An Improved Model of Ceramic Grinding Process and its Optimization by Adaptive Quantum inspired Evolutionary Algorithm

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***Abstract*— Surface grinding is a process extensively employed for removing material with specified surface finish and integrity. It is used in processing of advanced structural ceramics like silicon carbide, which are widely used in engineering applications. Ceramic grinding is a complicated process as ceramics have high hardness and low surface toughness, which adversely affect the output quality. Optimization of the process is thus essential for maintaining desired quality while maximizing productivity with minimum cost. This process has been modelled as a Continuous Nonlinear Constrained Optimization problem. Efforts have been reported to solve this problem by using Genetic Algorithms, Particle Swarms and Differential Evolution. However, most of these attempts have employed algorithms where the users / designers select parameters in the evolutionary operators. This paper improves the existing model with realistic modification and converts it into Mixed Integer Nonlinear Constrained Optimization problem. The same has been optimized by using an Adaptive Quantum inspired Evolutionary Algorithm, which is free from user selectable parameters in evolutionary operators, as they are determined adaptively. The proposed algorithm does not require mutation for maintaining diversity. Further, previous attempts have employed penalty factors or repair based techniques for handling constraints where as this effort employs feasibility rules which is again free from parameter tuning. The proposed algorithm is simple in concept, easy to use, faster and more robust than the known state of art methods available for solving such problems as shown by detailed analysis.**

Keywords-Adaptive; Ceramic; Constraint Handling; Evolutionary Algorithms; Process Optimization; Quantum

I. INTRODUCTION

Surface grinding is a complex machining process that is extensively used for material removal with specified surface finish, integrity, dimensional and form tolerance [15]. It is also used in processing of advanced structural ceramics for manufacturing industrial wear parts, bioceramics, cutting tools, and engine components, which find applications in many industries like mining, transmission electricity, aerospace, medicine, refinery, packaging, electronics, industrial and guided light wave transmission etc [1].

Advanced structural ceramics like Silicon Carbide (SiC) are useful as these are chemically inert and have high sublimation temperature. SiC has great potential to replace existing heat insulators as it has low thermal coefficient of expansion and higher thermal conductivity due to 9:1 ratio of covalent to ionic bonding. However, SiC suffers from low fracture toughness, which makes commercial utilization of SiC expensive [2]. The quality of products is largely dependent on the defects introduced during grinding of ceramics. These defects are primarily due to formation of cracks in layers close to the surface. The two types of cracks responsible for chip-formation and subsequent removal of material are median and radial cracks produced during grinding. The formation of cracks is due to the energy introduced in the layers close to the surface as explained by Malkin et al. [3].

The machining cost and rejection rate in ceramic processing are relatively high. Therefore, the optimization of grinding process of ceramics is essential for high quality and cost effective output. A suitable model of ceramic grinding process is necessary for understanding the complex process and its subsequent optimization. The grinding process can be optimized by suitably choosing the operating parameters like feed rate, depth of cut and grit size to maximize material removal rate under constraints of permissible surface roughness and number of defects.

Evolutionary Algorithms (EA) like Genetic Algorithms (GA) [4], Particle Swarms (PSO) [5], [16] and Differential Evolution (DE) [17] algorithms have been successfully employed for optimizing ceramic grinding process. EAs are population based stochastic search and optimization techniques inspired by nature's laws of evolution. They are popular due to their simplicity and ease of implementation. However, they suffer from issues like premature convergence, slow convergence, stagnation and are sensitive to the choice of the crossover and mutation operators and parameters. Quantum inspired Evolutionary Algorithm (QiEA) [10] has been proposed to overcome some of the limitations associated with EAs. They are developed by drawing some ideas from quantum mechanics and integrating them in the current framework of EA. This paper proposes to solve the ceramic grinding optimization problem by using a recently proposed framework for designing Adaptive QiEA (AQiEA) [7].

Constraint handling is also an important design decision in solving COPs with EAs, as no inherent mechanism is available in EAs to incorporate the constraints seamlessly without consuming extra computational effort. Many constraint-handling techniques have been suggested in literature [18], each having its own strengths and weakness that have been discussed in [8]. Feasibility Rules method has been used for handling constraints as it is free from parameter fine-tuning and simple in concept.

