Measurement Noise Analysis of Marginalized Particle Filter for Target Tracking Applications

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Abstract - The Marginalized Particle Filter (MPF) is considered to be an efficient state estimation technique, which is applicable when there is a linear Gaussian substructure present in the model. Typically, MPF consists of both Kalman Filter (KF) and Particle Filter (PF). MPF performs better in situations where KF and PF performance is not satisfactory. In an earlier study we found that the computational complexity of the MPF is less compared to PF. Also, we found that the MPF can achieve a given Root Mean Square (RMSE) error with less number of particles compared with Particle Filter making the filter faster and more efficient. In this paper we deal with measurement noise analysis of MPF against the more general PF for target tracking applications. The performance of the MPF and PF are analyzed with respect to different measurement noise levels. Also, the effect of the presence of noise spikes in the measurement is studied. A typical target tracking example is simulated using both MPF and PF. We conclude that MPF is better than PF in terms of complexity and accuracy, and can withstand measurement noise better.

Keywords - State Estimation; Target Tracking; Marginalized Particle Filter; Particle Filter; Gaussian

I. INTRODUCTION

Dynamic system state estimation has been a challenging problem in many different fields of science. In order to understand how a system performs, certain important parameters associated with the system needs to be accessed. Usually, these parameters are not directly available, which makes it necessary that they have to be estimated from some noisy measurements that may or may not be directly related. Bayesian approach gives a solution to such type of problems. In the present world, many engineering problems require an online mode of operation and are recursive in nature. Target tracking is considered an important element in many collision avoidances, surveillance, and guidance systems. Target tracking mainly deals with the estimation of certain parameters of an object based on some measurements. The performance of such systems depends on several factors. The most important factor is the dynamic model of the target. There are various types of models associated with target tracking. No matter how good the estimation method or filter employed, if the dynamic model does not accurately represent the actual system the result will not be satisfactory. Other factors that affect the optimal extraction of useful information about target states include the measurement model of the system and the quality of the filter itself [1], [2].

Target tracking can be modeled as a state estimation problem which consists of two models, the state transition model which shows how the different states of the target such as position, velocity etc. are evolving with respect to time and the measurement model which relates the current state with the current observations. When the model is linear and the noise associated with the model is Gaussian then Kalman Filter (KF) can be made use of. In practical scenarios, most of the systems are nonlinear. In such cases, both process models, as well as measurement model and either process model or measurement model, will be nonlinear. In that case Kalman Filter, can be remodeled using linearization technique which leads to Extended Kalman Filter (EKF). Extended Kalman Filter provides results which are better than Kalman Filter, but EKF is difficult to implement and suitable only when the nonlinearities are small. Other versions of KF are available which can be applied when the system is nonlinear. They are also found to give a reasonable performance when the nonlinearities are small [3], [4].

State estimation methods like Sequential Monte Carlo (SMC) methods also known as Particle filters [5], [6] can perform well when linearization and Gaussian approximations techniques yield low performance. In this method from the measurement, aposterior probability density function is evaluated. The performance of such filters depends on how effective the aposterior probability density function can be represented and propagated at every time step.

In Particle Filters, a set of weighted samples called particles are made use of in order to approximate the probability distribution. As the number of particles is sufficient, Particle Filter approaches the optimal estimate.
When the dimension of the state variable is small, Particle Filter (PF) performance is satisfactory. But as soon the system dimension increases the performance of the filter decreases as more number of the particle are required to accurately represent the probability density function. When the system is nonlinear and the noise associated with the model is Gaussian then a special type of Particle Filter known as the Gaussian Particle Filter [7] can be applied which will reduce the computational complexity of the standard Particle Filter. Another problem that affects the performance of PF is Degeneracy, in which the weights of many particles approaches zero. This problem can be avoided by a process known as resampling [8], [9], [10] where the particles are restructured. The performance of the filter depends on how effective resampling is performed. Different types of resampling technique’s can be used to reduce the problem of degeneracy. When the model is composed of a linear Gaussian [11] substructure the estimation process can be made more efficient. The basic idea of the Marginalized Particle Filter is to partition the state vector into the linear and nonlinear part and then perform the estimation process independently on linear and nonlinear parts and then combine the result so as to obtain the final result.

