

# An Image Matching Algorithm using Thin Plate Splines (TPS) Transformation Model

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**Abstract** — In this paper, through the analysis of the registration of edge feature points and the principle of distribution, the research on the image stitching and recovery is conducted by selecting regular images. Through the research, we can find that the image registration of edge feature points concludes two important steps: one is the criterion of feature matching; the other is the image edge feature points to be extracted. Through the estimation of the TPS transformation parameters between the stitching images, the final accurate TPS transformation parameters and the matching relation are obtained through the mutual approximation of the feature points set between the parameters. The validity of this study is verified by the example.

**Keywords** - image matching algorithm; edge detection; decision-making tree; TPS algorithm

## I. INTRODUCTION

Image stitching has been widely applied in the fields of medical detection, video surveillance and seabed exploration which have two or more than two images with overlap areas [1]. And the two important steps in image stitching method are to get the solutions to transform parameters and the image matching [2]. The solutions to the image matching and transformation with two images can be described as: based on the feature point sets on the given two images, find out the point corresponding relation of two point sets, and solve the transformation relation of the two point sets, make the points on an image be accurately mapped to another image. Over the years, in the process of the solution to the point matching, more and more people have begun to study the spectral graph theory so as to get good results[3-7].

In this paper, a piece of cut text image is taken as an example, in view of the complex recovery stitching of many vertical and horizontal chopped pieces, Matlab is used to read the image[8], the image pixel digital, get the pixel matrix, the image pixels are digitized[9], and the pixel matrix is obtained. The edge sequence of pixel matrix is extracted and data are preprocessed; the coordinates are converted into the values of either 0 or 1, DNA replication process is simulated according to the characteristic of the left side value and right side value of the adjacent images being equal. The equal values of the left side and that of the right side in other arrays are taken as the primary matches, search in turn according to the decision-making tree algorithm.

## II. RELATED INTRODUCTION TO IMAGE STITCHING

The organization of the lecture is as follows. After a general introduction of the effect of fault on the power

system, the usefulness and requirement of a fault current limiter is presented to the students which has been discussed in section II. The traditional ways of fixing fault currents in power system has been discussed in section III. In section IV, operating principle, design details, and experimental results of magnetic current limiter has been presented. The analysis and simulation results of high temperature superconducting fault current limiter has been discussed in section V. The lecture has been concluded in section VI.

The paper scraps which are more difficult for the research are selected in this study, its main content is the stitching of paper scraps, and the image registration is generally carried out in accordance with the following flow chart:



Figure 1. The flow chart of the matching principle between images

The paper scraps are scanned into images, the computer is used to read the image pixels, and process the source data; the pixel values are taken as the plane coordinates, the paper is cut by the shredder because of the printed word file from the same page. In the screening process, if two sets of data are equal in the same height, the two sets of data are taken as matching, and they are saved and recorded, then continue to find the next one until the search is completed. Considering some errors may occur, the obtained results are needed for manual correction.

III. ESTABLISHMENT AND SOLUTION OF IMAGE STITCHING MODEL

All information recorded in the image is concluded in the gray level value of image pixels. Select the characteristics of the gray level value of pixels, establish the criterion of the similar gray level information of registration image, how to get higher registration precision without the extraction of image features?

When a piece of paper is cut longitudinally, then a small number of paper pieces will be produced; for the analysis of both edges of each paper piece, because one word may be cut on two sheets of paper in shredding process by the shredder, it is needed to count the total number of cut words  $s$  on the two side of each sheet of paper  $i_n$  ( $n = 0 \dots 18$ ). If two sheets of paper are adjacent, they should comply with the principle of  $s_{m_r} = s_{i_l}$ , the first sheet of paper  $s$  is taken as the master plate to simulate replication process, when the search is conducted from image  $a$  to image  $b$ , the sequence on the right of the image  $b$  is taken as the master plate for next search.