The rest of the paper is organized as follows. Section 2 discusses the related work. Section 3 presents improvement in existing model. Section 4 describes design of the proposed algorithm. The testing methodology is presented in Section 5. The results are analysed and discussed in Section 6. Section 7 concludes with a brief summary and gives direction for future endeavour.

II. RELATED WORK

Modelling and simulation of grinding process with comparisons between different techniques were described in detail by Tonshoff et al. [19]. Konig et al. [20] have suggested optimum parameters by analysing results obtained from experiments performed to determine the conditions for maximizing the Material Removal Rate of surface grinding process while improving the strength of the ground components. However, mathematical models were not developed and so the optimization of process is difficult. Gopal et al. [4] have performed experiments on sintered SiC work pieces to find the significance of the grinding variables like grit size, depth of cut and work feed rate on the machining responses like surface finish, number of surface flaws and material removal rate. The experimental data has been used to develop a mathematical model of the grinding process using multiple regression analysis. The mathematical model of grinding process has been formulated as a continuous domain nonlinear constrained optimization problem. They have also attempted to optimize the grinding process with a real coded Genetic Algorithm (GA). GAs are mostly used for solving discrete domain problems and have been generally found to be inefficient for continuous domain optimization problem. Further, the proposed algorithm requires initialization of population with feasible solutions, which indicates that death penalty has been used as the constraint handling technique. Death penalty is known to consume more resources in finding feasible solutions [8], [18].

Ting et al. [21] optimized grinding process by using Particle Swarm Optimization (PSO) algorithm technique. They reported better results than GA [4]. Constraint handling was done through static penalty factor (pf) method with pf =10000. Lee et al. [17] investigated the grinding process optimization by employing PSO, Differential Evolution (DE) Algorithm and GA and using static penalty factor method for handling constraints with pf=10000. They found that PSO performs better than DE and GA techniques. Lee et al. [16] proposed a new technique for robustness analysis and tested their PSO with it.

Zhara et al. have also attempted to solve grinding process optimization problem by PSO and its improved variants in [5]. The NM-PSO uses Nelder Mead Simplex method hybridized PSO, constraint fitness priority-based ranking along with gradient repair for handling constraints. The constraint handling methods are complex and resource intensive, as they require computation of quasi inverse of matrix. Nelder Mead Simplex method is known to be resource hogging and conventional PSO do not guarantee global convergence [6].

III. PROBLEM FORMULATION

The ceramic grinding process has been modelled as maximization of Material Removal Rate (MRR), which is a product of Feed Rate (fr) and Depth of Cut (dc), subject to constraints of maximum permissible Number of Defects (NDmax) and Surface Roughness (SRmax) by Gopal et. al. in [4]. The number of defects (ND) is a function of fr and dc whereas the surface roughness (SR) is not only a function of both fr and dc but also of the size of the grit (M). The mathematical formulation of the problem is as given below [4]:

Maximize: MRR = (fr) * (dc)

Subject to: (1)

$ND= 29.67 *(dc)^{0.4167} *(fr)^{-0.8333} - NDmax \leq 0$

$SR= 0.145*(dc)^{0.1939} *(fr)^{0.7071} *(M)^{-0.2343} - SRmax \leq 0$

8.6 m/min < fr < 13.4 m/min

$5\ \mu m \leq dc \leq 30\ \mu m$

$120 \leq M \leq 500$

As per the model, MRR is directly proportional to the fr and dc so increasing either of them improves MRR. ND is directly proportional to dc but inversely proportional to fr i.e. dc and fr are two mutually competing factors that influence ND. The level of fr and dc will determine whether ND is increased or decreased. Similarly, SR is directly proportional to both dc and fr but inversely proportional to M i.e. dc and fr competes with M to influence the surface roughness. The level of fr, dc and M will determine whether SR is increased or decreased. The optimization problem is a nonlinear continuous constrained optimization problem, as the constraints are nonlinear and the three variables fr, dc and M are all continuous variables. The fr and dc can be taken as continuous as these parameters are set in the CNC grinding machines but the grinding wheel is generally purchased from the vendor and its specification is guided by the standards like ANSI / ASME. Though, these wheels can be custom designed but it would be more sensible if grit number M is taken as integer variable rather than continuous variable. Therefore, the modified model of ceramic grinding process is more realistic but the optimization problem becomes more complex as it is now a Mixed Integer nonlinear constrained optimization problem.

