

## A New Approach to the Mathematical Modelling of Econometric Dynamics of Enterprises' Demand for Labor

Nikolay Vladimirovitch Kuznetsov<sup>1</sup>, Deni Dokkaevich Saydulaev<sup>2</sup>, Gurgen Levonovich Gukasyan<sup>3</sup>

<sup>1</sup> State University of Management, Moscow, Russia.

<sup>2</sup> Chechen State University, Grozny, Russia.

<sup>3</sup> Peoples' Friendship University of Russia.

<sup>3</sup> The Institute of Oriental Studies of the Russian Academy of Sciences (RAS), Moscow, Russia.

**Abstract** – In this paper we propose using econometric models to determine the dynamics of hiring personnel at an industrial enterprise, in particular, we develop: i) a linear regression model to define the dependence of the enterprises' demand for labor on wages, and ii) a regression model of the employed labor force dependence on the expenditure to attract it.

**Key words:** economic and mathematical modeling, planning, econometric model, regression equation.

### I. INTRODUCTION

A labor market is an integral part of the economic system, because the labor force is formed, distributed and reproduced on the labor market. Today, when the human capital becomes one of the decisive factors in the development of any economy, the issues of the efficient labor market operation are becoming increasingly relevant.

Changes in the guidelines of the social development, which necessitate the introduction of innovations in all fields of human activity, lead to transformational shifts in the course of economic processes and approaches to their understanding. The labor market is no exception: approaches to the nature of labor, employment, motivation and other aspects of its operation have been drastically changed many times over the past decades [1].

Transformation of the labor market at the present stage of its development is due to the need for modernization of the economy and the formation of a labor force of new quality, taking the global trends in the social division of labor into account. The current state of the Russian labor market is described by large-scale structural and dynamic changes, more fierce competition among enterprises to attract skilled labor force, and establishment of new requirements for it by employers.

One of the most important categories in the labor market, which allows to realize its change in the game rules, is the enterprises' demand for labor, through which public demands and, accordingly, changes in the requirements for the educational, qualifying and professional levels of labor force are clearly manifested.

The enterprises' demand for labor gives a signal to the state regarding the promising fields of the investment, educational and social policy. With this in mind, it is very important not only to understand the nature of economic transformations, but also be able to evaluate their impact on

the formation of demand for labor and to identify the promising innovative shifts in its structure [2].

The current trends in the labor market development define the forms and methods of employers' competitive struggle in the labor market, and force them not just to define the requirements for the personnel at enterprises, but also to track and account for the requirements of job seekers for employment conditions at enterprises and the quality of workplaces, paying increased attention to the perception of the enterprise in the labor market, and measures to improve the image of employers [3].

Modern studies of the enterprises' demand for labor can be found in numerous academic papers, publications in periodicals, statistical yearbooks, etc. Many sources quote statistical data and indicators that allow to trace some general trends in the labor market development, but do not clearly identify or reflect the aspects of the labor market situation that define the ability of enterprises to attract labor and efficiently compete for it in the labor market.

Analysis of the labor market development in the context of forming the talent pool at enterprises can be found in the academic papers of I.V. Votyakova, M.I. Bukhalkov, N.M. Kuzmina, O.A. Babordina, Yu.G. Odegova, L.V. Kartashova, and others [4-6].

Studies of the state and trends in the labor market development allowed to identify some contradictory phenomena inherent in the contemporary Russian labor market, which define the ability to actively participate in the labor market.

Such peculiarities of the labor market development include the following:

- deficit of skilled labor, regardless of the specialty and region (so-called hunger for professionals);
- structural imbalances in the labor supply;
- low level of knowledge and practical skills in graduates of higher educational institutions and a shortage of qualified personnel in working specialties;

- general demographic trends, high level of employment in the informal sector of the economy; and
- changes in wages, which affect the ability to form the personnel of enterprises from external sources [7].

However, the general state of the labor market and the shift in key trends in its development define the necessity and expediency of further research in this field.

