

Introducing a New Toolbox for Theory of Connection

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Abstract - These days, graphs and graph theory is being used for modelling, simulation, and performance analysis of many physical systems, as physical systems are becoming more and more connected. However, ca. 75 years ago, Gabriel Kron introduced Non-Reinmannian Geometry for the modelling of physical systems. Based on Kron's work, the "Theory of Connection" was introduced as a mathematical tool for modelling of diverse systems (such as engineering, economic, and management). However, there were no tools (computer software) available to implement the theory to obtain executable models. In this paper, firstly, the Theory of Connection is introduced. Secondly, a new MATLAB toolbox is introduced as computer software for implementing the Theory of Connection to obtain executable models. Finally, as an application, the toolbox is used to solve a production planning and control problem. The scope and objective of this paper are to introduce the new toolbox so that various systems can be modelled and solved with this toolbox.

Keywords - Theory of Connection, Non-Reinmannian Geometry, modeling and simulation, production planning and control.

I. INTRODUCTION

The Theory of Connection (ToC) was introduced by Øyvind Bjørke as "Manufacturing Systems Theory" in 1995 [1]. ToC is based on Non-Reinmannian Geometry as it was formulated by Kron in his work in the thirties [2]. The advantage of using ToC is that the same approach can be used for modelling various types of systems. Bjørke's work presented some simpler models of manufacturing, electrical, mechanical, economical, hydraulic, and control systems, all modelled with the same approach [1]. Some of Bjørke's PhD students started using ToC, and there were some publications (such as [3], [4], [5], [6]). After a short series of publications, ToC was forgotten. Perhaps, this was due to the lack of tool or computer software to realize the theory. In this paper, a new MATLAB toolbox implemented by the first author of this paper is introduced as computer software for implementing ToC. The structure of this paper: section-II presents a short introduction to the ToC. Section-III presents an application example on production planning and control. Section-III discusses the implications of the research presented in this paper.

II. THEORY OF CONNECTION: A MODELING APPROACH BASED ON GEOMETRY

The Theory of Connection (ToC) makes use of geometry and algebra. For modelling, geometry is used, and then the model is converted into algebraic equations as to be solved by a computer. Also, the same approach can be used to model different types of systems. This section gives only a summary of ToC as it is impossible to compress the material presented in the textbook by Bjørke [1] into a conference paper. For a detailed study, the interested reader is referred to the textbook by Bjørke.

A. The Systemic Model by ToC

The modelling approach with ToC starts with the primitive (atomic) elements of a system, which possess unique properties. It is important to note that in isolation, primitive elements possess properties that do not vary. However, once the elements are connected into a system, the other elements in the connection may influence and change the properties of an element. In other words, the properties of an element

change when they are in a connected system. In addition to the primitive elements, and the connected system, a model may possess sources. The sources reside outside the model. However, the sources influence the dynamics of the model. Thus, the sources can be considered as the disturbances from the external environment.

B. Geometrical Space

ToC uses continuous geometrical n-space (a Euclidean space R_n) as the primary space for modelling. However, for functional spaces, three-dimensional space may suffice. A contravariant vector represents a point in the primary space R_n . Assuming a three-dimensional space, a contravariant vector, from the origin to a point, can be represented by:

$$x_p = \epsilon_1 \cdot x_{p1} + \epsilon_2 \cdot x_{p2} + \epsilon_3 \cdot x_{p3} \quad (1)$$

Where $(\epsilon_1, \epsilon_2, \text{ and } \epsilon_3)$ is the basis vector along the three axes.

The same point can be represented in the corresponding dual space of the vector space. The point, called a covariant vector, can be represented by:

$$x_d = x_{d1} \cdot \kappa_1 + x_{d2} \cdot \kappa_2 + x_{d3} \cdot \kappa_3 \quad (2)$$

Where $(K_1, K_2, \text{ and } K_3)$ is the basis co-vector along the three axes.

Fig.1 shows the connection between the primary space and the dual space.

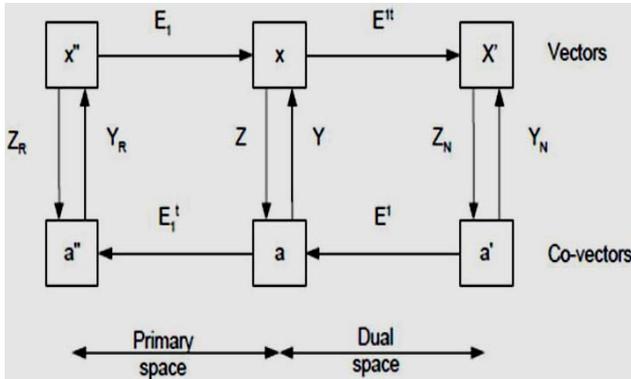


Fig. 1. Transformation of vectors and co-vectors (simplified Roth's diagram).