IV. PROPOSED ALGORITHM

Evolutionary Algorithms have been applied to solve complex constrained optimization problems, which cannot be attempted by deterministic optimization techniques. Such problems cannot be solved by traditional calculus-based methods due to lack of proper domain information and various restrictions. Further, such methods only perform local optimization [9].

EAs are population based stochastic search and optimization techniques inspired by the nature's laws of evolution. They are popular due to their simplicity and ease of implementation. However, they suffer from issues like premature convergence, slow convergence, stagnation and are sensitive to the choice of the crossover and mutation operators and parameters. Many efforts have been made by the researchers to overcome such limitation by establishing a good balance between the Exploitation and Exploration. A typical EA is designed using selection, crossover, mutation operators and local heuristics. The mutation operator is used for escaping from local minima i.e. improving exploration, while local heuristic is used for increasing the convergence rate i.e. improving exploitation. However, all such attempts tend to employ user selectable parameters in their algorithm design. Thus, the balance struck between the exploration and exploitation in a specific EA has a user bias rather than problem bias.

Quantum inspired Evolutionary Algorithms (QiEA) [10] was proposed to improve the balance between exploration and exploitation. QiEAs are evolutionary algorithms inspired by the principles of Quantum Mechanics. They are developed by drawing some ideas from quantum mechanics and integrating them in the current framework of EA. The important principles of Quantum Mechanics are Superposition, Entanglement, Interference and Measurement [11]. The principles mostly utilized in designing QiEA are superposition and measurement [12] and has been used for improving diversity as reported in the literature. Another interesting observation is the use of single qubit (quantum analogue of classical bit, and is governed by the principles of quantum mechanics) representation in almost all the efforts, thus ruling out use of any other principles of quantum mechanics such as entanglement.

This algorithm utilizes two qubits representation instead of one qubit representation, which helps in utilizing entanglement and superposition principles of quantum computing for improving the search. Further, a parameter free adaptive quantum crossover operator inspired by the quantum phase rotation has been designed to generate new population. The proposed AQiEA does not require mutation operator for avoiding premature convergence. However, a gradient-based local heuristic has been employed for improving convergence rate.

A. Quantum Inspiration

The smallest information element in quantum computer is a qubit, which is quantum analog of classical bit. The classical bit can be either in state 'zero' or in state 'one' whereas a quantum bit can be in a superposition of basis states in a quantum systems. It is represented by a vector in Hilbert space with |0> and |1> as the basis states. The qubit can be represented by the vector |ψ>, which is given by:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle. \quad (2)$$

where $|\alpha|^2$ and $|\beta|^2$ are the probability amplitudes of the qubit to be in state |0> and |1> and should satisfy the condition:

$$|\alpha|^2 + |\beta|^2 = 1. \quad (3)$$

The qubits can store, in principal exponentially more information than classical bits. However, these qubits exist in quantum computing systems and are constrained by several limitations like they collapse to one of the basis states upon measurement and can be evolved by using unitary transformations. The simulation of the qubits is inefficient on classical computers.

In QiEA, the probabilistic nature of qubits has been widely used for maintaining diversity [12]. A single qubit is attached to the solution vector and the solution is obtained by taking measurement or collapsing in binary coded as well as real coded QiEA i.e. RQiEA. The qubits associated with solution vector is also evolved by using quantum gate operators, which are influenced by phase rotation transformation used in Grover's Algorithm for searching unsorted database.

An interesting point of such implementation is that there is no direct correspondence between the solution vector and qubits especially in case of RQiEA [12]. The quantum rotation gates / operators also behave independent of the information from the problem and the solution domain assuming that the quantum behavior would help in reaching the solution. However, it should not be forgotten that such algorithms are to be run on classical computers without simulating any quantum phenomena. Further, it can be argued that increasing the diversity by collapsing the solution qubit may affect the exploitation of the solution as the solution found in the next iteration even of a good candidate solution may end up being far worse due to the probabilistic implementation.

