The Dynamics and Control of Flexible Wings with Two Control Surfaces

Dilmi Smain
dept. Aeronautic Saad Dahlab University
Aeronautical Science Laboratory
Blida, Algeria
e-mail: smain_dilmi@yahoo.fr

Bouzouia Brahim
Advanced Technology Development Center
Algeries, Algeria
e-mail: bouzouia@hotmail.com

***Abstract*— This paper investigates the problem of modeling the uncertainties for a one-dimensional two degrees of freedom section wing with leading and trailing edge flap control and active flutter suppression. The goal is to design an active flaps control capability to suppress the flutter instability and reducing the cycle oscillatory response to time-dependent external pulses. A new interpretation for the modeling of the airfoil is given which takes into account for the parametric model uncertainties on the mass of the airfoil, dynamic pressure, stiffness and damping. To this end, a combined Linear Quadratic Regulator LQR and Parameter Robust Linear Quadratic Regulator PRLQR control law is designed and its performances toward suppressing flutter and reducing the vibration level of the airfoil.**

Keywords: robust control; flexible wing; aeroservoelasticity; airfoil dynamic; quasi-steady aerodynamic; optimal control

I. INTRODUCTION

Aeroelasticity, and in particular flutter, has influenced the evolution of aircraft since the earliest days of flight. For modern high-speed aircraft, aeroelasticity phenomena have even more far-reaching effects upon the structural and aerodynamic design. The simultaneous presence of the aerodynamic, inertia, and elastic forces makes this a truly interdisciplinary problem, Aerodynamic lifting surfaces undergoing a manoeuvre may experience a self-excited oscillation, referred to as flutter that may often be destructive, wherein energy is absorbed from the fluid and leads to large-amplitude oscillations of the lifting body. Therefore, it is imperative that the occurrence of flutter phenomena on wings be suppressed, to avoid failure of the structure due to large deformation/deflection.

In modern aviation, properties of flight control systems are commonly included in the analysis as well since the closed loop nature of such systems can interact with aeroelastic phenomena. This study having the objective of analysing control systems considering aeroelastic interactions is commonly referred to as aeroservoelasticity.
Accurate multivariable state space models are therefore required to support control laws synthesis using modern control techniques (LQR and PRLQR).

Many studies have focussed in this domain like [1, 2] which present open and closed loop flutter analysis using Roger approximation. Ko, W. Strganac and J. Kurdila [1] investigate nonlinear, adaptative control problem for suppressing flutter in typical wing section with torsional nonlinearity, they studied the global stability of adaptive control techniques derived from full feedback linearization with the fourth order polynomial nonlinear pitch stiffness in the model.

S. Heinze [4] investigates the minimization of the introduced drag of highly flexible wing by using multiple control surfaces; S. Heinze and M. Karpel investigate the implementation of high bandwidth piezo electric actuator to control application using aeroelastic amplification.

J. Johansen [5] presents a report describing numerical investigation of two-dimensional unsteady airfoil flows with application to aeroelastic stability.

J. Douglas and M. Athans [8] present a linear quadratic regulator which is robust to real parametric uncertainty, by using the overbounding method of Petersen and Hollot.

II. AIRFOIL EQUATION OF MOTION

Consider the typical section shown in figure 1. The wing is mounted on flexible support which has a translation spring with stiffness K_h and a torsion spring with stiffness K_α. These springs are attached to airfoil at the shear center. Therefore, it is two degrees of freedom motion. Denoting h and α as plunge and pitch variable.

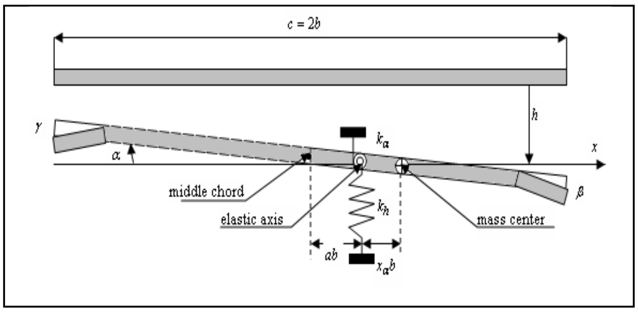

Figure 1. flexible wing with trailing and leading edge control surfaces

The aeroelastic equation of motion and the quasi-steady aerodynamic force $L(t)$ and moment $M(t)$, including both the leading and trailing edge control surfaces, are of the form [1]:

$$\begin{bmatrix} m & mx_\alpha b \\ mx_\alpha b & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L(t) \\ M(t) \end{bmatrix} . \quad (1)$$

Where m and b are the mass and semichord of the wig; I_α is the mass moment of inertia about the elastic axis; x_α represents the nondimensionalized distance between the center of mass and elastic axis; c_h and c_α are the pitch and plunge structural damping coefficients, respectively.

