Mathematical Modelling of Long-Term Venture Capital Funding of Start-Up Enterprises

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Abstract - This paper applies mathematical modelling techniques to explore the problems of long-term venture capital funding of start-up companies. We propose a novel approach to the use of traditional methods to evaluate the efficiency of venture investment. We develop mathematical models to assess the systemic risk of innovation activity of these companies in the long-term.

Keywords - mathematical modeling, venture capital funding; investment project; start-up.

I. INTRODUCTION

The cornerstone of the successful development of any state is the availability of innovations that are able to make the life of both an individual and the entire society easier. The key factor of innovation is the idea, which is able to turn into a popular product in the market under certain conditions. The financial support for the idea implementation is one of the factors of such a transformation. This especially actualizes the issues related to the search for and raising of funds for supporting and developing creative and at the same time risky business structures – start-ups, venture capital being their only efficient funding mechanism today.

The analysis of the literature sources \cite{1-4} revealed the need for a more thorough research into the problems of long-term venture capital funding of innovative companies, making investment decisions regarding determining the viability of the venture start-up firms.

The world has been experiencing a rapid spread of start-ups in recent years. Russia has already seen many examples of the implementation of such projects, which achieved good results. Such projects solve the problems of business development and the country's economy in general, since they introduce innovations and new innovative technologies in production. Investors face a problem of choosing methods of the efficiency evaluation that would give an accurate result. The distinctive feature of such an evaluation is that it requires to assess not the current but the future financial standing of the company, i.e. during the implementation. The main criteria for start-ups evaluation are such parameters as technology, market, team, investment, patents, media activity, etc.

However, the commercial success of innovation is not known in advance and cannot be predicted, although it is assumed that it is based on a significant financial multiplier. It is difficult because technological start-ups operate in the field of innovations, which, in turn, causes high risks regarding the perception of new products by the market. \textsuperscript{[5]}

The purpose of the study is the mathematical modeling of the long-term venture capital funding of start-ups.

II. TRADITIONAL METHODS OF EVALUATING THE EFFICIENCY OF THE VENTURE CAPITAL INVESTMENT

The main methods that venture investors use to evaluate start-up projects are presented in Table I.

Each of the proposed methods has its benefits and drawbacks. Let's review each of them.

DCF is found as the projected cash flow over 5-10 years reduced to the present value at the discount rate. The method is either not applicable or little applicable to start-ups at the initial stages of the life cycle and is used to assess companies with positive earnings. \textsuperscript{[6]}

The key weakness of the assessment of long-term investment based on the DCF method of finding the net present value of the project (NPV) and the internal rate of return (IRR) is inadequacy of its adjustment due to future uncertainty. Besides, firstly, the projecting assumptions of poor quality that underlie the method and the lack of alternative estimates when finding NPV and DCF may not just reduce the quality of the analysis, but also appear potentially harmful (NPV analysis is a rather inflexible
method, because the current value of both cash proceeds and outflows is static). Secondly, it is not taken into account that the investor takes an active part in the start-up management during funding (the NPV method does not take the investor’s ability to influence the funding process into account). Thirdly, the weighted average cost of capital is usually taken as the discount rate, which can be altered depending on the risk, so the use of a single discount rate throughout the life cycle of a start-up is not quite justified. Since the unique features of the cycle of the venture capital funding development during the start-up transition to the next stage depend on changes in risks, the discount rate is subject to adjustment. [2, p. 454].

<table>
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<th>Method</th>
<th>Formula</th>
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<td><strong>DCF (Discounted Cash Flow)</strong></td>
<td>Net present value: [ NPV = \sum_{i=1}^{n} CF_i \left(1 + r\right)^{i} - \sum_{n=0}^{m} \frac{1}{(1 + r)^n} ]</td>
<td>( IC_n ) is the investments made by the end of the ( n )-th period; ( n ) is the length of the net income accumulation period; ( CF_i ) is the cash flow; ( r ) is the individual discount rate for a venture project.</td>
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<td>Profitability index:</td>
<td>[ PI = \sum_{i=1}^{n} CF_i \left(1 + r\right)^{i} - \sum_{n=0}^{m} \frac{1}{(1 + r)^n} ]</td>
<td>( NPV ) is the net present value; ( i_1 = ) discount rate at which ( NPV &gt; 0 ); ( i_2 = ) discount rate at which ( NPV &lt; 0 ).</td>
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<tr>
<td>Internal rate of return:</td>
<td>[ IRR = \frac{NPV_{11}}{NPV_{11} - (-NPV)_{22}^{*} \left(i_2 - i_1\right)} ]</td>
<td>( IC ) is the volume of capital investments in the project; ( CF ) is the average net cash flow.</td>
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<td>Payoff period:</td>
<td>[ PP = E/CF ]</td>
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**Method of Multipliers (or Comparative Assessments)**
- **ExitValue** = \( E/P \): Start-up capitalization to sales volume ratio (Price/Sales).
- **ExitValue** = \( EV/EBITDA \): Start-up capitalization to earnings ratio (Price/Earnings).
- **Comparable Sales Method**
  - Average start-up launch cost index: \( L_{st} = \sum_{i=1}^{n} p/n \) \( n \) is the number of start-ups assigned to segments, \( p \) is the cost of one launch.
  - Start-up launch cost index depending on the segment: \( L_{seg} = \sum_{i=1}^{n} p_{seg}/n \) \( n \) is the number of start-ups assigned to a specific segment, \( p_{seg} \) is the cost of launching one project in this segment.
  - Launch cost index for start-ups participating in specific programs/projects: \( L_{prc} = \sum_{i=1}^{n} p_{prc}/n \) \( n \) is the number of start-ups participating in programs/projects, \( p_{prc} \) is the cost of launching one project, which participates in programs/projects.

