System Computing and Simulation for Big Data Dissonances: A LMI-based Criterion for Fuzzy Large-Scale Systems

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Abstract. **This paper proposes a novel artificial intelligence based EBA (Evolved Bat Algorithm) controller for large-scale nonlinear system which has high speed computation due to its LMI (Linear Matrix Inequality) type criterion. The proposed membership functions are adopted and stabilization criterion of the closed-loop T-S fuzzy systems are obtained through a new parametrized LMI which is rearranged by machine learning membership functions.**

Keywords - large-scale systems, Takagi-Sugeno fuzzy model

I. INTRODUCTION

Big data sources utilizing web data are particularly prone to dissonance effect. In order to exclude it, the mathematical models of many physical and engineering systems are frequently of high dimension, or possessing interacting dynamics phenomena. The information processing and requirements to experiment with these models for control purposes are usually excessive. It is therefore natural to seek techniques that can reduce the computational effort. The methodologies of large-scale systems provide such techniques through the manipulation of system structure in some way. Thus, there has been considerable interest in the research area of modeling, analysis, optimization and control of large-scale systems [1]. Recently, many approaches have been used to investigate the stability and stabilization of large-scale systems, as proposed in the literature [2-3].

During the past several years, fuzzy-rule-based modeling has become an active research field because of its unique merits in solving complex nonlinear system identification and control problems. This approach can obtain more flexibility and more effective capability of handling and processing uncertainties in complicated and ill-defined systems. Unlike conventional modeling, fuzzy rule-based modeling is essentially a multi-model approach in which individual rules are combined to describe the global behavior of the system [4].

In this paper, we consider a fuzzy large-scale system composed of *J* subsystems with interconnections and each subsystem is represented by the so-called Takagi-Sugeno fuzzy model [11]. One critical property of control systems is stability and considerable reports have been issued in the literature on the stability problem of fuzzy dynamic systems. However, a literature survey indicates that the stability problem of fuzzy large-scale systems has not yet been resolved. Hence, a stability criterion in terms of Lyapunov's direct method is proposed to guarantee the asymptotic stability of fuzzy large-scale systems.

II. SYSTEM DESCRIPTION AND STABILITY ANALYSIS

Consider a fuzzy large-scale system ***F*** which consists of *J* interconnected fuzzy subsystems F_j, $j = 1, 2, \cdots, J$. The *j*th fuzzy subsystem F_j is of the following form:

$$F_j : \begin{cases} x_j(k+1) = \sum_{i=1}^{r_j} h_{ij}(k) A_{ij} x_j(k) + \phi_j(k) & (1a) \\ \phi_j(k) = \sum_{\substack{n=1 \\ n \neq j}}^{J} C_{nj} x_n(k), & (1b) \end{cases}$$

where A_{ij} is a constant matrix with appropriate dimensions, $x_j(k)$ is the state vector, C_{nj} is the interconnection between the *n*th and *j*th subsystems, r_j is the number of fuzzy implications and $h_{ij}(k)$ is the normalized weight. Each isolated subsystem (i.e., $C_{nj} = 0$) of ***F*** is represented by a Takagi-Sugeno fuzzy model composed of a set of fuzzy implications and the final output of this fuzzy model is described as [5]:

$$x_j(k+1)=\sum_{i=1}^{r_j} h_{ij}(k)A_{ij}x_j(k) \quad . \tag{2}$$

Lemma 1 [5]**:** The *j*th isolated subsystem (2) is asymptotically stable if there exists a common positive definite matrix P_j such that

$$Q_{ij} \equiv A_{ij}{}^T P_j A_{ij} - P_j < 0 \quad \text{for } i=1,2,\cdots,r_j . \tag{3}$$

Based on Lemma 1, a stability criterion is derived below to guarantee the asymptotic stability of the fuzzy large-scale system ***F***.

Theorem 1*:** The fuzzy large-scale system ***F is asymptotically stable, if each isolated subsystem of ***F*** is asymptotically stable and the following inequality is fulfilled [6-10]:

$$\sum_{i=1}^{r_j} h_{ij}^2(k)\lambda_M(Q_{ij})+\sum_{i<f}^{r_j} h_{ij}(k)h_{fj}(k)[\lambda_M(Q_{ij})+\lambda_M(Q_{fj})] \;+\sum_{\substack{n=1\\ n\neq j}}^{J}[\sum_{i=1}^{r_j} h_{ij}(k)m_{ijn}+\sum_{i=1}^{r_j} h_{in}(k)m_{inj} \;+(J-1)\lambda_M(P_n)\left\|C_{jn}\right\|^2]$$
$$<0 \text{ for } \quad 1\le i<f\le r_j, \quad j=1,2,\cdots,J, \tag{4}$$

where *f* is a positive integral, $\lambda_M(Q_{ij})$ is the maximal eigenvalue of Q_{ij} defined in Eq. (3) and:

$$m_{ijn} \equiv \left\|A_{ij}^T P_j C_{nj}\right\|+\left\|C_{nj}^T P_j A_{ij}\right\| \;,\; m_{inj} \equiv \left\|A_{in}^T P_n C_{jn}\right\|+\left\|C_{jn}^T P_n A_{in}\right\| \;.$$