Let \( x_t \) be the state vector represented as,
\[
\begin{bmatrix} x_t^n \\ x_t^l 
\end{bmatrix}
\]
Let \( x_t^n \) and \( x_t^l \) are state variable vector with linear and nonlinear dynamics respectively.

In many engineering applications like passive target tracking, bearing only tracking, positioning, collision avoidance, Spiling Ballistic Missile state estimation etc. [12], [13], [14], this type of models are important. This structure can be made use of to obtain estimates with less variance compared to that obtained with a traditional Particle Filter. This technique of splitting the state vector into the linear and nonlinear part is known as marginalization or Rao-Blackwellisation. Filter based on this technique is known as a Marginalized Particle Filter (MPF) [15], [16]. MPF make use of the substructure present in the model to improve the performance of the standard Particle Filter. In Marginalized Particle Filters, Kalman Filter is used to estimate the state variables that are linear in nature and nonlinear state variables are estimated using Particle Filters. Thus, MPF is a combination of both Kalman Filter and Particle Filter.

The main contribution of this paper is measurement noise analysis of Marginalized Particle Filter. Here it assumed that there are no signal propagation delays and also the measurement and process noise are independent of each other. The performance of Particle Filter under the influence of signal propagation delay and dependent noise processes is discussed in [17], [18].

This paper demonstrates how the measurement noise affects the MPF performance. In Section 2 of the paper, a basic description of the Marginalized Particle Filter is given. To illustrate how the performance of the filter is affected by different measurement noise levels and noise spikes, a typical target tracking example is given in Section 3. Section 4 gives the details of related work. In Section 5, a comparison of the standard Particle Filter and Marginalized Particle Filter is made using the simulation results obtained from the above example and Section 6 discuss the conclusions of the above simulation results.

II. RELATED WORK

Complexity analysis of Marginalized Particle Filter is discussed in [19], where the complexity of MPF is calculated from theoretical point of view in terms of floating point operations. Also numerical complexity analysis, constant time and constant velocity RMSE simulation of MPF is compared with that of PF. Presence of noise in the measurement data is a limiting factor in the performance of the filter. Noise tolerance property is an important aspect in terms of filter performance. Marginalized Particle Filter is found to be more noise tolerant when compared to Particle Filter. A detailed analysis of the Noise tolerance property of MPF and PF including Constant Execution Time simulation, Constant RMSE simulation and Different Measurement noise covariance’s in the case of Non-Maneuvering trajectory is given in [20].

III. MARGINALIZED PARTICLE FILTER (MPF)

Marginalized Particle Filter reduces the variance of the estimates compared to that of Particle Filter by making use of the Linear Gaussian substructure present in the model. Here the filter marginalizes out the linear variables present in the model and it will be estimated using Kalman Filter.

Consider the general model [16] given below:
\[
\begin{align*}
x_{t+1}^n &= f_t^n(x_t^n) + A_t^n(x_t^n)x_t^l + G_t^n(x_t^n)v_t^n \\
x_{t+1}^l &= f_t^l(x_t^n) + A_t^l(x_t^n)x_t^l + G_t^l(x_t^n)v_t^l \\
y_t &= h_t(x_t^n) + C_t(x_t^n)x_t^l + e_t
\end{align*}
\]
where \( x_{t+1}^n, x_{t+1}^l, x_t^n, x_t^l \) represents the nonlinear and linear state variables for \((t + 1)^{th}\) and \(t^{th}\) time interval, \( y_t \) represents the measurement variable, \( A_t^n(x_t^n), A_t^l(x_t^n), G_t^n(x_t^n), G_t^l(x_t^n), C_t(x_t^n) \) are constant matrix, \( f_t^n(x_t^n), f_t^l(x_t^n), h_t(x_t^n) \) are the functions related to nonlinear, linear and measurement variables, \( v_t, e_t \) are the process noise and measurement noise affecting the
system which is assumed to be Gaussian white noise, with distribution

\[ v_i = N(0, Q_i) \]

\[ z_i = Hx_i + v_i \]

where \( v_i \) represents the process noise related to linear and nonlinear state dynamics, \( Q_i \) and \( R_i \) represent the covariance matrix of process noise and measurement noise respectively.