This algorithm takes a different approach for designing the QiEA by using not only the qubit representation and the associated superposition principle but also the entanglement principle. Entanglement is one of the fundamental principles and if two qubits are entangled then performing any quantum operation on one of the qubits will affect the state of the other qubit also i.e. there is a relation between the two qubits, which can be utilized for computation purposes. The proposed AQiEA uses two qubits per solution vector to utilize the entanglement principle. An important point to remember is that the entanglement principle is being integrated in a classical algorithm, so the implementation would be classical. The AQiEA tries to overcome the limitations associated with the classical EA.

The classical EA being a black box optimization technique has only objective function value as the domain information regarding a specific problem. This feedback is mostly used in the selection phase and not for directly controlling the crossover or the mutation operators or even the local heuristic. Therefore, the feedback through the objective function value is not being utilized properly. The algorithm uses the second qubit, which stores the information regarding the objective function value of the solution vector. This provides information regarding the solution domain as well as the problem function domain made available simultaneously. The information stored in the first and second qubits are entangled to harness the power of the important entanglement principle. The first qubit influences the second qubit as the probability amplitude of the first qubit will determine the objective function value and hence the probability amplitude of the second qubit. The second qubit influences the first qubit as the parameter free adaptive quantum inspired rotation crossover operator used for evolving the first qubit uses the probability amplitude of the second qubit. Any operation performed on either of the two qubits would affect the other and so they are entangled in the classical implementation.

B. Proposed Representation of Quantum Registers

The first set of qubits $|\psi_{1i}>$ store the current value of the i^{th} variable as amplitude α_{1i} whose value [0, 1]. The upper and lower limits of variables are scaled between zero and one. The amplitude β is not stored as it can be computed from equation 2. Therefore, the number of qubits per quantum register QR_1 is equal to the number of variables. The quantum register stores the qubits. The number of quantum registers has also been made a function of the number of variables in the specific problem. Thus giving it a problem bias rather than user bias. The number of QR_1 is five times the number of variables i.e. 15. The structure of QR_1 is shown below:

$$\begin{array}{ll} QR_{11} = & [\alpha_{111}, \alpha_{112} \ldots \alpha_{11n}] \\ \ldots\ldots\ldots\ldots & \\ QR_{15N} = & [\alpha_{15n1}, \alpha_{15n2} \ldots \alpha_{15nn}] \end{array} \quad (4)$$

The second set of qubits $|\psi_{2i}>$ store the ranked and scaled objective function value of the i^{th} solution vector as amplitude α_{2i} whose value [0, 1]. The fittest vector's objective function value is assigned one and the worst vector is assigned value zero. The rest of the solution vectors' objective function value is ranked and assigned in between the zero and one. Another alternative was normalizing the solution vector's objective function value between zero and one and assigning it to α_{2i}. However, it has been found that amplitude amplification due to quantum phase rotation led to similar problems as are faced in Grover's algorithm.

C. Adaptive Quantum Rotation based Crossover (AQCO)

Quantum gates are used for evolving the qubits in quantum computing paradigm. Quantum phase rotation gate was used in Grover's algorithm for amplitude amplification for searching the marked element in the unsorted database. Most of the efforts have used rotation gates for evolving the qubits. A quantum rotation inspired adaptive and parameter tuning free crossover operator is designed for the AQiEA. The second qubits' amplitude is used for determining the angle of rotation for evolving the first qubit. Equation (5) is used for the purpose:

$$|\psi_{1i}(t+1)> = |\psi_{1i}(t)> + f(|\psi_{2i}(t)>, |\psi_{2j}(t)>) * (|\psi_{1j}(t)> - |\psi_{1i}(t)>) \quad (5)$$

where t is iteration number, $|\psi_{1j}>$ can be the best solution vector or any other randomly or deterministically selected solution vector. All solution vectors are rotated towards the best solution vector when $|\psi_{1j}>$ is the best solution. When the solution vector is randomly or deterministically picked then the inferior solution is rotated towards the better solution. In case of the best solution, it is rotated away from the inferior solution. Therefore, the rotation crossover operator balances the exploration and exploitation and converges the solution vector adaptively towards global optima by using the three strategies viz. Rotation towards Best (R-I), Rotation away from Worse (R-II) and Rotation towards Better (R-III).