The multiplier method is used for companies that have successfully entered the market and are at the growth stage, and therefore it is not completely unsuitable for the early stage of start-ups. [7] It is used to estimate the value (Exit Value) at the “exit” based on the projected values of its operating indicators and comparison with similar start-ups. Using the P/S ratio allows to formalize the market valuation of start-ups that do not yet have profit but already conduct sales.

It is more efficient to use the guideline transaction method at the stages of the start-up projects’ inception and launch. It is applied similarly to the multiplier method [8].

**III. MATHEMATICAL MODEL OF ASSESSING THE SYSTEMIC RISK OF INNOVATION ACTIVITY OF START-UPS**

Special attention in ensuring the efficiency of venture investments should be paid to assessing the systemic risk in the activities of start-ups. The completeness, reliability and accuracy of information are a prerequisite for making adequate decisions in market conditions for all business entities. Uncertainty is a significant obstacle to achieving the efficiency of entrepreneurial activity; it leads to significant costs of funds and time and irrational allocation of resources.

A risk is an uncertainty that is evaluated in some way. Lacking the complete information about future processes, business entities tend to minimize risk losses if they operate in a situation of uncertainty.

Today, there are main methods of risk minimization, which include diversification, risks combination (insurance), distribution, and information tracking. Nevertheless, there are risks that cannot be eliminated, because they are either systemic and are caused by external factors, or are dependent on the operation of a particular enterprise (industry). This phenomenon has been named “systemic risk”, which is assessed using the coefficient determining the level of fluctuations in the performance of the enterprise (industry) in relation to the performance of the market or the economy in general. Assessment of the systemic risk will allow to project the efficiency of start-ups.

Systemic risk \( \beta \) determines the level of fluctuations or
deviations in the performance of an enterprise (industry) in relation to the performance of the market or the entire economy.

The systemic risk of investing in a start-up is found using the following formula:

$$\beta_i = \frac{\text{cov}(D_m, D_i)}{\sigma(D_m)} = \frac{\sum_{j=1}^{n}(D_{mj} - \bar{D}_m) \cdot (D_{ij} - \bar{D}_i)}{\sum_{j=1}^{n}(D_{mj} - \bar{D}_m)^2}$$  (1)

where \( n \) is the number of time intervals in the period under review,
\( D_m \) is the return on shares of the \( i \)-th start-up and the medium-term profitability,
\( D_m \) is the medium-term profitability,
\( \text{cov}(D_m, D_i) \) is the covariance of the return on shares of the \( i \)-th start-up and average market yield,
\( \sigma(D_m) \) is the variance of the average market return on shares;
\( D_{ij} \) is the average return on shares for the \( j \)-time interval;
\( D_{mj} \) is the average market return on shares for the \( j \)-time interval;
\( \bar{D}_i \), is the average return on shares of the \( i \)-th start-up for the entire period under review, and
\( \bar{D}_m \) is the average return on shares for the entire period under review.

The use of \( \beta \) is associated with an important question: how fully do the coefficients found based on the data from the previous period reflect the forecasts for the future period?

Research [2] indicates that indicator \( \beta \) is unstable, which leads to the conclusion that the value of \( \beta \) for the previous period may reflect the risk in the future period. Objectively, there is also a trend of \( \beta \) tending to 1 over time. Therefore, it is expedient to calculate \( \beta \) as a weighted average value for the previous period using the following formula:

$$\beta = a \cdot \beta_{pret} + b \cdot 1.0$$  (2)

where \( a, b \) are the weight coefficients, which each firm defines based on its own considerations, generally conceals the method of their selection, and even changes them over time;

\( \beta_{pret} \) is the preliminary value of the systemic risk for a start-up.

A scale is used for characteristic \( \beta \), see Table 2. It is also recommended to use the range of values from 0 to 2 to evaluate coefficient \( \beta \) expertly.

An important practical application of \( \beta \) is its use to assess the level of payoff of risky investments in a start-up by the expected return. In other words, \( \beta \) allows to find the profitability of the risky start-up share \( D'_i \) depending on the average market return \( D_m \) at a given time and the profitability of risk-free investments \( D_0 \). The following formula is used:

$$D'_i = D_0 + \beta \cdot (D_m - D_0)$$  (3)

where \( D_0 \) is the rate of return with zero risk – for instance, the Central Bank’s rate on government debt securities.

For practical purposes, \( \beta \) can be conveniently calculated using the following formula:

$$\beta = \frac{V_{R_i R}}{\sigma_R^2}$$  (4)

where \( V_{R_i R} \) is the correlation coefficient of the return on shares \( R_i \) and \( R \), describing the \( i \)-th industry and the economy as a whole, respectively, which is found using the following formula:

$$V_{R_i R} = \frac{n}{n-1} \left( \bar{R}_i \bar{R} - \bar{R}_i \cdot \bar{R} \right)$$  (5)

Where:
\( n \) is the number of time intervals in the period under review;
\( R \) is the expectation describing the entire economy; and
\( R_i \) is the expectation describing the \( i \)-th industry.

IV. CONCLUSION

The analysis results in the conclusion that the main problem investors face is the need to calculate the real value of future return the investor will earn at the exit from the investment. The reviewed methods exclude the possibility for an investor to influence the course of funding the project with a certain value, as well as the value of strategic prospects.

The use of different methods or even the same one can result in different valuation in different cases. Therefore, investors need to pay attention to the following, in the first place: the very idea and capabilities of the team representing the start-up, its interest in getting a return, ability to make decisions; competitiveness and demand for the start-up, as well as possible risks, and much more.

REFERENCES