The aerodynamic forces acting on an airfoil in unsteady motion in an incompressible fluid was derived based on the Theodorsen's theory [7] as:

$$L(t) = q2bs_p c_{l\alpha}\left[\alpha + \left(\frac{\dot{h}}{V}\right) + \left(\frac{1}{2} - a\right)b\left(\frac{\dot{\alpha}}{V}\right)\right] + q2bs_p c_{l\beta}\beta + q2bs_p c_{l\gamma}\gamma \quad (2)$$

$$M(t) = q2b^2 s_p c_{m_\alpha}\left[\alpha + \left(\frac{\dot{h}}{V}\right) + \left(\frac{1}{2} - a\right)b\left(\frac{\dot{\alpha}}{V}\right)\right] + q2b^2 s_p c_{m_\beta}\beta + q2b^2 s_p c_{m_\gamma}\gamma \quad (3)$$

Where $q = \rho V^2/2$ is the dynamic pressure, V is the freestream velocity, a is nondimensional distance from the midchord to the elastic axis, s_p is the wing span, c_{l_α} and c_{m_α} are the lift and moment coefficients per angle of attack, c_{l_β} and c_{m_β} are the lift and moment coefficients per trailing edge flap deflection, c_{l_γ} and c_{m_γ} are the lift and moment coefficients per leading edge flap deflection.

In order to modeling the uncertainties for the nominal airfoil model, let takes into account for the parametric model uncertainties on the mass, dynamic pressure, stiffness and damping as:

$$m = m_{nom}(1 + \delta m) . \quad (3)$$

$$q = q_{nom}(1 + \delta q) . \quad (4)$$

$$c_h = c_{h-nom}(1 + \delta c_h) . \quad (5)$$

$$k_\alpha = k_{\alpha-nom}(1 + \delta k_\alpha) . \quad (6)$$

Where $\delta_{()} < 1$ is the perturbation parameter.

Substituting (3) into (1), yields

$$m\ddot{h} + mx_\alpha b\ddot{\alpha} + c_h\dot{h} + k_h h + w_1 = -L(t) . \quad (7)$$

$$mx_\alpha b\ddot{h} + I_\alpha\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha + w_2 = M(t) . \quad (8)$$

Where

$$w_1 = \left(m\ddot{h} + mx_\alpha\ddot{\alpha}\right)\delta m = \delta m z_1 . \quad (9)$$

$$w_2 = mx_\alpha b\ddot{h}\delta m = \delta m z_2 . \quad (10)$$

Substituting (4) into (2) and (3) respectively, yields the unsteady aerodynamic loads which is the combination of the quasi-steady dynamic pressure, named as the nominal dynamic pressure, and the perturbation with the unsteady dynamic pressure:

$$L(t) = q_{nom}\,2bs_p c_{l\alpha}\left[\alpha + \left(\frac{\dot{h}}{V}\right) + \left(\frac{1}{2} - a\right)b\left(\frac{\dot{\alpha}}{V}\right)\right] + q_{nom}\,2bs_p c_{l\beta}\beta + q_{nom}\,2bs_p c_{l\gamma}\gamma + w_3 \quad (11)$$

or

$$w_3 = \delta_q z_3 . \quad (12)$$

$$z_3 = 2q_{nom}bs_p c_{l\alpha}\left[\alpha + \left(\frac{\dot{h}}{V}\right) + \left(\frac{1}{2} - a\right)b\left(\frac{\dot{\alpha}}{V}\right)\right] + 2q_{nom}bs_p c_{l\beta}\beta + 2q_{nom}bs_p c_{l\gamma}\gamma . \quad (13)$$