A. Realization of Decision-Making Tree Algorithm

Here, an example is taken to illustrate the stitching recovery process of paper scraps 0,12,15. First, respectively count the cut words on both edges of the three images and they are denoted as  $s$ . Second, specify paper scrape 0 to be the initial position, the right edge  $s_r$  is taken as the criterion and compared with the left edge  $s_l$  on paper scraps 12,15; if they are equal, which is provisionally determined to be matched and saved up in order. Finally, if  $s_{0_r} = s_{12_l}$ ,  $i_{12}$  is kept and  $s_r$  in  $i_{12}$  is compared with  $s_l$  in the next array. If they are not equivalent, then skip to the scrap 12 and is compared with the data in the next array until the search is completed.

B. Supplementation of Model Algorithm and Operation Samples

Taking the fewer samples into account, there may appear one to many mapping in the screening process, all which can meet the requirements should be kept and all initial reservations should be taken as parental genes to continue, until the whole samples are connected. It can simply see from the original data that the left side of the scrap of paper  $i_8$  to be  $s = 0$ , which should be the first reconstruction image.

Explanation of Image 2: the relationship of Array  $ab$  is  $8(5)-14(5)- \{12(4),18(4)\}$ , 12(4) and 18(4) exist in Array  $C$  at the same time, so it can be regarded that 14(5) is more prominent than others, 8(5) is adjacent. The model is constantly ramifying and merging until all are retrieved.

When you can manually complete the selection of the first recovery plan, the array  $s_r$  in the first array is used to filter the arrays  $s_l$  and write the equivalent arrays into the

second array, repeat the operation of Array  $a$  in view of each item in the second array respectively, and then Array  $C$  is obtained. If each item in Array  $C$  is the same as that in the previous array, the operation ends, the further analysis is conducted and merges it with the duplicate item in Array  $b$ .

When the first recovery plan can't be determined, any paper scrap can be specified to be the beginning, and then conduct the above operation until the search process is forced to stop, indicating the specified first one is not the real first recovery plan, then match the left  $s_l$  of the array with the right  $s_r$  of other arrays, inversely deduce until the search ends.

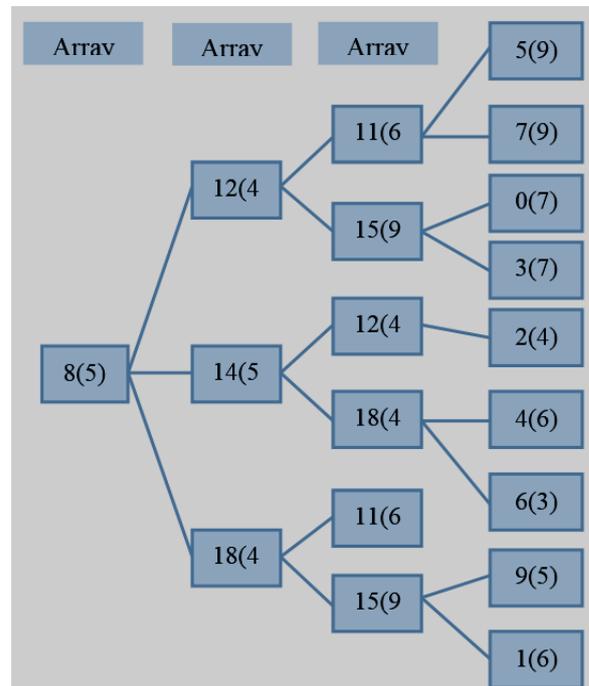


Figure 2. The paper scraps recovery explained by the examples

C. Model of Paper Scraps Recovery Cut Vertically and Horizontally and Establishment of Algorithm

(1) Principle of boundary detection model

The decision-making tree algorithm discussed is a simple algorithm in view of less data, which is not suitable for large scale data processing. Because there may exist a line of words cut in the cutting process, Matlab is needed to transform the image into the pixel gray level matrix with mathematical meaning in establishing the model, the system tacitly approves that the image is divided into  $180 \times 72$  cells.