Furthermore, \( x^a_p \), linear state variables are Gaussian.

\[ x^a_p \sim N(\bar{x}_p, \bar{P}_o) \]

The probability density of \( x^a_p \) can be known and arbitrary. The linear state variables from \( p(x_i | y_i) \) is marginalized out and applying Bayes theorem gives \( \alpha^* = \{x^a_p\}_{i=0}^t \cdot \)

\[ (x^a_p, x^a_y | y_i) = p(x^a_p | x^a_y, y_i) p(x^a_y | y_i) \]

where the probability density \( p(x^a_p | x^a_y, y_i) \) is analytically tractable. Therefore the probability density \( p(x^a_p | x^a_y, y_i) \) can be estimated using the Kalman Filter and \( p(x^a_y | y_i) \) can be estimated using Particle Filters. The Marginalized Particle Filter can obtain estimates which have less variance when compared to that obtained using Particle Filter even if the same number of particles are used. Here the dimension of \( p(x^a_y | y_i) \) is less than \( p(x^a_p | x^a_y, y_i) \) i.e., dimension of Particle Filter is less than that of Marginalized Particle Filter which is the reason behind the improvement in its performance. Another reason for the reduction in the variance is that Kalman Filter which is used for estimating linear state variables is an optimal filter. The general formulation of the Marginalized Particle Filter is well explained in [16].

IV. TYPICAL TARGET TRACKING EXAMPLE

In order to analyze the performance of the Marginalized Particle Filter a typical target tracking example [15], [19], [20] is considered. Here an aircraft’s position and velocity are estimated using a two dimensional model with constant acceleration. In this example, it is assumed that the height of the aircraft is constant, i.e. a level flight is considered. In this tracking problem, range and bearing angle are considered to be the measurements which are applied to the filter. From the dynamic state-space model, it is clear that the model contains linear state equations and nonlinear measurement equations.

The dynamic state-space model of the target tracking example is given below. As a level flight is considered, the height component is discarded.

\[
\begin{bmatrix}
1 & 0 & 0 & T^2/2 & 0 \\
0 & 1 & 0 & T & 0 \\
0 & 0 & 1 & 0 & T \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[ x_{t+1} = x_t + v_t + e_t \]

where the state vector \( x_t = (p_x, p_y, v_x, v_y, a_x, a_y)^T \), i.e position, velocity and acceleration. The sample time is assumed to be 1 Sec, i.e. \( T = 1 \) sec. The measurement noise \( e_t \) is Gaussian with zero mean and covariance \( R = \text{cov} e = \text{diag}(100, 0.5) \). The process noises are assumed Gaussian with zero mean and covariance

\[ Q^* = \text{cov}(v^*) = \text{diag}(50, 50) \]

From the dynamic state-space model, it is clear that the two position states \( [p_x, p_y] \) are nonlinear and the remaining states \( [v_x, v_y, a_x, a_y] \) are linear. Therefore, by marginalizing out the linear state variables,

\[ x^a_p = \begin{bmatrix} p_x \\ p_y \end{bmatrix}, x^a_i = \begin{bmatrix} v_x \\ v_y \\ a_x \\ a_y \end{bmatrix} \]

This model is similar to the model mentioned in [16] with the terms \( A_x, x^a_p \) zero. Comparing with the model we have,

\[ G^* = I_{2x2}, G^t = I_{4x4}, A_x = \begin{bmatrix} 1 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0.5 \end{bmatrix}, A^t = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]
The algorithm of Marginalized Particle Filter for the above target tracking model with nonlinear measurement equation is discussed in [19], [20].

V. SIMULATION RESULTS

In this section, the performance of the Marginalized Particle Filter is analyzed in the aircraft target tracking model described in equation (7) using Monte Carlo (MC) simulations. Here standard Particle Filter and Marginalized Particle Filter are applied to the above problem and their performance is compared. Performance of improved versions of Particle Filters and Marginalized Particle Filter for various tracking applications is discussed in [21-24]. Different scenarios are considered. For simulation, an aircraft trajectory (assuming level flight) and corresponding measurements/observations (range and bearing angle) have been generated according to (7a) and (7b) for a period of 600 time samples. The parameters given in Table 1 are used for generating the model unless specified.