$$M(t) = q_{nom}2b^2 s_p c_{m_\alpha}\left[\alpha + \left(\frac{\dot{h}}{V}\right) + \left(\frac{1}{2} - a\right)b\left(\frac{\dot{\alpha}}{V}\right)\right] + q_{nom}2b^2 s_p c_{m_\beta}\beta + q_{nom}2b^2 s_p c_{m_\gamma}\gamma + w_4 . \quad (14)$$

or

$$w_2 = \delta_q z_4 . \quad (15)$$

$$z_2 = 2q_{nom}b^2 s_p c_{m_\alpha}\left[\alpha + \left(\frac{\dot{h}}{V}\right) + \left(\frac{1}{2} - a\right)b\left(\frac{\dot{\alpha}}{V}\right)\right] + 2q_{nom}b^2 s_p c_{m_\beta}\beta + 2q_{nom}b^2 s_p c_{m_\gamma}\gamma . \quad (16)$$

For the uncertainty in the structural damping coefficient in plunge due to variable viscous damping, substituting (5) into (1) we define:

$$w_5 = \delta c_h z_5 . \quad (17)$$

$$z_5 = c_{h-nom}\dot{h} . \quad (18)$$

For the similar purposes, substituting (6) into (1) the pitch stiffness uncertainty due to structural nonlinearity can be expressed as:

$$w_6 = \delta k_\alpha z_6. \quad (19)$$

$$z_6 = k_{\alpha-nom}\alpha. \quad (20)$$

Combining from (7) to (20), the MIMO robust aeroelastic system can be expressed as the state space form:

$$\begin{aligned} X &= AX + BU \\ Y &= CX + DU \end{aligned}. \quad (21)$$

Where $X = \begin{bmatrix} h & \alpha & \dot{h} & \dot{\alpha} \end{bmatrix}^T$ is the state vector, $U = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & \beta & \gamma \end{bmatrix}^T$ is the input vector and $Y = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 & h & \alpha \end{bmatrix}^T$ is the output vector. The system elements are defined such that $A \in R^{4\times4}$ is the state matrix, $B \in R^{4\times8}$ is the input matrix, $C \in R^{8\times4}$ is the output matrix and $D \in R^{8\times8}$ is the input to output coupling matrix. The matrices A, B, C and D are given in Appendix.

III. FEEDBACK CONTROL DESIGN

In this section we interest to present an overview of the linear quadratic regulator (LQR) and parameter robust linear quadratic regulator (PRLQR) used in the control design.

A. LQR controller

In the context of the optimal control law, a linear quadratic regulator (LQR) is initially used for the nominal model. Its goal is to find the optimal gain matrix K such that the state-feedback law $u(t) = -Kx(t)$ subject to minimise the quadratic cost function [3]

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt. \quad (22)$$

Where $Q = Q^T \geq 0$ and $R = R^T > 0$ are the weighting matrices. The corresponding optimal controller is given by

$$K = R^{-1} B^T P. \quad (23)$$

Where, the Riccati matrix P is determined by the solution of the following steady state Riccati equation.

$$PA + A^T P - PBR^{-1}B^T P + Q = 0. \quad (24)$$

B. PRLQR controller

The PRLQR method design a controller to guarantee stability and robustness by applying the feedback

$$u(t) = -K_c x(t), \qquad K_c = \frac{1}{\rho} B^T P. \quad (25)$$

Where P is the positive definite solution, if it exists, to the modified Robust Riccati equation given by [3]:

$$PA + A^T P + (Q_0 + \gamma N^T N) - P\left(\frac{1}{\rho} BB^T - \frac{1}{\rho} MM^T\right)P = 0. \quad (26)$$

Where, M and N are two constants matrices which represent the structure of the uncertainty.
Then the PRLQR controller is the optimal controller when we are minimizing the cost functional

$$J = \int_0^\infty \left(x^T Q_0 x + x^T \gamma N^T N x + x^T \frac{1}{\gamma} PMM^T P x + \rho u^T u\right) dt. \quad (27)$$

Where the term $PMM^T P/\gamma$ is to finding the worst possible disturbance coming in the direction defined by the M matrix, which depends on which parameters are uncertain and the term $\gamma N^T N$ is the uncertain energies or the energies of the system in the direction of the uncertainties (worst case) of the state space.