The existing feature point extraction method is used to digitalize the image, extract effective and enough information, that is, realize from continuation to dissociation, and the construct the feature space. In terms of the establishment of the mathematical model, based on the

theory establishment of discrete geometry, and then extract the feature points, which can realize the establishment of the mathematical model of paper scraps recovery from dissociation to continuation process. In the process of matching, the TPS transformation idea is used to convert the complexity of the problem, the high-dimensional feature vector is converted to a low dimensional feature vector, making full use of the stable characteristics under transformation to match.

|     | C   | D   | E   | F   | G   | H   | I   | J   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 83  | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 84  | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 85  | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 86  | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 87  | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 88  | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 89  | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 90  | 58  | 67  | 69  | 70  | 71  | 72  | 72  | 73  |
| 91  | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 92  | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 93  | 19  | 204 | 204 | 204 | 204 | 204 | 204 | 204 |
| 94  | 20  | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 95  | 16  | 255 | 255 | 255 | 233 | 105 | 116 | 126 |
| 96  | 11  | 255 | 255 | 255 | 232 | 0   | 0   | 0   |
| 97  | 7   | 255 | 255 | 255 | 242 | 0   | 0   | 0   |
| 98  | 3   | 255 | 255 | 255 | 243 | 0   | 0   | 0   |
| 99  | 0   | 254 | 255 | 255 | 246 | 0   | 0   | 0   |
| 100 | 0   | 252 | 255 | 255 | 246 | 0   | 0   | 0   |
| 101 | 3   | 255 | 255 | 255 | 246 | 0   | 0   | 0   |
| 102 | 9   | 255 | 255 | 255 | 246 | 0   | 0   | 0   |
| 103 | 16  | 255 | 255 | 255 | 246 | 0   | 0   | 0   |
| 104 | 23  | 255 | 255 | 255 | 243 | 0   | 0   | 0   |
| 105 | 29  | 255 | 255 | 255 | 242 | 0   | 0   | 0   |
| 106 | 36  | 255 | 255 | 255 | 231 | 0   | 0   | 0   |
| 107 | 43  | 255 | 255 | 255 | 219 | 55  | 66  | 77  |
| 108 | 50  | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 109 | 56  | 255 | 255 | 255 | 255 | 255 | 255 | 213 |

Figure 3. The image pixel coordinate matrix explained by the examples

Explanation of Figure 3: on the basis of the principle of three original colors, the pixel distribution of blank photos is the numerical matrix of 255, when there appear broken words or symbols at the edge of the image, the pixel value read by the computer changes, the change range is 0-255, in which, 255 indicates pixel matrix unit is marked in the margin, 0 indicates the pixel cell is on the image words, the values between 0-255 indicate cells have incomplete contact with words.

Regardless of the incomplete contact between the pixel cells and words, all pixels on the edge of the image are converted into the matrix of either 0 or 1, wherein, 0 indicates pixels matrix has the unchanged region, 1 indicates the pixel values in the region change. The first array in each set of data is extracted, Line 1 Array 72, 10 are selected from Line 180 coordinates horizontally and longitudinally and restructure them into the image pixel table.

|    | A | B | C | D | E | F | G | H | I | J |
|----|---|---|---|---|---|---|---|---|---|---|
| 1  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 4  | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 5  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6  | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 7  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9  | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 10 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |

Figure 4. Randomly selected 10 \* 10 sets data coordinates after the transformation of data 0-1 in figure

(2) Edge feature detection

Image edge is one of the basic characteristics of images, Roberts operator can concurrently process the edge detection so as to reduce the amount of calculation, improve the computing speed. In this study, Roberts edge detection operator is selected for discussion. Roberts regards the gradient approximate solution as the difference problem in the perpendicular direction in any pair. The difference is called the edge detection operator. The direction template of edge detection operator is as follows:

$$f_x : \begin{Bmatrix} 1 & 0 \\ 0 & -1 \end{Bmatrix} \quad f_y : \begin{Bmatrix} 0 & 1 \\ -1 & 0 \end{Bmatrix} \quad (1)$$

If the gradient magnitude:

$$R(i, j) = f(i, j) - f(i+1, j+1) + f(i, j+1) - f(i+1, j) \quad (2)$$

then, through the difference, the approximate value  $R(i, j)$  of continuous gradient magnitude of Roberts operator at different point  $(i+1/2, j+1/2)$  can be obtained. If the threshold T is appropriately selected, assuming  $R(i, j) > T$ , then it is regarded  $R(i, j)$  to be the edge point. The edge detection algorithm based on Roberts operator is fast, and the positioning accuracy is relatively high.