The function $f(|\psi_{2i}(t)>, |\psi_{2j}(t)>)$ controls gross and fine search. Presently, $f(|\psi_{2i}(t)>, |\psi_{2j}(t)>)$ generates a random number either between α_{2i} and α_{2j} or $|\alpha_{2j}|^2$ and $|\alpha_{2i}|^2$. The value $|\alpha_{2j}|^2 - |\alpha_{2i}|^2$ is smaller than $\alpha_{2j} - \alpha_{2i}$, thus the later is used for the gross search and the former for the fine search. The salient feature of the new quantum rotation inspired crossover operator is the adaptive change of each variable in the solution vector and at the same time, it is problem driven rather than being an arbitrary choice of the user.

D. Constraint handling

Constraint handling is an important issue in constrained optimization. The choice of the technique has serious impact on the quality of the solutions, as EAs are generically unconstrained optimization algorithms. Many approaches have been suggested in literature [8] for handling constraints of which penalty factor methods, feasibility rules, and their derivatives are most commonly used techniques. These techniques penalize infeasible solutions by introducing penalty parameter(s). These parameters require fine-tuning, which itself becomes an optimization problem often solved by the developers through trial and error methods. If the penalty parameter is small, the resulting solution may be infeasible and if it is too large, the solution found is usually too far from the optimum.

Feasibility Rules method has a set of rules for comparing the two solution vectors which give preference to feasible solutions over infeasible ones. It ensures that the search takes place with emphasis on the feasibility of solution. It is free from fine-tuning of the penalty parameters and at the same time is robust in performance [13]. Therefore, Constraint handling has been implemented using the Feasibility Rules. It has been implemented as comparison between two individuals in the population and is stated as follows [14]:

1) If the two solution vectors being compared are both feasible, the one with better objective function value is considered fitter.
2) If one solution vector is feasible and the other is unfeasible, the feasible one is fitter.
3) If both solution vectors are infeasible, the one with lower level of constraint violation or degree of infeasibility is fitter.

E. Proposed AQiEA

The pseudo code is given below followed by the flow chart and description of the proposed AQiEA.

Pseudo code

1) Initialize Quantum inspired Register QR_1.
   While (!termination_criteria) {
2) Compute fitness of QR_1 using Feasibility Rules.
3) Assign QR_2 on the basis of the fitness of QR_1.

4) Perform Adaptive Quantum Rotation based Crossover on QR_1.
5) Select next generation using Feasibility Rules.
If (random_number < 0.1)
6) Apply Local Heuristic on Best Individual. }