TABLE I. PARAMETER VALUES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Monte Carlo Simulations</td>
<td>100</td>
</tr>
<tr>
<td>Initial Position ([p_x, p_y]) in m</td>
<td>[-1000<em>10, 1000</em>5]</td>
</tr>
<tr>
<td>Initial Velocity (in (m/s))</td>
<td>56</td>
</tr>
<tr>
<td>Acceleration (in (m/s^2))</td>
<td>0.4</td>
</tr>
<tr>
<td>Initial state covariance (P_u)</td>
<td>diag(0.01,0.01,0.01,0.01,0.01)</td>
</tr>
<tr>
<td>Measurement Noise Covariance (R)</td>
<td>diag(100,0.5)</td>
</tr>
<tr>
<td>Process Noise Covariance (Q^n)</td>
<td>diag(50,50)</td>
</tr>
<tr>
<td>Process Noise Covariance (Q^l)</td>
<td>diag(5,4,0.08,0.08)</td>
</tr>
</tbody>
</table>

Two aircraft trajectory are considered. One where the aircraft follows a linear path and another where aircraft performs some amount of maneuvering. MPF and PF are applied for the aircraft trajectory and its performance is analyzed for:

1. Different Measurement Noise Covariance’s
2. Effect of Measurement Noise Spike

The non-manuevering trajectory is given in Fig.1a, where the aircraft is assumed initially at coordinates \([-1000*10, 1000*5]\) traveling with a constant acceleration of \(0.4 \, m/s^2\). The maneuvering trajectory is given in Fig.1b, where the aircraft is assumed initially at coordinates \([-1000*10, 1000*5]\) traveling with a constant acceleration of \(0.4 \, m/s^2\).

The performance of the filters is compared by using Root mean square error (RMSE) which is given by

\[
\left( \frac{1}{N} \sum_{i=1}^{N} \left( \frac{1}{N_{MC}} \sum_{j=1}^{N_{MC}} \| \hat{x}_t^j - \hat{x}_t^i \|^2 \right) \right)^{1/2}
\]

(11)

where \(N\) is the number of samples and \(N_{MC} = 100\) is the number of Monte Carlo simulations used. Monte Carlo simulations are carried out to find a stable estimate.

![Figure 1. Different target trajectories, black line (solid line), an observer at the origin is shown as *: a) True Non-Maneuvering Target Trajectory b) True Maneuvering Target Trajectory.](image_url)

A. Different Noise Covariance Simulation

In this section the measurement noise covariance is varied and its effect is studied on both Marginalized Particle Filter and Particle Filter. Here three situations are considered. First, the bearing angle covariance is kept constant at \(0.4 \, \text{deg}^2\) and the range noise covariance is varied. Second, the range covariance is kept constant at \(1000 \, m^2\) and the bearing angle noise is varied. Third, both
the range noise covariance and bearing angle covariance are varied. These situations are applied for Maneuvering trajectory. Both PF and MPF are applied in the case of Maneuvering aircraft trajectory shown in Fig 1b.

**TABLE II. RESULT FROM VARYING RANGE NOISE COVARIANCE, IN THE CASE OF THE PARTICLE FILTER WITH 1000 PARTICLES. (CONSTANT BEARING ANGLE NOISE COVARIANCE – 0.4 deg²)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Particle Filter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range Noise Covariance (m²)</td>
<td>200 600 2400</td>
</tr>
<tr>
<td>RMSE X Position (m)</td>
<td>152.64 155.68 298.85</td>
</tr>
<tr>
<td>RMSE Y Position (m)</td>
<td>59.61 60.82 343.07</td>
</tr>
<tr>
<td>RMSE X Velocity (m/s)</td>
<td>15.99 16.21 18.94</td>
</tr>
<tr>
<td>RMSE Y Velocity (m/s)</td>
<td>6.93 7.04 10.9</td>
</tr>
</tbody>
</table>

**TABLE III. RESULT FROM VARYING RANGE NOISE COVARIANCE, IN THE CASE OF THE MARGINALIZED PARTICLE FILTER WITH 1000 PARTICLES. (CONSTANT BEARING ANGLE NOISE COVARIANCE – 0.4 deg²)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Marginalized Particle Filter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range Noise Covariance (m²)</td>
<td>200 600 2400</td>
</tr>
<tr>
<td>RMSE X Position (m)</td>
<td>127.64 128.53 129.94</td>
</tr>
<tr>
<td>RMSE Y Position (m)</td>
<td>48.4 49.23 53.76</td>
</tr>
<tr>
<td>RMSE X Velocity (m/s)</td>
<td>14.15 14.27 15.26</td>
</tr>
<tr>
<td>RMSE Y Velocity (m/s)</td>
<td>6.14 7.44 7.59</td>
</tr>
</tbody>
</table>