(3) Keren improvement algorithm

The model of affine four parameter transformation selected in the study is as follows:

$$x' = x + a_1x + a_2y + a_3 \quad (3)$$

$$y' = y + a_1y + a_2y + a_3 \quad (4)$$

Wherein,  $a_3$  is the horizontal translation,  $a_4$  is the vertical translation, the rotation angle  $\theta \approx -180 / \pi \cdot \arcsin a^2$ . The image transformation relation between reference image  $f(x, y)$  and target image  $g(x, y)$  is as follows:

$$g(x, y) = f(x + a_1x + a_2y + a_3, y + a_1y + a_2y + a_3) \quad (5)$$

Conduct the two-dimensional Taylor series expansion,

$$g(x, y) \approx f(x, y) + (a_1 + a_2y + a_3) \frac{\partial f}{\partial x} + (a_1y - a_2x + a_4) \frac{\partial f}{\partial y} \quad (6)$$

The error function is obtained:

$$E(a_1, a_2, a_3, a_4) = \sum [f(x, y) + (a_1 + a_2y + a_3) \frac{\partial f}{\partial x} + (a_1y - a_2x + a_4) \frac{\partial f}{\partial y} - g(x, y)]^2 \quad (7)$$

$E(a_1, a_2, a_3, a_4)$  as for  $a_1, a_2, a_3, a_4$ , respectively for partial derivatives, assuming the partial derivatives are zero, then it can be obtained that

$$X = c^{-1} \cdot v \quad (8)$$

In the extraction process of feature points of Roberts operator, two thresholds are needed to determine: one is the threshold of interest value, the other is the threshold of the mean absolute value of difference operator; the determination of registration accuracy depends on the different influence of two values to a great extent. If the selection of a smaller threshold will greatly increase the

amount of calculation in the process of selecting feature points, on the contrary, the selection of a too large threshold will increase the miss-selection probability and error probability in selecting feature points.

*D. TPS Transformation Model*

Roberts matching algorithm is used to obtain  $m(m \leq n)$  for the estimation of PTS model parameters at the matching point. The requirements of bending energy function shown by the minimal nature of TPS interpolation function  $f(x, y)$  are as follows:

$$I_f = \iint \left\{ \frac{\partial f^2}{\partial x^2} + 2 \left( \frac{\partial f^2}{\partial x \partial y} \right) + \frac{\partial f^2}{\partial y^2} \right\} dx dy \quad (9)$$

At the same time, the closed form solution may exist, shown as follows:

$$f(x, y) = a_1 + a_x x + a_y y + \sum_{i=1}^m w_i U(\|(x_i, y_i) - (x, y)\|) \quad (10)$$

Wherein, the kernel function is  $U(r) = r^2 \log^2 r$ . At the same time, the solutions of the linear equation of  $\begin{bmatrix} K & P \\ P^T & 0 \end{bmatrix} \begin{bmatrix} w \\ a \end{bmatrix} = \begin{bmatrix} z \\ 0 \end{bmatrix}$  are the coefficients  $a$  and  $w$  of TPS, in  $K_{ii} = U(\|(x_i, y_i) - (x, y)\|)$ , the Line  $i$  of  $P$  is  $(1, x_i, y_i)$ , the column vector of  $z_i$  and  $w_i$  consists of  $z$  and,  $a = (a_1, a_x, a_y)^T$ .

*E. Algorithm process*

X and Y are respectively the feature point sets of the image  $I_1$  and  $I_2$  to be stitched, the image algorithm of TPS transformation model is used to stitch the image, the specific process is shown in the following workflow.

IV. ALGORITHM OF THE MODEL

**Step1** Matlab reads the image, record the pixel gray level matrix.

**Step2** Extract the first array and the last array of the matrixes.

**Step3** Data integration: All data are incorporated in the same file, the data of the two adjacent arrays indicate a picture pixel matrix, singular array indicates the pixel array on the left side of the image, dual array represents the pixel array on the right side of the image.

