the decision as a sequential, Stackelberg game, with the manufacturer as the leader and the retailer as the follower, firstly the manufacturer determine the optimal wholesale price according to the related information, then the retailer determine the optimal retail price according to manufacturer’s wholesale price and the customers’ demand quantity. We solve the Stackelberg equilibrium solution of supply chain by inverse order method, firstly we analyze to the retailer’s decision, the retailer’s decision:

\[
\max(\pi_r(p)) = \mu - \frac{\rho}{2} \sigma^2 = (p - w)(a - gp) - \frac{\rho}{2} (p - w)^2 \sigma^2 . 
\]  
(8)

s.t \( p \geq w \)

As \( \frac{d \pi_r(p)}{dp} = -\rho \sigma^2 - 2g < 0 \), so the retailer’s certainty equivalent profit function \( \pi_c \) is the concave function of price \( p \), by

\[
\frac{d \pi_r(p)}{dp} = -\rho(p - w)\sigma^2 + a - 2gp + gw = 0 
\]  
(9)

we have

\[
p^* = w + \frac{a - gw}{\rho \sigma^2 + 2g}. 
\]  
(10)

Substituting (9) into (1), and find mathematical expectation to (1), we can obtain the market’s expected demand

\[
E[Q(p^*)] = (a - gw)\theta. 
\]  
(11)

Where \( \theta = \frac{\rho \sigma^2 + g}{\rho \sigma^2 + 2g} \).

Substituting (9) into (2), and find mathematical expectation to (2), we can obtain the manufacturer’s expected profit function

\[
E[\pi_m(w)] = (w - c)E(Q) = (w - c)(a - gw)\theta. 
\]  
(12)

So we have the following theorem:

**Theorem 1** In the decentralized supply chain system, the manufacturer is constrained by initial fund \( B \), the manufacturer’s initial fund equilibrium point \( B_* = \frac{(a - cg)\theta}{2} \), and the manufacturer’s optimal pricing strategy is

\[
w^* = \begin{cases} 
\frac{a - B}{g} & \text{if } 0 < B < B_* \\
\frac{a - c}{2} & \text{otherwise}
\end{cases}. 
\]  
(13)

**Proof**: Obviously, the optimization model represented by (12) equal to find the maximizing to quadratic function in the closed interval, its axis of symmetry
and by \((a-gw)c\theta_i \leq B\), we have \(w \geq \frac{a}{g} - \frac{B}{c\theta_i}g\). So

If \(w_i > w_i^*\), that is \(\frac{a}{g} - \frac{B}{c\theta_i}g > \frac{a}{2g}\), so when

\[w \geq \frac{a}{2g},\]

\[\frac{a}{g} - \frac{B}{c\theta_i}g \leq \frac{a}{2g} + \frac{c}{2}\] so when

\[B \geq B^*_n = \frac{(a - cg)c\theta_i}{2},\]

\[w^* = \frac{a}{2g} - \frac{c}{2}\]

Thus we can obtain the optimal decision of manufacturer and retailer in the decentralized supply chain system (see Table I).

Based on Table 1, we can obtain the following conclusions:

**Theorem 2.** In the decentralized supply chain system, when the manufacturer’s initial fund \(0 < B < B^*_n\), the manufacturer will improve the optimal wholesale price to retailer, that is to say \(\frac{a}{g} - \frac{B}{c\theta_i}g > \frac{a}{2g}\), but the manufacturer’s expected profit is less than that of the fund is sufficient.

**Theorem 3.** In the decentralized supply chain system, when the manufacturer’s initial fund \(0 < B < B^*_n\), the retailer will improve the optimal retail price, that is to say \(\frac{ac - B}{cg} > \frac{(6a + cga)c\sigma^2}{4g(\rho\sigma^2 + 2g)}\), but the retailer’s certainty equivalent profit is less than that of the fund is sufficient.

It can be seen from Theorem 2 and Theorem 3 that when the manufacturer’s initial fund \(0 < B < B^*_n\), insufficient fund leads to the benefit lost of supply chain, in order to satisfy the market demand as much as possible, increase the fill rate and its profit, for the manufacturer with insufficient fund, it is necessary to financing.