**TABLE IV. RESULT FROM VARYING BEARING ANGLE NOISE COVARIANCE, IN THE CASE OF THE PARTICLE FILTER WITH 1000 PARTICLES. (CONSTANT RANGE NOISE COVARIANCE – 1000 m²)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Particle Filter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing Angle Noise Covariance (deg²)</td>
<td>0.5 1 2.5</td>
</tr>
<tr>
<td>RMSE X Position (m)</td>
<td>163.98 267.98 494.16</td>
</tr>
<tr>
<td>RMSE Y Position (m)</td>
<td>58.48 76.91 305.44</td>
</tr>
<tr>
<td>RMSE X Velocity (m/s)</td>
<td>17.13 21.07 27.73</td>
</tr>
<tr>
<td>RMSE Y Velocity (m/s)</td>
<td>7.57 8.91 15.73</td>
</tr>
</tbody>
</table>

Table II and III show that as the range noise covariance increases, the increase in the RMSE in the case of the Marginalized Particle Filter is less compared to that of a Particle Filter. Similar results are obtained where the bearing angle covariance is varied, keeping range covariance constant from Table IV and V. From Table VI & VII, it is clear that as the performance of the Marginalized Particle Filter is better than Particle Filter when the range and bearing angle measurement noise level increases. From the tables 2-7 it can be concluded that the performance of the Marginalized Particle Filter is better than Particle Filter when the measurement noise covariance is increased.

**TABLE V. RESULT FROM VARYING BEARING ANGLE NOISE COVARIANCE, IN THE CASE OF THE MARGINALIZED PARTICLE FILTER WITH 1000 PARTICLES. (CONSTANT RANGE NOISE COVARIANCE – 1000 m²)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Marginalized Particle Filter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing Angle Noise Covariance (deg²)</td>
<td>0.5 1 2</td>
</tr>
<tr>
<td>RMSE X Position (m)</td>
<td>155.83 195.57 280.00</td>
</tr>
<tr>
<td>RMSE Y Position (m)</td>
<td>52.42 70.42 105.61</td>
</tr>
<tr>
<td>RMSE X Velocity (m/s)</td>
<td>15.71 18.40 21.7</td>
</tr>
<tr>
<td>RMSE Y Velocity (m/s)</td>
<td>7.01 8.33 9.36</td>
</tr>
</tbody>
</table>

**TABLE VI. RESULT FROM VARYING RANGE AND BEARING ANGLE NOISE COVARIANCE, IN THE CASE OF THE PARTICLE FILTER WITH 1000 PARTICLES.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Particle Filter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range Noise Covariance (m²)</td>
<td>1000 2000 3000</td>
</tr>
<tr>
<td>RMSE X Position (m)</td>
<td>169.89 232.812 400.37</td>
</tr>
<tr>
<td>RMSE Y Position (m)</td>
<td>63.31 88.66 115.83</td>
</tr>
<tr>
<td>RMSE X Velocity (m/s)</td>
<td>17.54 21.98 26.48</td>
</tr>
<tr>
<td>RMSE Y Velocity (m/s)</td>
<td>7.81 9.8 11.89</td>
</tr>
</tbody>
</table>

**TABLE VII. RESULT FROM VARYING RANGE AND BEARING ANGLE NOISE COVARIANCE, IN THE CASE OF THE MARGINALIZED PARTICLE FILTER WITH 1000 PARTICLES.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Marginalized Particle Filter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range Noise Covariance (m²)</td>
<td>1000 2000 3000</td>
</tr>
<tr>
<td>RMSE X Position (m)</td>
<td>149.49 199.63 319.17</td>
</tr>
<tr>
<td>RMSE Y Position (m)</td>
<td>56.18 73.47 95.93</td>
</tr>
<tr>
<td>RMSE X Velocity (m/s)</td>
<td>14.96 18.6 22.99</td>
</tr>
<tr>
<td>RMSE Y Velocity (m/s)</td>
<td>6.92 8.03 9.70</td>
</tr>
</tbody>
</table>

This result is similar to the one discussed in [20] where the both MPF and PF are applied to a non-maneuvering trajectory. Thus, it can be concluded that MPF is more tolerant to measurement noise level. This property of MPF is very useful in application where measurement noise levels are relatively high and requires a faster operation as MPF requires only less number of particles when compared to standard PF.