The matrices Y and Z are called the property matrices. These two matrices (along with their transformations) possess properties of the individual primitive elements. Matrix E (and its transformations) is called the connection matrix. The connection matrix represents the connections between the primitive elements. The relationship between the matrices $Y, Z,$ and E is represented in Roth's diagram, as devised by Kron [2], and reproduced and explained by Bjørke [1].

C. ToC Toolbox of Functions

ToC was originally implemented in the APL language. IBM developed APL (APL stands for "A Programming Language") in the sixties, and it is a symbolic language. In this paper, a new MATLAB based toolbox of functions is presented. The ToC toolbox contains functions that fall into four groups:

- 1) Establishing the primitive system: functions for creating primitive elements, and connecting them into the connected system.
- 2) Establishing the connected system: Extracting topological structure, in terms of the matrices $E, Z,$ and Y .
- 3) Simulation and Analysis: Functions for solving the algebraic equations.
- 4) The utilities: Functions for saving the model, loading the model, and printing the model details, and the simulation results.

Table-I presents the toolbox of functions for creating elements, and grouping of elements (primitive system).

TABLE I. FUNCTIONS FOR CREATING THE PRIMITIVE SYSTEM

Function Name	Purpose
element	To create a primitive element with an element ID number and with its properties.
Primitive_system	To group a set of primitive elements together. Thus, property matrices such as Y and Z can be created.

Table-II shows the functions of extracting the topological structure from the connected system. The functions for analysis and solving a model is shown in Table-III.

TABLE II. FUNCTION FOR EXTRACTING TOPOLOGICAL STRUCTURE

Function Name	Purpose
branin row	To indicate the connections between different elements as row-1 and row-2.
connect system	To make the connected system. Thus, connection matrices like V and E can be created.
Topology	Similar to the function connect system, but more powerful for universal solution.

TABLE III. FUNCTIONS FOR SOLVING

Function Name	Purpose
Property_matrix	Transforms property matrix into different reference frames.
Transpose	Geometric transformation based on reference frames.
Remove_reference	To remove a reference frame from the operating space.
Source	To identify sources with characteristic (property) value.
Step_input	Step input as a source.
Sine_input	Sine input as a source.

Finally, Table-IV presents the utility functions such as loading a model, saving a model, and printing the details of a model.

TABLE IV. UTILITY FUNCTIONS

Function Name	Purpose
Print_sys	To print the results (temporary, partial, or full results).
Save_sys	To save the results (temporary, partial, or full results).
Load_sys	To load partial results for further processing.

Fig.2 below shows where these functions are used in connection with the Roth's diagram.

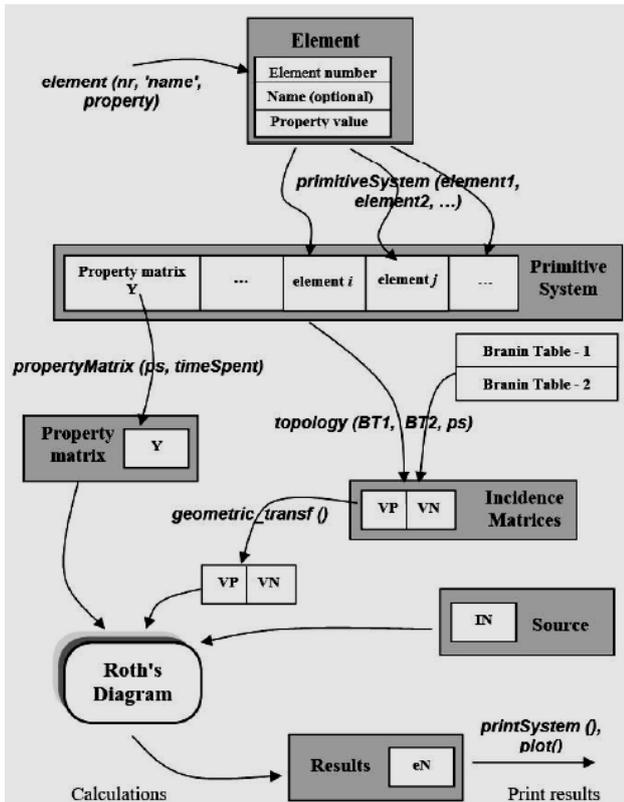


Fig. 2. Overview of the main functions and data structures.