**Step4** Edge registration operation: Compare the value of singular array of the first pixel matrix with horizontal numerical value of dual arrays in other arrays, keep them if they are the same; stop the comparison if the values on two sides are not equal in the inspection process, and continue to find the next pair of arrays for comparison.

**Step5** Recycle the process: find the value from Image  $i_m$  to Image  $i_n$ ,  $l_r$  of  $i_n$  is taken as the initial position, repeat Procedure step4 until the search is completed.

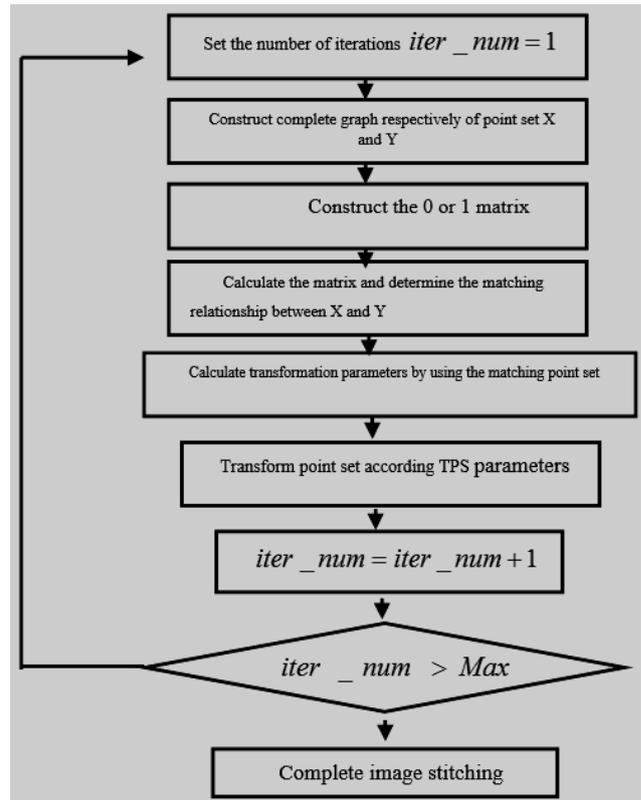


Figure 5. TPS Workflow

V. ANALYSIS OF ALGORITHM FAILURE AND ITS SOLUTION

**Step1** The arbitrarily specified image is taken as the initial position, find the image according to the above algorithm. When the numerical value on left side of an image complies with that of side numerical value of the previous image, both are 0, then it is regarded the search order from left to right of the 11\*19 matrix of a row of images ends. And compare the numerical value on the left side of the specified image with the initial position with the numerical value on right side of other arrays (except for the determined array), deduce and find the image whose left numerical value are zero in line with the conditions, which is the left boundary.

**Step2** Taking into account the fact that we can manually extract the image with the numerical value of zero on the left side or the right side of the matrix, the image is taken as the border of the recovery plan. Repeat Procedure step1, revise Procedure step2, the data on the first row and the last row are added to be the basis for determining the lateral boundary. Other operations remain unchanged. If Procedure step2 is implemented, Procedure step3 is not needed to implement.

**Step3** According to the description of above algorithm, it is needed to avoid the right values being zero, find the image with 0 value on the left for stitching, at this time, it is needed for manual intervention, which is mainly to conduct array division for the continuous 19 images as an array.