**B. Effect of Measurement Noise Spike**

The presence of noise in the measurement has been always a limiting factor in the performance of the filter. Sometimes there may be a spike of noise present in the measurement for some period of time. In such situations the
chances of filter getting diverged are more and also the error in the estimated parameters may increase more than expected. In this section, noise spikes are introduced in the measurement data and both MPF and PF are applied and the corresponding RMSE of X and Y position are compared. Two types of effects are studied, First, the duration of noise spike is varied and its effect is studied in which three situations are considered, a noise spike in range measurement, noise spike in angle measurement and noise spike present in both angle and range measurements. Here the duration of spikes is increased and its effect is analyzed. Secondly, the magnitude of the noise spike is varied and its effect in RMSE is studied.

From the simulation, it will be clear that the presence of a noise spike in the measurement has more effect in PF than in MPF and finally leading to divergence of the filters. These situations are applied for both non-maneuvering and maneuvering trajectory. Here noise spikes are introduced in the measurement data at three time periods.

Fig 2 shows the presence of noise spikes in the measurement data that is given to both the filters in the case of a non-manuevering trajectory. Fig 2 indicates that three spikes of noise are present at 150th, 250th & 250th time sample respectively. Fig 2(a) & Fig 2(b) shows the spikes present in the respective time samples in angle and range measurement data.

In order to get an overall picture of how Marginalized Particle Filter and Particle Filter will work when noise spikes are present in the measurement, a simulation is done by continuously varying the duration of the spike from 1 sec to 50 secs assuming that noise spikes are present at three different time slots.

In this simulation three scenarios are considered, first the duration of noise spike in the range measurement is varied assuming that angle measurement is not affected with any noise spike, secondly the duration of the noise spike in the angle measurement is varied with zero noise spike in the range measurement. Thirdly the duration of noise spike in both range and angle measurement is varied. In all the three situations, the RMSE of X and Y position is plotted with respect to the duration of the noise. It is expected that as the duration of spike increase the RMSE error increases in both the filters but the increase should be more in the case of the Particle Filter. Simulation is carried out using 600 particles and 100 Monte Carlo simulation is done in order to get a stable output.
From Fig 3 it can be brought into attention that as the duration of noise spike in range measurement is increased from 1 sec to 50 secs, the RMSE of both filters increases but the effect it produces will be more in the case of Particle Filter which is indicated by green line than Marginalized Particle Filter indicated by red line. Similar results can be noticed in the other two simulation scenarios.

In Fig 4, the red line indicates the RMSE of Marginalized Particle Filter when the duration of noise spike in angle measurement is increased and the green line indicates the Particle Filter. From the figure, it is clear that the effect of duration of noise spike in the measurement is more in the case of Particle Filter when compared to that of the Marginalized Particle Filter.
Fig. 5. RMSE of X and Y position when the duration of noise spike in range and angle measurement is varied. (Non Maneuvering Target), a) RMSE of X position, b) RMSE of Y position.

Fig. 6. RMSE of X and Y position when the magnitude of noise spike present in the range measurement is varied. (Assuming no noise spike is present in the angle measurement – Non Maneuvering Target), a) RMSE of X position, b) RMSE of Y position.
Fig 6 shows the performance of Marginalized Particle Filter and Particle Filter when the magnitude of noise spike present in the range measurement is varied. Here the green line represents the RMSE of the Particle Filter and the red line shows the performance of Marginalized Particle Filter. From the simulation, it is clear that the error of Particle Filter is more when compared to Marginalized Particle Filter.

Figure 7. RMSE of X and Y position when the magnitude of noise spike present in the angle measurement is varied. (Assuming no noise spike is present in the range measurement – Non Maneuvering Target), a) RMSE of X position, b) RMSE of Y position.

From Fig 7, it is clear that as the magnitude of noise spike present in the angle measurement is increased the effect it produces on Particle Filter is more than Marginalized Particle Filter. This is similar to the result obtained in the previous situation.