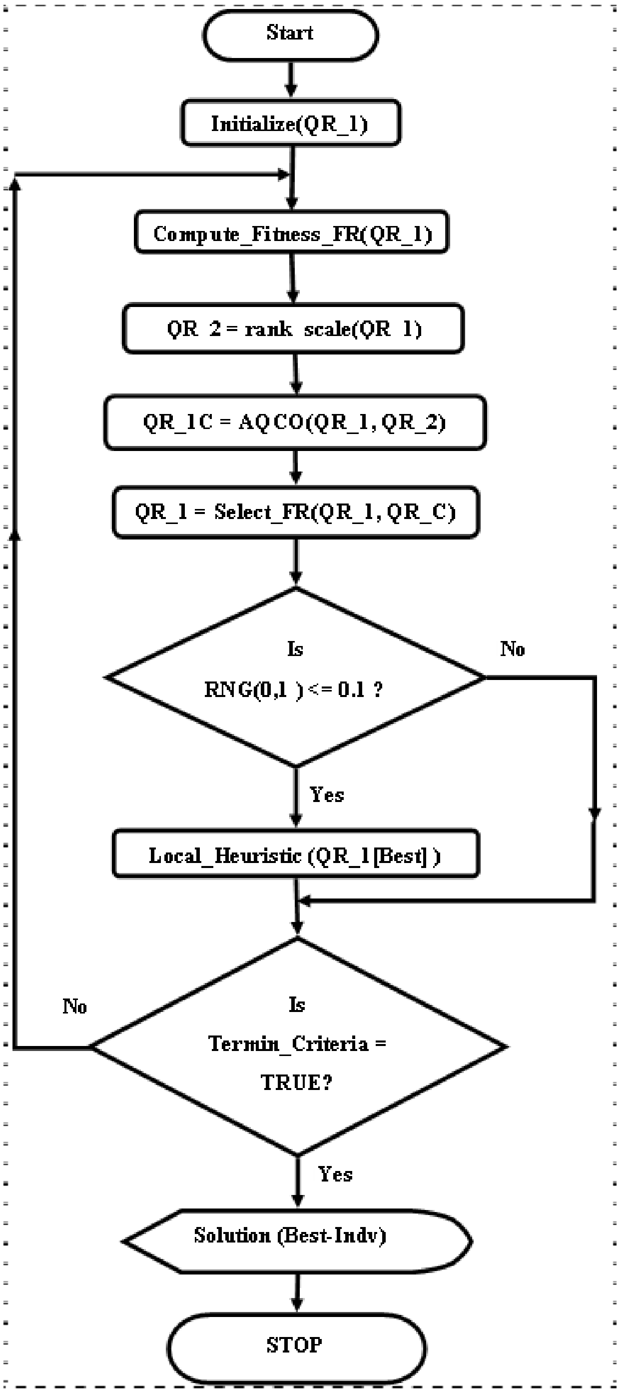

Figure 1. Flowchart of AQiEA

Description

1) Initialize the first set of qubits in the quantum register QR_1 randomly. QR_1 stores α`s corresponding to the solution vectors scaled between [0, 1] of the population.
2) Compute the fitness of each solution vector of QR_1 by using feasibility rules [14].
3) QR_2 stores the ranked and scaled objective function value of the ith solution vector in QR_1 as amplitude α_{2i} whose value [0, 1]. The fittest vector's objective function value is assigned one and the worst vector is assigned value zero. The rest of the solution vectors' objective function value is ranked and assigned in between the zero and one.
4) Adaptive Quantum Crossover is performed by using all the three strategies R-I, R-II and R-III on QR_1.
5) The solution vector for the next generation are selected by comparing individual parents with their best child and applying tournament selection i.e. the fitter one makes it to the next generation.
6) Local Heuristic is applied on the used with a probability of 0.1 on the best solution found so far. It is based on the gradient search with step size of 0.001.
7) Termination criterion is based on the maximum number of iteration, which in this implementation is 500.

V. Testing Methodology

The ceramic grinding optimization problem has been solved using the AQiEA for nine different combinations of constraints for both the continuous and mixed integer models. One hundred independent runs are performed for each combination of constraints using the AQiEA for both the models. AQiEA is implemented in 'C' programming language on an IBM Workstation with Pentium-IV 2.4 GHz processor, 2 GB RAM and Windows XP platform. The testing has been performed for determining the stability and efficiency of the proposed algorithm on both the models. The stability has been determined by analyzing statistically the quality of the solutions produced for each combination of the constraints in hundred independent runs and also by Post optimal Robustness Analysis. The efficiency has been determined by the number of function evaluations required to reach the optimum.

VI. Results and Discussions

Table I presents the median value of the results obtained from the experimental study of the implemented algorithm for all the combinations of the constraints in the continuous domain model. The results are promising as the AQiEA is able to reach the reported optimum in literature [5] in all the 100 runs, while meeting all the constraints in all the nine cases. This clearly shows that the proposed technique is robust and efficient for the optimization of the ceramic grinding process.

It is observed that the best quality would be obtained by grinding SiC at a depth cut of 5.607 µm and feed rate of 13.40 m/min with a grit size of 500.0000 mesh, which is the maximum permissible size of the mesh. It is also observed that for a higher number of permissible flaws and with an increase in roughness value from 0.3µm to 0.4µm, MRR could be improved by approx. 24% to 70%. Thus, a specified MRR can be achieved with constraints on the surface roughness and the surface damage. However, there are certain combinations in which no improvement in MRR is observed even if the quality is compromised by increasing surface roughness limit e.g. case 1, 4 and 7 or by increasing the number of flaws in case 5 and 8. For these combinations, sacrificing surface roughness alone does not improve MRR as the limit on the number of defects dominates the process as shown by Equation (1). Further, feed rate, fr, upper limit is reached more quickly that prevents MRR from improving at same level of number of defects.