### Table I. The Supply Chain’s Pricing Strategy and Profits with Capital Constraint of Manufacturer

<table>
<thead>
<tr>
<th>Pricing and profits</th>
<th>The initial fund of manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 &lt; B &lt; B_n)</td>
<td>(B &gt; B_n)</td>
</tr>
<tr>
<td>The retail price (p)</td>
<td>(\frac{ac - B}{cg})</td>
</tr>
<tr>
<td>The wholesale price (w)</td>
<td>(\frac{a}{g} - \frac{B}{c\theta_i}g)</td>
</tr>
<tr>
<td>The retailer’s certainty equivalent profit (\pi_r)</td>
<td>(\frac{(a - cg)c\sigma^2}{c^2(\rho\sigma^2 + 2g)})</td>
</tr>
<tr>
<td>The manufacturer’s expected profit (E(\pi_m))</td>
<td>(\frac{(ac - 1)c\sigma^2 B + gb(ac - 2B)}{c^2(\rho\sigma^2 + 2g)} - B)</td>
</tr>
</tbody>
</table>

**IV. The Supply Chain’s Pricing Strategy After Manufacturer’s Financing**

In this mode, the manufacturer’s initial fund \(0 < B < B^*_n\), in order to obtain the more profits, the manufacturer financing to the outer financing institution, it assumed that the financing institution is risk neutral, it is indifference to risk, the financing rate provided by the financing institution is \(r\) \((0 < r \leq 1)\), the amount of financing fund for the manufacturer is \(cE(Q) - B\), to calculate conveniently, the article studied the financing behaviour in a sales cycle, after a sales cycle, the manufacturer need to pay the principal and interest \((cE(Q) - B)(1 + r)\) to financing institution, so in the decentralized supply chain system, the manufacturer’s expected profit:

\[E(\pi_m, (w)) = (w - c)E(Q) - (cE(Q) - B)r\]

Substituting (10) into (14), we can obtain the manufacturer’s optimization model with initial fund \(B\) constraint is as follows:

\[
\max E(\pi_m, (w)) = r\left[ B - c(a - \frac{g(\rho^2 + a + gw)}{\rho^2 + 2g}) - (c - w)(a - \frac{g(\rho^2 + a + gw)}{\rho^2 + 2g}) \right]
\]

s.t \(w \geq c\), \((a - gw)c\theta_i \leq B\)

Then we have the following theorem:

**Theorem 4.** In the decentralized supply chain system, if the manufacturer’s initial fund is insufficient, that is to say, \(0 < B < B^*_n\), in the above mentioned financing mode, the manufacturer has financing rate equilibrium point \(r_n = \frac{c\theta_i(a - gc) - 2B}{c^2\theta_i g}\), and the manufacturer’s optimal pricing strategy is determined by
Based on the above conclusions, we can obtain the following conclusions:

**Theorem 5** In the decentralized supply chain system, if the manufacturer’s initial fund is insufficient, and the financing rate 0 < r < r_s = \frac{c\theta(a - gc) - 2B}{c^2\theta g}, the manufacturer’s optimal expected profit is no less than the one without financing; the manufacturer’s optimal wholesale price to retailer is no higher than the one without financing. And the retailer’s optimal certainty equivalent profit is always no less than the one without financing, the retailer’s optimal retail price is no higher than the one without financing.

**Proof:** let the manufacturer’s optimal expected profit when 0 < r < r_s minus the one when r \geq r_s, its difference 

\begin{align*}
[(4B - ac)g + c^2g^2(1 + r) + \rho\sigma^2(2B - ac) + \rho\sigma^2cg(1 + cr)]^2 \geq 0
\end{align*}


Obviously, when 0 < r < r_s = \frac{c\theta(a - gc) - 2B}{c^2\theta g}, we have

w_1 > w_s, that is \frac{a - B}{\theta gc} > \frac{a + c(1 + r)}{2g}, namely

0 < r < r_s = \frac{c\theta(a - gc) - 2B}{c^2\theta g}, we have w' = \frac{a + c(1 + r)}{2g}.

Corresponding, by theorem 4, we can obtain the optimal expected profit of manufacturer is as follows.

\begin{align*}
E(\pi_s) = \begin{cases} 
(B - c\theta)\theta + \left[\frac{a}{2g} - \frac{(1 + r)c}{2}\right]\theta, & 0 < r < r_s \\
B\rho\sigma^2(ac - B) + B_g(ac - 2B) & c^2g(\rho\sigma^2 + g), \quad \text{otherwise}
\end{cases}
\end{align*}

Where

\begin{align*}
\theta_1 = a \frac{g(\rho\sigma^2[a + c(1 + r)g] + 2ag + g[a + c(1 + r)g])}{2g(\rho\sigma^2 + 2g)}.
\end{align*}