D. ToC Modeling Approach

The ToC approach consists of three steps.

Step-1: Establishing the Primitive System. The individual primitive elements are identified and declared with the function *element*. Function *element* assigns a unique element number to the primitive element, along with a name, property value, and optionally a comment.

After declaring all the primitive elements, the elements are grouped as a primitive system using the function *primitive_system*.

Step-2: Establishing the Connected System. The connected system is established with the function *connect_system* and the Branin table. The function *connect_system* returns the matrices VP, Y, and VN. Y is the property matrix that consists of the property values of the elements. VP and VN are the incidence matrices, reflecting the directed connections between the elements. The function *remove_reference* removes the reference node from the VP, VN matrices. Thus, this function returns the overall incidence matrix V.

Step-3: Simulations. Roth's diagram shown in Fig.1 is the one that is used for solving the problem. Roth's diagram connects all the involved tensors and matrices. Fig.2 shows the functions used for solving and the data structure.

III. APPLICATION EXAMPLE

In this section, a production planning and control (PPC) problem is given as an application. The intention behind working through this PPC problem is to prove that the ToC approach for modelling physical systems will show the fundamental geometric transformations between the primary and dual geometrical spaces, and algebraic computations. The PPC problem is intentionally selected to be a simple problem so that the underlying approach can be clearly shown. The PPC problem is taken from [1].

A. The PPC Problem

The PPC system is shown in Fig.3 represents two production methodologies, the push (make to stock) and the pull (make to order).

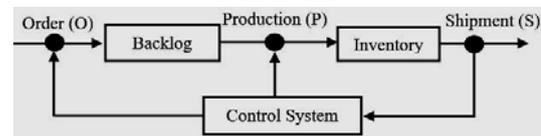


Fig. 3. Simplified PPC system.

The push methodology eliminates customer waiting time; however, it creates inventory. The pull works the opposite way as it avoids inventory but causes customer waiting time.

The basic equations are:

$$\Delta B(t) = O(t) - P(t) \tag{3}$$

$$\Delta I(t) = P(t) - S(t) \tag{4}$$

The timed functions involved in the equations given above are:

- B(t): Backlog of orders.
- O(t): Incoming orders.
- P(t): Production.
- I(t): Inventory.
- S(t): Shipment.

B. Problem Analysis

An analysis of the PPC system is presented in Fig.7. Fig.7 shows the four states of the system and the connections between them. The four states in the system are, at time t:

- x1: Orders queue.
- x2: Stock level.
- x3: Shipment forecast.
- x4: Production schedule.

The parameters (in number of periods) in Fig.7 are:

- A: Needed Stock.
- B: Build up of stock.

- C: Build up the production.
- D: Change in shipment for distribution.
- E: Backlog to be distributed.

C. Problem Solving

The block diagram in Fig.3 does not show the connections between the different states explicitly. Therefore, from the block diagram in Fig.3, the connections between the states are extracted and presented in the state diagram (aka the property graph) in Fig.4. Fig.4 shows how the four states are connected by the eight elements.

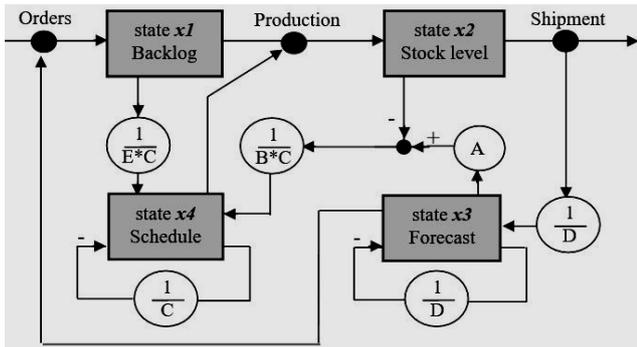


Fig. 4. The block diagram of the PPC system.

The eight elements are created first, and they are grouped into the primitive system, and then the solution process is started. Then, the connection matrices are obtained from the topology structure. From the connection matrices and from the property matrix, the solution is obtained by using Roth’s diagram.