As such, the applied aspect of studies of trends in enterprises' demand for labor defines the urgency of the research.

Everyone decides on whether to work, stay at their workplace or seek for another one depending on the level of the proposed wage, total family income, family circumstances, free time, debt obligations, time spent on training or retraining, social status, etc.

Employers also adjust the headcount in the internal labor market, taking the level of wages that exists in the labor market into account. Having predicted the supply and demand for labor, the enterprise can decide on the number of employees, the level of their qualifications and their positions. As such, the enterprise should have an optimal strategic plan for personnel and its use, as well as adopted human resources policy, which contains a system of recruitment, training, retraining, and remuneration.

Therefore, interdependence and mutual influence between such elements of the labor market – labor force and wages – is obvious.

The goal of the study is to analyze the possibility of using econometric models to study the demand of enterprises for labor in terms of the ratio between the employed labor and wages.

## II. LITERATURE REVIEW

Current studies of the enterprises' demand for labor can be found in numerous academic papers, publications in periodicals, statistical yearbooks, etc. Many sources cite statistical data and indicators that allow tracing some general trends in the labor market development, but do not clearly identify or reflect the aspects of the labor market situation that define the ability of enterprises to attract labor and efficiently compete for it in the labor market.

Analysis of the labor market development in the context of human resources' formation at enterprises is presented in the academic papers of I.V. Votyakova, M. I. Bukhalkov, N. M. Kuzmina, O. A. Babordina, Yu.G. Odegov, L.V. Kartashova, etc [4-6].

Studies of the labor market condition and its development trends have identified a number of contradictory phenomena inherent in the contemporary Russian labor market, which determine the ability to act as an active player in the labor market. These labor market features include the following:

- deficiency of trained manpower regardless of specialty and region (the so-called hunger for professionals);
- structural imbalances in labor supply;
- low level of knowledge and practical skills of graduates, as well as lack of skilled workers;
- general demographic trends, high level of employment in the informal economy sector;
- wage payment dynamics, which affect the possibility of formation of enterprises personnel at the cost of external sources [7].

However, the general state of the labor market and changes in key development trends determine the need and feasibility of further research in this direction.

The scientific papers of domestic and foreign scientists reflect the issue of practical solutions to many problems related to planning and organizing labor and wages at enterprises and organizations using methods and techniques of mathematical economic model.

Thus, O.V. Kornilova investigates the optimal ratio of the constant and variable portions of wages, focusing on the dependence of the variable portion of wages on external factors [8]. Bereslavskaya V.A. and the co-authors developed a mathematical economic model describing the dependence of operations and production staff's wages fund on the micro- and macroeconomic factors [9]. Aghion P. and co-authors proposed the mathematical economic model to optimize labor remuneration at the enterprise, based on the laborers' skills [10]. Masich M.A. and E.V. Kapliuk consider it appropriate to carry out forecasts of wages expense using multifactor additive models [11]. To assess the impact of wages differentiation on the receipt of insurance premiums, E.V. Pakhomov uses the numerical integration method [12].

So, as one can see, the methodological tools of mathematical economic modeling are widely used by scientists when solving various problems in the course of research, planning, organization and analysis of labor remuneration, employment, labor demand and labor supply, as well as labor resources' effectiveness at enterprises and organizations. Also, considering today's realities and changing economic priorities, a number of approaches and techniques proposed by scientists require a certain supplement, clarification, extension, and refining.

Within this context, the aim of the present study is to analyse the possibilities of using econometric models to study the demand of enterprises for labour in terms of the ratio between the number of hired labour and the level of wages.

## III. A NEW ECONOMETRIC MODEL OF DEMAND FOR LABOUR

The efficient study of the demand for labor requires to study the dependence of changes in hired employees on such an independent variable as wages offered to an employee.

The econometric model may be a tool in such a study, understood as regression equations that establish the relationship between the employed labor and wages.