IV. RESULTS AND DISCUSSION

The aeroelastic system is simulated with the parameters data presented in table I. The nominal aeroelastic system is simulated without considering any perturbation.

TABLE I. PARAMETERS DATA OF THE FLEXIBLE WING

Parameters	***data***
airspeed m/s	13
Nondimensional distance a	-0.6719
nondimensional distance x_α	0.5721
semichord of the wing m	0.1905
wing span m	0.5945
density of air kg/m^3	1.225
mass of the wing kg	4.340
total mass moment of inertia kg m^2	0.1419
plunge stiffness N/m	2844.4
pitch stiffness Nm/rad	3.525
plunge damping Ns/m	27.43
pitch damping Ns/rad	0.036

Fig. 2 shows the open-loop plunging and pitching time-histories of the nominal aeroelastic system operating in three different flight speeds ($V1 = 13m/s$, $V2 = 13.5m/s$; and $V3 \approx V_{Flutter} = 14m/s$) with initial condition ($h = 0.01m$,

$\alpha = 0.1$ rad). With the increase of the flight speed, an increase of the nominal aeroelastic response amplitude is experienced.

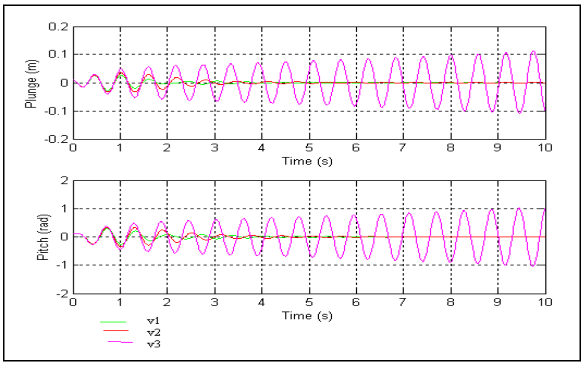

Figure 2. Open-loop nominal aeroelastic response due to initial condition ($h = 0.01m$, $\alpha = 0.1rad$), for selected flight speeds ($V1 = 13m/s$, $V2 = 13.5m/s$, $V3 = 14m/s$)

The computation results from the nominal aeroelastic stability margin obtained herein via the complex eigenvalues problem:

Flutter margin: 15.8525 N/m^2
Airspeed margin: 0.96 m/s
Critical dynamic pressure: 119.3650 N/m^2.
The computation results from the nominal controlled aeroelastic system gives:
Flutter margin: 134.1926 N/m^2
Airspeed margin: 6.70 m/s
Critical dynamic pressure: 237.7051 N/m^2.

The increased stability margin, critical pressure and critical airspeed are noticed from above data comparing to the nominal aeroelastic system.

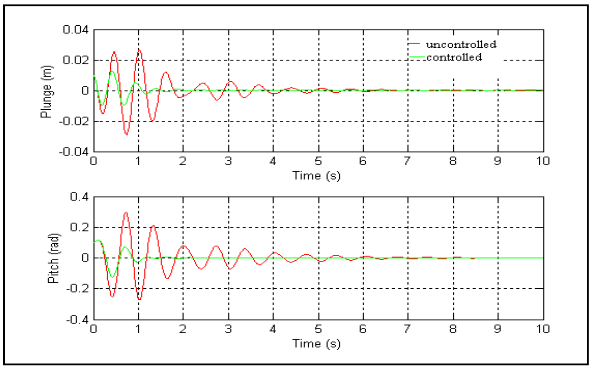

Figure 3. Uncontrolled and controlled plunging/pitching response due to initial conditions (h=0.01m, $\alpha = 0.1$ rad) , for selected flight speed (V=13m/s)

Figs. 3, 4 display uncontrolled and controlled plunging and pitching aeroelastic response time-histories of the nominal system when considering the given initial conditions ($h = 0.01m$, $\alpha = 0.1$ rad). Two flight speeds ($V = 13m/s$; $V = V_F = 13.96m/s$) are considered. A substantial beneficial effect of the LQR control is visible.