Proof*:** Let the Lyapunov function for the fuzzy large-scale system ***F be defined as

$$V(k)=\sum_{j=1}^{J} V_j(k)=\sum_{j=1}^{J} x_j(k)P_j x_j(k) \tag{5}$$

where P_j is the solution of Eq. (3). Taking the backward difference of $V(k)$, we have

$$\Delta V(k)=V(k+1)-V(k)$$
$$=\sum_{j=1}^{J}\{[\sum_{i=1}^{r_j} h_{ij}(k)A_{ij}x_j(k)+\phi_j(k)]^T P_j[\sum_{f=1}^{r_j} h_{fj}(k)A_{fj}x_j(k)+\phi_j(k)]-x_j^T(k)P_j x_j(k)\}$$
$$=\sum_{j=1}^{J}\sum_{i=f=1}^{r_j} h_{ij}^2(k)x_j^T(k)(A_{ij}^T P_j A_{ij}-P_j)x_j(k)\;+\sum_{j=1}^{J}\sum_{i\neq f}^{r_j} h_{ij}(k)h_{fj}(k)x_j^T(k)(A_{ij}^T P_j A_{fj}-P_j)x_j(k)$$
$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(k)x_j^T(k)A_{ij}^T P_j\phi_j(k)+\sum_{j=1}^{J}\sum_{f=1}^{r_j} h_{fj}(k)\phi_j^T(k)P_j A_{fj}x_j(k)+\sum_{j=1}^{J}\phi_j^T(k)P_j\phi_j(k)$$
$$=m_1+m_2+m_3+m_4+m_5 , \tag{6}$$

Where:

$$m_1 \equiv \sum_{j=1}^{J}\sum_{i=f=1}^{r_j} h_{ij}^2(k)x_j^T(k)(A_{ij}^T P_j A_{ij}-P_j)x_j(k) \le \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}^2(k)\lambda_M(Q_{ij})x_j^T(k)x_j(k) , \tag{7}$$

$$m_2 \equiv \sum_{j=1}^{J}\sum_{i\neq f}^{r_j} h_{ij}(k)h_{fj}(k)x_j^T(k)(A_{ij}^T P_j A_{fj}-P_j)x_j(k)$$
$$=\sum_{j=1}^{J}\sum_{i<f}^{r_j} h_{ij}(k)h_{fj}(k)x_j^T(k)(A_{ij}^T P_j A_{fj}+A_{fj}^T P_j A_{ij}-2P_j)x_j(k)$$
$$=\sum_{j=1}^{J}\sum_{i<f}^{r_j} h_{ij}(k)h_{fj}(k)x_j^T(k)\{-[(A_{ij}-A_{fj})^T P_j(A_{ij}-A_{fj})]+[A_{ij}^T P_j A_{ij}+A_{fj}^T P_j A_{fj}-2P_j]\}x_j(k)$$

$$\leq \sum_{j=1}^{J}\sum_{i<f}^{r_j} h_{ij}(k)h_{fj}(k)[\lambda_M(Q_{ij})+\lambda_M(Q_{fj})]x_j^T(k)x_j(k), \tag{8}$$

$$m_3 \equiv \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)x_j^T(k)A_{ij}^T P_j C_{nj} x_n(k) \leq \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)\left\|x_j(k)\right\|\left\|A_{ij}^T P_j C_{nj}\right\|\left\|x_n(k)\right\|, \tag{9}$$

$$m_4 \equiv \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)x_n^T(k)C_{nj}^T P_j A_{ij} x_j(k) \leq \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)\left\|x_n(k)\right\|\left\|C_{nj}^T P_j A_{ij}\right\|\left\|x_j(k)\right\|, \tag{10}$$