Analysis of the process of attracting labor through econometric methods consists of the following:

- defining factors affecting the employment in various positions;
- creating a statistical database;
- defining regression dependencies (creating regression models); and
- assessing the conformity of models, their interpretation and application in practice.

Based on the statistical data provided by the HR department of the enterprise, a linear regression model (Di(Pi)) is created that establishes the dependence of employment in the enterprise (di) on wages in the organization (pi), (i = 1, n), n is the number of periods under study:

$$d_i = a_0 + a_1 p_i + l_i \tag{1}$$

Where:

- a0 is the constant of the regression equation, showing the labor employed;
- a1 is the regression coefficient;
- li is the deviation of the actual labor (di) employed at the enterprise from the estimate (mathematical expectation) (yi) of wages at the enterprise in the i-th year.

Regression parameters can be estimated in many ways, the simplest and most universal being the least squares method (LSM).

LSM defines the regression parameters in accordance with the condition that the maximum approximation that the regression parameters can provide is possible in the case of the minimum sum of squares of differences li between the actual value of the expenditure and their estimate, i.e.:

$$\sum_{i=1}^n l_i^2 \Rightarrow \min \tag{2}$$

The volume of the residual variation (2) is the functional Q(a0,a1) of the parameters of the regression equation:

$$Q(a_0, a_1) = \sum_{i=1}^n l_i^2 = \sum_{i=1}^n (d_i - y_i)^2 = \sum_{i=1}^n (d_i - a_1 p_i - a_0)^2 \tag{3}$$

In accordance with the LSM, the regression parameters a0 and ai represent the solution of the system of normal equations:

$$\begin{aligned} \frac{\partial Q(a_0, a_1)}{\partial a_0} &= -2 \sum_{i=1}^n (d_i - a_1 p_i - a_0) p_i = 0 \\ \frac{\partial Q(a_0, a_1)}{\partial a_1} &= -2 \sum_{i=1}^n (d_i - a_1 p_i - a_0) = 0 \end{aligned} \tag{4}$$

which is as follows:

$$\begin{aligned} a_1 &= \frac{n \sum_{i=1}^n p_i d_i - \sum_{i=1}^n p_i \sum_{i=1}^n d_i}{n \sum_{i=1}^n p_i^2 - \left( \sum_{i=1}^n p_i \right)^2} \\ a_0 &= \frac{\sum_{i=1}^n d_i - a_1 \sum_{i=1}^n p_i}{n} \end{aligned} \tag{5}$$

The reliability of the estimate of the average amount of labor employed at the enterprise di is demonstrated by the standard (mean-square) regression error s, which is an indicator of the variance of the values of expectation yi of the regression indicator.

Value s is a dimensional one and represents the absolute standard deviation of the actual values of the wages at the enterprise relative to the regression and is found using the following formula:

$$s = \sqrt{\frac{\sum_{i=1}^n (d_i - y_i)^2}{n - 2}} \tag{6}$$

At the same time, the average sample value of the employed labor is as follows:

$$\bar{d} = \frac{1}{n} \sum_{i=1}^n d_i \tag{7}$$

The coefficient of determination in this model is as follows:

$$R^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{\sum_{i=1}^n (d_i - \bar{d})^2} \tag{8}$$

As such, the suggested model allows to quantify and explain the dependence of changes in the employed labor on the size of the proposed wages in practice.

At the same time, it is possible to demonstrate the effect of factors that are beyond the limits of the model presented (working conditions not appropriate for some candidates, negative relationships in the team, etc.) on the employment of labor.

The significance of coefficients a0 and a1 is controlled by studying their ratio to the corresponding standard deviations Sa0 and Sa1:

$$t_n = \frac{a_n}{S_{an}} \tag{9}$$

Standard deviation San is found as follows:

$$S_{an} = \sqrt{\frac{s^2}{\sum_{i=1}^n (x_i - \bar{x})^2}} \tag{10}$$

Random variables t0 and t1 have Student t-distribution with the number of degrees of freedom n-2. The critical value tcrit is found using tables of t-distribution.