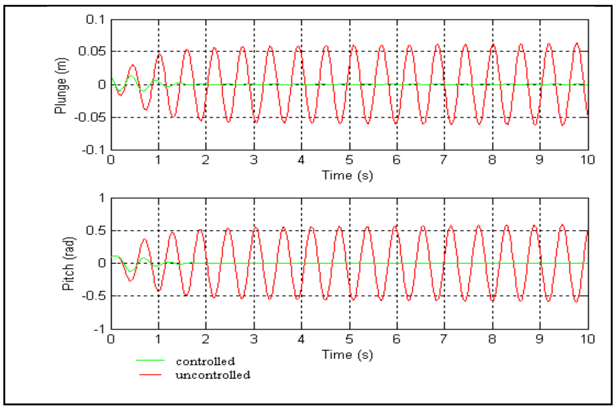

Figure 4. Uncontrolled and controlled plunging/pitching response due to initial conditions ($h = 0.01m$, $\alpha = 0.1$ rad) , for selected flight speed ($V = V_{Futter} = 13.96m/s$)

The robust aeroelastic system is simulated by considering the perturbation to the mass, unsteady aerodynamic loads and the perturbations to the variable structural damping and the structural stiffness.

The computation results from the robust aeroelastic system considering the perturbation on the dynamic pressure gives :

Flutter margin: 0.8087 N/m^2
Airspeed margin: 0.0507 m/s
Critical dynamic pressure: 104.3212 N/m^2

Note that the robust aeroelastic system has to reduce flutter margin, critical dynamic pressure and critical airspeed comparing to the nominal aeroelastic system.

The computation results from robust aeroservoelasic system with the PRLQR controller gives:

Flutter margin: 13.7478 N/m^2
Airspeed margin: 13.8364 m/s
Critical dynamic pressure: 117.2603 N/m^2.

We note the increased stability margin, critical pressure and critical airspeed but not significant.
Fig. 5 shows the evolution of the damping and frequency of the uncontrolled robust system with dynamic pressure, we can see that the flutter appear near 104 N/m^2 which is done by computation results.

Fig. 6 shows, that the robust aeroelastic system has also a limit cycle oscillation and reach the stability. When the PRLQR controller is activated the motion of the wing damped quickly and smoothly.

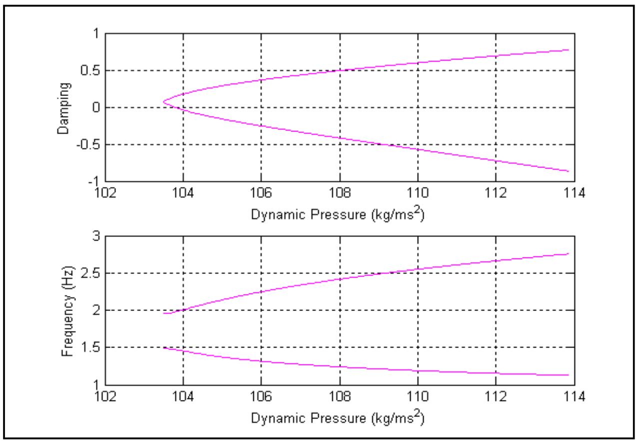

Figure 5. Systeme damping and frequency vs. dynamic pressure of robust aeroelastic model

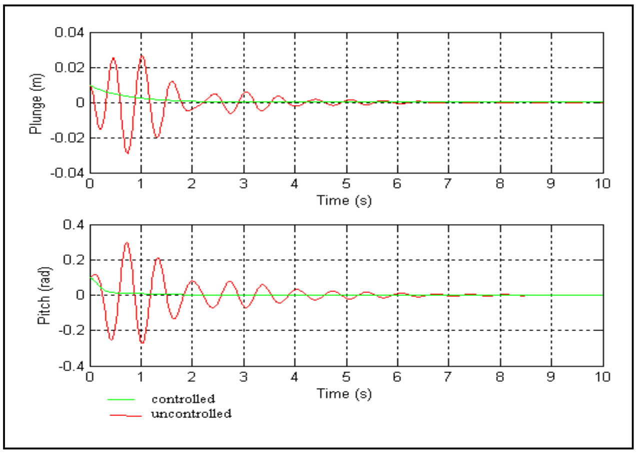

Figure 6. Robust uncontrolled and controlled plunging/pitching response due to initial conditions ($h = 0.01m$, $\alpha = 0.1$ rad) , for selected flight speed ($V = 13m/s$)