AQiEA is also tested on the Mixed Integer model and Table 2 presents the results. The optimum values of MRR are same as in Continuous model and no constraints are violated in all the nine combinations. Thus, there is no degradation in performance of the algorithm due to change in the nature of the model.

Further, the results in Table I and Table II also indicate that a constant grit size of M = 500 mesh can be employed in all the nine combinations where as in the previous attempts several different sizes of grit have been employed like in [21] for GA all nine combinations has arrived at different grit size for each case. PSO proposed in [16] arrives at six different grit sizes i.e. 500 mesh for case 1, 2, 3 and 6, 258.2402 mesh for case 4, 430.7846 mesh for case 5, 238.4232 mesh for case 7, 295.0714 mesh for case 8 and 452.2157 mesh for case 9.

NM PSO [5] used seven different grit sizes for nine cases viz. of 481.0194 mesh for case 1, 500.0000 mesh for case 2, 3, 6, 387.2869 mesh for case 4, 493.2479 mesh for case 5, 436.5734 mesh for case 7, 388.5176 mesh for case 8 and 400.0047 mesh for case 9. It is well known that more the different types of meshes more would be procurement, operation and maintenance cost for same flexibility in the manufacturing. This further indicates that the proposed technique provides better results than the previous attempts [4], [5].

Table III presents statistical analysis of the efficiency of AQiEA on the Mixed Integer model. The efficiency of an stochastic algorithm is reflected as the number of function evaluations (NFE) required to reach optimum within fixed accuracy level of 0.0001, Feasibility Rate (the percentage of total number of independent runs in which AQiEA produced even a single feasible solutions) and Success Rate (the percentage of total number of independent runs in which AQiEA produced solutions near optimum within fixed level of accuracy).

The results in Table III show that AQiEA produces feasible solutions in all the runs of all the cases and reaches near optimum in all the runs of all the cases within 6200 NFEs, which is more efficient than any other known state of art technique [5], [16]. However, in certain runs the best individual able to reach the optimum even more quickly.

Further, convergence graphs have been included for cases where MRR reached saturation with respect to the number of defects. Figure 2 shows convergence graphs for median run of case 1, 4 and 7, which indicates that as the constraints on SR were relaxed, it became easier for the AQiEA to reach the optimum. However, the level of the optimum remained fixed with respect to ND.

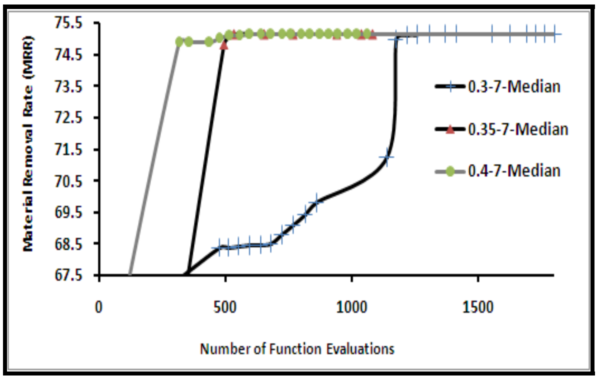

Figure 2. Convergence graph for Exp. case – 1, 4, and 7

Figure 3 shows the convergence graph for median run of case 5 and 8, which indicates similar findings. Figure 4 shows the convergence graph for median run of case 9, which indicates that AQiEA converges quickly if the constraints are relaxed. Same is true for the remaining cases also so they have been excluded.