Fig 8 shows the effect of the Marginalized Particle Filter and Particle Filter when the magnitude of noise spike present in both range and angle measurement is varied. From the figure, it is clear that as the magnitude increases the RMSE of the Particle Filter will be more when compared to that of the Marginalized Particle Filter. Hence, it can be concluded that Marginalized Particle Filter was able to withstand more noise spike than Particle Filter in the case of non-maneuvering trajectory. In practical situation, the target will follow a maneuvering trajectory most of the time and hence the same situations that are demonstrated above is applied in the case of maneuvering trajectory as shown in Fig. 1.b. as well. If a similar result is
obtained as in the case of non-maneuvering trajectory, then it strengthens the result that Marginalized Particle Filter is more noise tolerant when compared to Particle Filter thereby making it a good choice when a Gaussian linear substructure is present in the model.

Fig. 9. Angle and Range Measurement showing the presence of noise spike at three locations in the case of maneuvering trajectory, a) Presence of noise spike in the angle measurement, b) Presence of noise spike in the range measurement.

Fig. 10. RMSE of X and Y position when the duration of noise spike in range measurement is varied. (Assuming no noise spike in angle measurement – Maneuvering Target), a) RMSE of X position, b) RMSE of Y position.

Fig.10. shows the X and Y position RMSE of both Particle Filter and Marginalized Particle Filter when the duration of noise spike present in the range measurement is varied. Here the green line indicates the RMSE of the Particle Filter and the red line indicates the RMSE of the Marginalized Particle Filter. From the figure, it can be noted that the effect of duration of noise spike in the range measurement is more in the case of Particle Filter than in Marginalized Particle Filter.
Fig. 11. RMSE of X and Y position when the duration of noise spike in angle measurement is varied. (Assuming no noise spike in range measurement – Maneuvering Target), a) RMSE of X position, b) RMSE of Y position.

Fig. 12. RMSE of X and Y position when the duration of noise spike in range and angle measurement is varied. (Maneuvering Target), a) RMSE of X position, b) RMSE of Y position.

Fig. 11 shows the effect of duration of noise spike in angle measurement. It is assumed that noise spike is not present in the range measurement data. As the duration of noise spike increases the RMSE also increases in the case of both filters but the effect is less in the case of the Marginalized Particle Filter.

Fig. 12 shows the performance of filters when the duration of noise spike present in both range and angle measurement is varied. In this case also Marginalized Particle Filter is better than Particle Filter. Now, the magnitude of noise spike present in the measurement data is varied and its effect is analyzed in the case of Maneuvering Trajectory.
Figure 13. RMSE of X and Y position when the magnitude of noise spike present in the range measurement is varied. (Assuming no noise spike is present in the angle measurement –Maneuvering Target), a) RMSE of X position, b) RMSE of Y position.

In Fig.13, the green line indicates the effect of the magnitude of noise spike present in the range measurement in the case of the Particle Filter and the red line indicates the effect in the case of the Marginalized Particle Filter. From the figure, it is clear that RMSE is less in the case of the Marginalized Particle Filter.

Figure 14. RMSE of X and Y position when the magnitude of noise spike present in the angle measurement is varied. (Assuming no noise spike is present in the range measurement –Maneuvering Target), a) RMSE of X position, b) RMSE of Y position.

From Fig 14, it is clear that as the magnitude of noise spike present in the angle measurement is increased the effect it produces in the case of the Marginalized Particle Filter is less when compared to that of the Particle Filter.
Fig.15. shows the performance of both Marginalized Particle Filter and Particle Filter when the magnitude of noise spike present in both range and angle measurement is varied. As the magnitude of noise spike increases the root mean square error in both X and Y position increases. From Fig.10-15, it is clear that the effect is more in the case of Particle Filter than in the case of the Marginalized Particle Filter. This result is similar to the one obtained in the case of non-maneuvering trajectory. This result strengthens the conclusion that Marginalized Particle Filter is more noise tolerant than Particle Filter.

VI. CONCLUSION

In this paper, various simulation scenarios are considered and the effect of measurement noise on the performance of the Marginalized Particle Filter is compared with that of the Particle Filter. From the simulation, it is clear that when the measurement noise covariance increases the MPF outperformed PF, which means that MPF is more noise tolerant to measurement noise. Also, it has been seen that the presence of a spike of noise in the measurement data affects the performance of the filter. From the simulation, it can be seen that Marginalized Particle Filter was able to withstand the measurement noise more than Particle Filter. Thus, it can
be concluded that Marginalized Particle Filters are more tolerant to noise present in the measurements.

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REFERENCES