V. CONCLUSION

For a physical flight wing, to determine its nonlinear structural functions and unsteady aerodynamic loads as well as variable structural damping and mass are difficult and make the system large and difficult to be controlled. The previous method requires the knowledge of structural nonlinear functions and ignores the uncertainties from unsteady aerodynamics loads, mass and variable structural damping, which results in great difficulties and large errors in aeroservoelastic modeling. This research presents an approach to predict the robust stability of a mathematical model developed for a typical airfoil with leading and trailing edge control surfaces. For the robust aeroelastic system, an uncertainty on the dynamic pressure, stiffness and damping was developed. To control the surface deflection and meet the desired specification on the plunge, pitch response and flutter suppression, an adapted controller was used. The computation results show a decrease in the performance of the system when the uncertainty is considered. Simulation results demonstrate an increase over the critical flutter speed of an aeroelastic (uncontrolled) system may be achieved when LQR controller is activated. The improved effectiveness on the active flutter suppression is observed by using the extended PRLQR controller, thesis approach produced also the ability to compensate the limit cycle oscillation perfectly and meet the desired performance and robustness specifications for a bounded uncertainties.

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Appendix A

The system variables are given by:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

Where

$$a_{31} = (-I_\alpha k_h)/d$$

$$a_{32} = \left[m x_\alpha b k_{\alpha-om} - q_{nom} 2 b s_p \left(I_\alpha c_{l\alpha} + m x_\alpha b^2 c_{m_\alpha} \right) \right] / d$$

$$a_{33} = \left[-I_\alpha c_{h-nom} - q_{nom} 2 b s_p \left(I_\alpha c_{l_\alpha} + m x_\alpha b^2 c_{m_\alpha} \right) \left(\frac{1}{V} \right) \right] \Big/ d$$

$$a_{34} = \left[m x_\alpha b c_\alpha - q_{nom} 2 b s_p \left(I_\alpha c_{l\alpha} + m x_\alpha b^2 c_{m_\alpha} \right) \left(\frac{1}{2} - a \right) \left(\frac{b}{V} \right) \right] \Big/ d$$

$$a_{41} = (m x_\alpha b k_h)/d$$

$$a_{42} = \left[-m k_{\alpha-nom} + 2 q_{nom} b^2 s_p \left(m c_{m\alpha} + m x_\alpha c_{l\alpha} \right) \right] / d$$

$$a_{43} = \left[m x_\alpha b c_{h-nom} + q_{nom} 2 b^2 s_p \left(m x_\alpha c_{l_\alpha} + m c_{m\alpha} \right) \left(\frac{1}{V} \right) \right] \Big/ d$$

$$a_{44} = \left[-m c_\alpha + q_{nom} 2 b s_p \left(m b c_{m_\alpha} + m x_\alpha b c_{l_\alpha} \right) \left(\frac{1}{2} - a \right) \left(\frac{b}{V} \right) \right] \Big/ d$$

$$d = m I_\alpha - (m x_\alpha b)^2$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} & b_{37} & b_{38} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} & b_{47} & b_{48} \end{bmatrix}$$

Where

$$b_{31} = (I_\alpha)/d$$

$$b_{32} = (m x_\alpha b)/d$$

$$b_{33} = (-I_\alpha)/d$$

$$b_{34} = (-m_W x_\alpha b)/d$$

$$b_{35} = (-I_\alpha)/d$$

$$b_{36} = (m_W x_\alpha b)/d$$

$$b_{37} = \left[-q_{nom} 2 b s_p \left(I_\alpha c_{l_\beta} + m_W x_\alpha b^2 c_{m_\beta} \right) \right] / d$$

$$b_{38} = \left[-q_{nom} 2 b s_p \left(I_\alpha c_{l_\gamma} + m_W x_\alpha b^2 c_{m_\gamma} \right) \right] / d$$

$$b_{41} = b_{43} = b_{45} = (m x_\alpha b)/d$$

$$b_{42} = b_{46} = -(m)/d$$

$$b_{44} = (m)/d$$

$$b_{47} = \left[q_{nom} 2 b s_p \left(m b c_{m_\beta} + m x_\alpha b c_{l_\beta} \right) \right] / d$$

$$b_{48} = \left[q_{nom} 2 b s_p \left(m b c_{m_\gamma} + m x_\alpha b c_{l_\gamma} \right) \right] / d$$