$$\begin{aligned}
m_5 &\equiv \sum_{j=1}^{J} \phi_j^T(k)P_j\phi_j(k) = \sum_{j=1}^{J}\{[C_{1j}x_1(k)+C_{2j}x_2(k)+\cdots]^T P_j[C_{1j}x_1(k)+C_{2j}x_2(k)+\cdots]\} \\
&\leq \sum_{j=1}^{J}[(J-1)\lambda_M(P_j)\left\|C_{1j}x_1(k)\right\|^2+(J-1)\lambda_M(P_j)\left\|C_{2j}x_2(k)\right\|^2+\cdots] \\
&= \sum_{j=1}^{J}\sum_{n\neq j}^{J}[(J-1)\lambda_M(P_j)\left\|C_{nj}x_n(k)\right\|^2] \\
&= \sum_{j=1}^{J}\sum_{n\neq j}^{J}[(J-1)\lambda_M(P_n)\left\|C_{jn}x_j(k)\right\|^2] \leq \sum_{j=1}^{J}\sum_{n\neq j}^{J}(J-1)\lambda_M(P_n)\left\|C_{jn}\right\|^2\left\|x_j(k)\right\|^2.
\end{aligned} \tag{11}$$

From Eqs. (9-10), we have:

$$\begin{aligned}
m_3+m_4 &\leq \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)\left\|x_j(k)\right\|[\left\|A_{ij}^T P_j C_{nj}\right\|+\left\|C_{nj}^T P_j A_{ij}\right\|]\left\|x_n(k)\right\| \\
&\leq \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)m_{ijn}[\left\|x_j(k)\right\|^2+\left\|x_n(k)\right\|^2] \\
&= \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)m_{ijn}\left\|x_j(k)\right\|^2+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{in}(k)m_{inj}\left\|x_j(k)\right\|^2.
\end{aligned} \tag{12}$$

Substituting Eqs. (7-12) into Eq. (6) yields:

$$\begin{aligned}
\Delta V(k) \leq& \sum_{j=1}^{J}x_j^T(k)\{\sum_{i=1}^{r_j}h_{ij}^2(k)\lambda_M(Q_{ij})+\sum_{i<f}^{r_j}h_{ij}(k)h_{fj}(k)[\lambda_M(Q_{ij})+\lambda_M(Q_{fj})] \\
&+\sum_{n\neq j}^{J}[\sum_{i=1}^{r_j}h_{ij}(k)m_{ijn}+\sum_{i=1}^{r_j}h_{in}(k)m_{inj}+(J-1)\lambda_M(P_n)\left\|C_{jn}\right\|^2]\}x_j(k).
\end{aligned}$$

According to Eq. (4), we have $\Delta V(k)<0$ and then the fuzzy large-scale system $\boldsymbol{F}$ is asymptotically stable.

Evolved Bat Algorithm (EBA) is proposed based on the bat echolocation fuzzy complex system in the natural world. Unlike other swarm intelligence algorithms, the strong point of EBA is that it only has one parameter, which is called the medium, which needs to be determined before employing the algorithm to solve problems. Choosing different medium determines different searching step size in the evolutionary process. In this study, we choose the air to be the medium because it is the original existence medium in the natural environment where bats live. The operation of EBA can be summarized in the following steps:

- Initialization: the artificial agents are spread into the solution space by randomly assigning coordinates to them.
- Movement: the artificial agents are moved.
- A random number is generated and then it is checked whether it is larger than the fixed pulse emission rate.
- If the result is positive, the artificial agent is moved using the random walk process.

$$x_i^t = x_i^{t-1} + D,$$

where x_i^t indicates the coordinate of the i-th artificial agent at the t-th iteration, x_i^{t-1} represents the coordinate of the i-th artificial agent at the last iteration, and D is the moving distance that the artificial agent goes in this iteration.

DOI 10.5013/IJSSST.a.19.06.46 ISSN: 1473-804x online, 1473-8031 print

$$D = \gamma \cdot \Delta T$$

where γ is a constant corresponding to the medium chosen in the experiment, and $\Delta T \in [-1, 1]$ is a random number. $\gamma = 0.17$ is used in our experiment because the chosen medium is air.

$$x_i^{t_R} = \beta\left(x_{\text{best}} - x_i^t\right), \quad \beta \in [0, 1]$$

where β is a random number; x_{best} indicates the coordinate of the near best solution found so far throughout all artificial agents; and $x_i^{t_R}$ represents the new coordinates of the artificial agent after the operation of the random walk process.

III. CONCLUSIONS

In this paper, a stability criterion in terms of Lyapunov's direct method is derived to guarantee the asymptotic stability of fuzzy large-scale systems with high speed computation governed by LMI based criterion. If each isolated subsystem is asymptotically stable and the stability condition is fulfilled, then the fuzzy large-scale system is asymptotically stable.

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