The adequacy of the model can be assessed using Fisher F-test, for which  $F_{crit}$  is found from the F-distribution tables.

The obtained value of F-test is as follows:

$$F = \frac{R^2}{1 - R^2} (n - 2) \quad (11)$$

Comparison of the critical and estimated values of F-test will allow to make a conclusion about the adequacy of the presented model.

The elasticity coefficient is used to assess the dimensionless effect of wages  $p_i$  on the amount of employed labor  $d_i$ , defined using the following formula:

$$K_{p_i}^e = \frac{(a_0 + a_1 p_i)' p_i}{y_i} = a_1 \frac{p_i}{y_i} \quad (12)$$

$K_{p_i}^e$  demonstrates how much (in %) the amount of employed labor  $d_i$  in the  $i$ -th period will change, in case of a 1% change in the wages  $p_i$ .

The size of wages at the enterprise has a significant effect on the size of employed labor, which is explained by the fact that wages are the main motivating factor of working at the enterprise.

There is a similar possibility to analyze the dependence of the amount of labor involved on advertising, personnel adaptation, etc.

The use of this method allows to create a regression equation that defines the dependence of employed labor  $d_i$  on the cost of its attraction  $c_i$ . It can be demonstrated in a similar way that such a model of the dependence of employed labor on the cost of its formation ( $D_i(C_i)$ ) is also adequate and described by high reliability.

#### IV. DISCUSSION

The 'Wage' is of great social and economic importance since it affects the productivity of labourers, their well-being and welfare, as well as the moral environment in the team. The use of inefficient labour remuneration leads to a decrease in the labour quality and productivity, violation of workplace discipline, and the emergence of social and labour conflicts at the enterprise. The higher is the level of labour remuneration, the more workers are ready to be employed. At the same time, the demand for labour is inversely proportional to the level of wages, because employers are willing to pay high wages only in case of a significant increase in the productivity of employees, which in turn leads to the release of a certain portion of the "unnecessary" labourers, and a decrease in the employment level.

Another factor that affects the demand for labour is the cost of the marginal product. Hiring an additional employee will be justified only if the income from the sale of the products produced by him (minus material costs and profit margins) is higher than his monthly wages. Accordingly, the wages of the additional employee will be equal to the cost that he has put through his work to the total product of the enterprise. If the wages of the potential additional worker exceed the possible cost of the product which is in addition

created by him, his employment is inexpedient. Thus, the increase in the level of labour remuneration provided constant prices for the enterprise's products leads to the fact that wages become higher than the income from the sale of the marginal product. Therefore hiring additional workers is inappropriate, and consequently, the demand for labour in the labour market decreases. On the contrary, a decrease in wages or an increase in prices for the enterprise's products contributes to an increase in demand for labour, because the cost of the marginal product created by an additional employee exceeds the wages increment.

Despite the fact that the main factor determining the value-based supply of labour is its price in the form of wages, it is impossible not to mention economic phenomena such as the substitution effect of and the income effect. The substitution effect is an alternative choice between free time and a set of goods and services that a person can purchase for his wages. With a rise in wages, the employee is able to broaden the range of purchased goods and services, and therefore is ready to substitute part of free time by work. But the further growth of wages and improvement of living standards leads to the fact that in addition to the well-being, the person seeks to have more free time for leisure and recreation, that is, the income effect is manifested.

Thus, under any conditions, wages act as a supply and demand regulator in the job market.

#### V. CONCLUSION

The following can be summarized:

1. The econometric model ( $D_i(P_i)$ ) can be used in the real conditions to study the enterprises' demand for labor. It helps define the size of correction of wages for the employed labor in accordance with the HR policy at the enterprise. Differentiating wages allows to manage the amount of the employed labor and, thus, the actual headcount.

2. The potential amount of employed labor is mainly determined by the amount of labor remuneration. In the opinion of the authors, other factors are the costs of attracting labor, working conditions, relationships in the team, image of the enterprise, etc.

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