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ 0 & c_{32} & c_{33} & c_{34} \\ 0 & c_{42} & c_{43} & c_{44} \\ 0 & 0 & c_{53} & 0 \\ 0 & c_{62} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Where

$$c_{11} = -k_h$$

$$c_{12} = -2 q_{nom} b s_p c_{l_\alpha}$$

$$c_{13} = -c_{h-nom} - 2 q_{nom} b s_p c_{l_\alpha} / V$$

$$c_{14} = -2 q_{nom} b s_p c_{l_\alpha} \left(\frac{1}{2} - a \right) \left(\frac{b}{V} \right)$$

$$c_{21} = -k_{h-nom} m x_\alpha b / d$$

$$c_{22} = m x_\alpha b \left[m x_\alpha b k_{\alpha-nom} - 2 q_{nom} b s_p c_{l_\alpha} - 2 q_{nom} m x_\alpha b^3 c_{m_\alpha} \right] / d$$

$$c_{23} = m x_\alpha b \left[-c_{h-nom} - 2 q_{nom} b s_p c_{l_\alpha} \left(\frac{1}{V} \right) - 2 q_{nom} m x_\alpha b^3 s_p c_{m_\alpha} \left(\frac{1}{V} \right) \right] \Big/ d$$

$$c_{24} = m x_\alpha b \left[m x_\alpha b c_\alpha - 2 q_{nom} b s_p c_{l_\alpha} \left(\frac{1}{2} - a \right) \left(\frac{b}{V} \right) - 2 q_{nom} m x_\alpha b^3 s_p c_{m_\alpha} \left(\frac{1}{2} - a \right) \left(\frac{b}{V} \right) \right] \Big/ d$$

$$c_{32} = 2 q_{nom} b s_p c_{l_\alpha}$$

$$c_{33} = 2q_{nom}bs_p c_{l_\alpha} / V$$

$$c_{34} = 2q_{nom}bs_p c_{l_\alpha}\left(\frac{1}{2} - a\right)\left(\frac{b}{V}\right)$$

$$c_{42} = 2q_{nom}b^2 s_p c_{m_\alpha}$$

$$c_{43} = 2q_{nom}b^2 s_p c_{m_\alpha} \quad /V$$

$$c_{44} = 2q_{nom}b^2 s_p c_{m_\alpha}\left(\frac{1}{2} - a\right)\left(\frac{b}{V}\right)$$

$$c_{53} = c_{h-nom}$$

$$c_{62} = k_{\alpha-nom}$$

$$D = \begin{bmatrix} -1 & 0 & -1 & 0 & -1 & 0 & d_{17} & d_{18} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} & d_{27} & d_{28} \\ 0 & 0 & 0 & 0 & 0 & 0 & d_{37} & d_{38} \\ 0 & 0 & 0 & 0 & 0 & 0 & d_{47} & d_{48} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Where

$$d_{17} = -2q_{nom}bs_p c_{l_\beta}$$

$$d_{18} = -2q_{nom}bs_p c_{l_\gamma}$$

$$d_{21} = mx_\alpha bI_\alpha / d$$

$$d_{22} = (mx_\alpha b)^2 / d$$

$$d_{23} = -mx_\alpha b / d$$

$$d_{24} = -d_{22}$$

$$d_{25} = d_{23}$$

$$d_{26} = d_{22}$$

$$d_{27} = mx_\alpha b\left[-2q_{nom}bs_p c_{l_\beta} - 2q_{nom}mx_\alpha b^3 s_p c_{m_\beta}\right] / d$$

$$d_{28} = mx_\alpha b\left[-2q_{nom}bs_p c_{l_\gamma} - 2q_{nom}mx_\alpha b^3 s_p c_{m_\gamma}\right] / d$$

$$d_{37} = -d_{17}$$

$$d_{38} = -d_{18}$$

$$d_{47} = 2q_{nom}b^2 s_p c_{m_\beta}$$

$$d_{48} = 2q_{nom}b^2 s_p c_{m_\gamma}$$