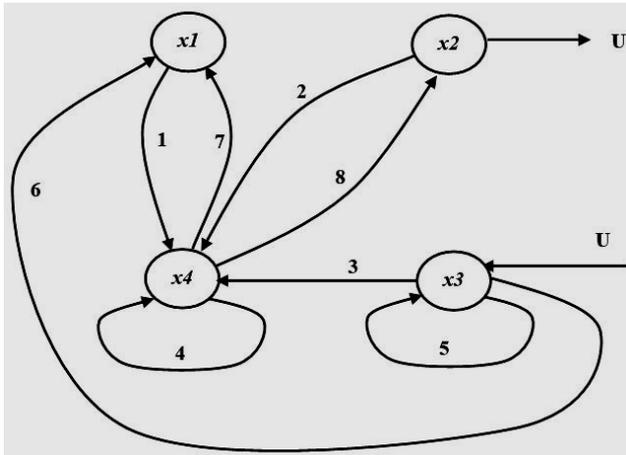


Fig. 5. The state diagram of the PPC system

Due to brevity, the simulation code for solving the PPC problem is not shown in this paper. However, for reproducibility, the complete code is given in the website [7]. Simulation results are shown in Fig.6 and Fig.7, when two different sources were applied.

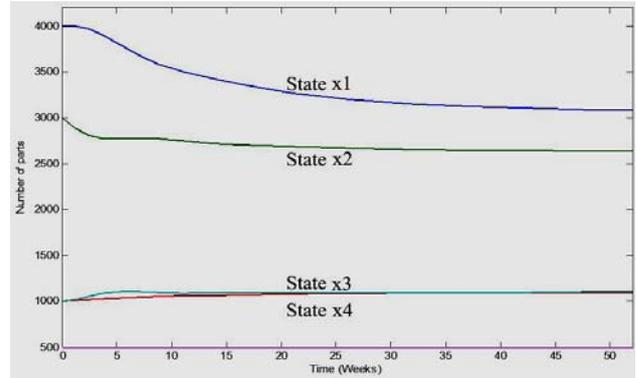


Fig. 6. Case-1: A Step response on the Stock Control System.

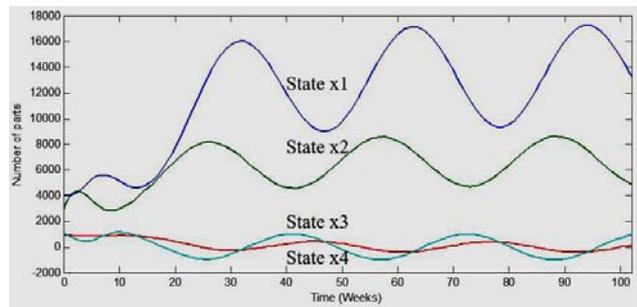


Fig. 7. Case-2: A Sinus response on the Stock Control System. of ToC.

D. Discussion

Fig.6 (for case 1) shows the system response when a step function representing 100 units of increase in sale is applied as the source.

Fig.7 (for case 2) shows the system response when the step function (as in case 1) was superimposed with a sine function with $\omega = 0.2$.

Fig.7 shows that there is a large increase in backlog and stock level where the backlog increased up to 4 times the initial value.

The simulations of the PPC problem show that the designed stock control system can handle slow varying disturbances, but it will not properly handle high-frequency disturbances.

IV. CONCLUDING REMARKS

From the PPC problem solved in the previous section, it is clear that ToC combines the computational power of algebra with the easiness of geometry. ToC approach starts with geometry by establishing the connection between the elements of a system. Then, with the help of Roth’s diagram, it moves from one reference space to another space to simplify the analysis. Based on the work of Kron, the application of geometry as the foundation makes ToC different from the other graphical modelling techniques (e.g., Automata, System Dynamics, and Petri Nets), which do not make use of the geometrical spaces. ToC approach is also convenient as the same set of steps can be used for

modelling and simulations of any systems (be it engineering, economics or management). However, unlike Petri nets, ToC is only useful for microsystems modelling. This is because ToC does not provide any mechanism for "modular" model development, which is inevitable for modelling large real-life systems [8]. For example, modular Petri Net is convenient for modelling large real-life discrete event systems. Lacking the modular model developing capability can be considered as the disadvantage of TOC.

This paper presents a MATLAB toolbox for modelling diverse systems with the Theory of Connection (ToC) based approach. Though ToC is a unique modelling approach (geometry-based modelling), only a few attempts were made to utilize this approach. By presenting the MATLAB based toolbox of functions for modelling and simulation, the authors of this paper believe the interest in ToC will be renewed.

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