VI. RESULTS OF MODEL RECOVERY

TABLE I. CALCULATION OF RECOVERY PLAN

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 49  | 54  | 65  | 143 | 186 | 2   | 57  | 192 |
| 61  | 19  | 78  | 67  | 69  | 99  | 162 | 96  |
| 168 | 100 | 76  | 62  | 142 | 30  | 41  | 23  |
| 38  | 148 | 46  | 161 | 24  | 35  | 81  | 189 |
| 71  | 156 | 83  | 132 | 200 | 17  | 80  | 33  |
| 14  | 128 | 3   | 159 | 82  | 199 | 135 | 12  |
| 94  | 34  | 84  | 183 | 90  | 47  | 121 | 42  |
| 125 | 13  | 182 | 109 | 197 | 16  | 184 | 110 |
| 29  | 64  | 111 | 201 | 5   | 92  | 180 | 48  |
| 7   | 208 | 138 | 158 | 126 | 68  | 175 | 45  |
| 89  | 146 | 102 | 154 | 114 | 40  | 151 | 207 |
| 178 | 118 | 190 | 95  | 11  | 22  | 129 | 28  |
| 131 | 79  | 63  | 116 | 163 | 72  | 6   | 177 |
| 147 | 191 | 50  | 179 | 120 | 86  | 195 | 26  |
| 122 | 103 | 130 | 193 | 88  | 167 | 25  | 8   |
| 202 | 198 | 15  | 133 | 170 | 205 | 85  | 152 |
| 73  | 160 | 203 | 169 | 134 | 39  | 31  | 51  |
| 124 | 144 | 77  | 112 | 149 | 97  | 136 | 164 |
| 187 | 66  | 106 | 150 | 21  | 173 | 157 | 181 |
| 37  | 75  | 55  | 44  | 206 | 10  | 104 | 98  |
| 174 | 0   | 137 | 53  | 56  | 93  | 153 | 70  |
| 155 | 140 | 185 | 108 | 117 | 4   | 101 | 113 |
| 131 | 79  | 63  | 116 | 163 | 72  | 6   | 177 |

Through the above calculation, the following results can be obtained:

**ResearchGate** is a social networking site for scientists and researchers to share papers, ask and answer questions, and find collaborators. According to a study by Nature it is the largest academic social network. However, it has been widely criticized for emailing unsolicited invitations to the coauthors, and for being "intransparent and irreproducible".

ResearchGate was founded in 2008 by virologist and computer scientist Ijad Madisch. It started in Boston, and moved to Berlin, Germany, shortly afterwards. In 2009, the company began a partnership with Seeding Labs in order to supply third-world countries with surplus lab equipment from the United States.

According to *The New York Times*, the website began with few features, then developed over time based on input from scientists. From 2009 to 2011, the site grew from 25,000 users to more than 1 million. In 2013, it closed Series C financing for \$35M from investors including Bill Gates. The company grew from 12 employees in 2011 to 120 in 2014.

Figure 6. Recovery plan

VII. CONCLUSION

In view of the stitching with smaller tasks, the use of decision-making tree algorithm can visualize the relationship of each sub-image, which is of simple operation and high precision. As for the large data processing, convert the large data to the either 0 or 1 matrix, and randomly select some parts to replace the whole, realize the dimension reduction of the model, therefore, reduce the amount of calculation. A semi-auto stitching method is proposed based on the line feature of the scrap paper words, which mainly

introduces the principle of image recovery and stitching by relying on boundary text messages, the algorithm does not depend on the geometric features of the scrap paper, so it is of simple implementation and better reliability. For the more strict requirements in the statistical data accuracy, the existence of limit state is not considered in the study, that is, non-synchronization of the pixel change may exist on both sides of the adjacent two pictures. In view of the defects of the model, the improvement direction is proposed. When the array is compared with another array, set the meaning of the threshold, when the sequence on the right side of the Array A and the side sequence of Array B in the same pixel coordinates equal probability is greater than the threshold, it is considered to be the matching items, calculate the minimum value of the threshold so as to guarantee the accuracy of the search results.

Discussion on the decision-making tree: the traditional decision tree is a process of constant ramifying and the process ends until all elements are reflected on the same relationship chain, and priority isn't considered. According to the explanation of Fig 2, the relationship of Array ab is 8(5)-14(5)-{12(4),18(4)}, 12(4) and 18(4) also appear in Array c, then, it can be regarded that 14(5) has more outstanding priorities than others, is adjacent to 8(5), in this case, Line 1 and Line 3 in Array b are not considered, which can change the growth rule of traditional decision-making tree, therefore, the constant branches are converted into branches and the merger is conducted at the same time until all the samples are retrieved.

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