And by (9), the retailer’s optimal retail price is determined by

\begin{align*}
p' = \begin{cases} 
\frac{a + c(1 + r)}{2g} + \frac{2ag - g[a + c(1 + r)g]}{2g(\rho\sigma^2 + 2g)}, & 0 < r < r_s \\
\frac{a - B}{\theta gc}, & \text{otherwise}
\end{cases}
\end{align*}

Finally, the retailer’s optimal certainty equivalent profit \( \pi_s' \) is

\begin{align*}
\pi_s' = \begin{cases} 
\frac{[cg(1 + r) - a]^2}{8(\rho\sigma^2 + 2g)}, & 0 < r < r_s \\
\frac{(\rho\sigma^2 + 2B)^2}{2c^2(\rho\sigma^2 + g)^2}, & \text{otherwise}
\end{cases}
\end{align*}

It can be seen that in the condition of manufacturer’s insufficient initial fund, if the financing rate provided by the financing institution 0 < r < r_s, the manufacturer will accept the financing service, otherwise the manufacturer will reject it.
when $B = 5500, r_c = 0.619$ is the critical point of pricing $p$; when $B = 6000, r_c = 0.3571$ is the critical point of pricing $p$, we can obtain the supply chain’s optimal decision and profits before and after manufacturer’s financing (see Table II).

Table II gives the optimal wholesale price of manufacturer to retailer, the optimal retail price of retailer, the manufacturer’s optimal expected profit and the retailer’s optimal certainty equivalent revenue before and after retailer’s financing when the retailer’s initial fund $B$ and the financing rate $r$ changes. It can be seen from Table 2 that when $B = 5500$, $r_c = 0.619$ is the critical point of pricing $p$.

According to Table II, the manufacturer initial fund $B$ take values 5000, 5500, 6000, respectively, the financing rate $r$ provided by the financing organization takes values from 0 to 1.0 with step value 0.01. The effect of financing rate $r$ on the optimal wholesale price of manufacturer to retailer, the manufacturer’s optimal expected profit, the optimal retail price of retailer, the retailer’s optimal certainty equivalent revenue is as Fig. 1- Fig. 4.

It is observed from Figure 1-Figure 4 that as long as the financing rate:

$$0 < r < r_c = \frac{c\theta(a-gc) - 2B}{c^2\theta^2g},$$

the manufacturer’s optimal expected profit, the retailer’s optimal certainty equivalent revenue are all increased after financing service, but both the optimal wholesale price of manufacturer to retailer and the optimal retail price of retailer are all below the level of before financing, so the consumers get the more consumer surplus. The optimal wholesale price of manufacturer to retailer and the retailer’s optimal retail price are all increased with the financing rate, and the manufacturer’s optimal expected profit and the retailer’s optimal certainty equivalent revenue are all increased after financing service.

When $B = 5500, r_c = 0.881$ is the critical point of pricing $p$;
and the retailer’s optimal certainty equivalent revenue are all increased compared with before financing. Although the wholesale price of manufacturer to retailer decreased after financing, the retailer lowered the retail price in retail channel, so the retailer satisfied the more market demand, lower price attracts more consumers, the manufacturer and retailer obtained more profits because of the more market demand, and it also increased the total profit of supply chain.

VI. CONCLUDING REMARKS

The paper studied the supply chain system which are composed of one risk neutral manufacturer and one risk averse retailer, and sells a single product, the manufacturer faced with the problem of insufficient initial fund. We studied the supply chain’s financing and pricing strategy under the condition of manufacturer’s insufficient initial fund. The research result showed that as long as the financing rate provided by the financing organization less than a certain threshold value, the financed manufacturer can increase its expected revenue, the retailer’s certainty equivalent revenue is also increase; the retailer’s optimal retail price and manufacturer’s optimal wholesale price are all below the optimal pricing of unfinanced supply chain. Thus it also better meet the market demand, the consumers also benefit from financing service, the whole supply chain’s profit also significantly improved, financing service creates new values for the manufacturer and retailer.

There are several directions for further analysis that will achieve a better understanding of supply chain. For instance, one may consider the more complicated supply chain system, further discuss the influence of insufficient fund, financing service on the pricing decision of supply chain, in addition, it would be interesting to examine the financing and pricing strategy of the insufficient manufacturer under the condition of asymmetric information, moreover, with the monetary market’s continuously development and innovation to supply chain, to the enterprise with capital constraint, financing channel is also increasing, and it is insightful to
consider the supply chain’s pricing strategy under other financing mode.

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