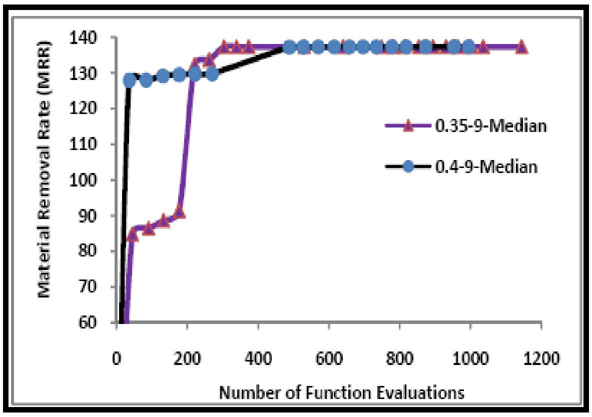

Figure 3. Convergence graph for case –5 and 8

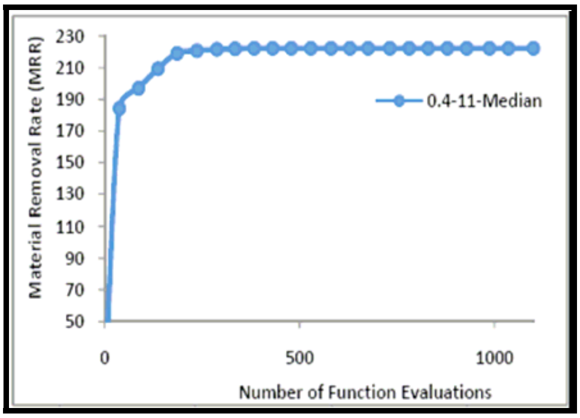

Figure 4. Convergence graph for case – 9

A. *Comparative Study*

A comparative study of the proposed AQiEA with the existing state of art algorithms like Genetic Algorithm (GA) [4], DE [17], PSO [16] and Nelder Mead Particle Swarm Optimization (NM-PSO) [5] has been performed. Table IV presents statistical comparison on the best, mean, worst values of MRR, its standard deviation and the number of function evaluations (NFE) obtained in the experiments with continuous model of ceramic grinding optimization problem.

The proposed algorithm beats GA proposed in [4] in terms of robustness of the solution quality and the efficiency also as it takes almost three times less number of function evaluations to reach the optimum in cases 1, 4, 5, 7 , 8 and 9 and almost two times faster in the remaining cases.

The proposed algorithm beats DE proposed in [17] in case 2, 3, 6 and 9 and matches the result in rest of the cases but is also around three times faster in cases 1, 4, 5, 7 , 8 and 9 and almost two times faster in the remaining cases.

The proposed algorithm beats PSO proposed in [16] in terms of robustness of the solution quality in cases 2, 3, 5, 6, 8 and 9 and matches the result in rest of the cases. However, it is also around three times faster in cases 1, 4, 5, 7, 8 and 9 and almost two times faster in the remaining cases.

The proposed algorithm matches the results obtained by the hybrid Nelder-Mead PSO (NM-PSO) proposed in [5] in all the cases except case 6 where it is better than NM-PSO when compared with respect to the mean and worst MRR. Further, it has better solution quality as the standard deviation of AQiEA is less than NM-PSO in all the nine cases. However, it is more than twice as fast as the NM-PSO since it requires less than half the number of function evaluations in all the cases except cases 2, 3 and 6. Further, NM-PSO [5] uses quasi inversion of matrix for repairing the infeasible solution, which is resource intensive. Hence, the actual efficiency of the AQiEA is far better than NM-PSO. Therefore, the AQiEA is now the best-known algorithm for solving ceramic grinding optimization problem.

AQiEA is more robust and efficient than other techniques for ceramic grinding optimization as it is able to have a better balance between exploration and exploitation in the search process due to adaptive selection of degree of rotation in the crossover operator. In case of other techniques, such parameters are generally fixed and dependant on the choice of the developer. It is well known that adaptive parameters perform better than static ones [23].

B. *Post-optimal Robustness Analysis*

Robustness analysis of the algorithm is proposed in [16] to investigate the stability of the algorithm, as it is a highly desirable feature. It is performed by investigating the variation of objective function with respect to change in a particular variable [22]. The method can only study the influence of change in one variable at a time. The equation given below has been derived in [16] to give an estimate of the robustness of the algorithm:

$$\|\Delta MRR/MRR\| \leq \|\Delta fr/fr\| + \|\Delta dc/dc\| + \|\varepsilon\| \qquad (6)$$

where Δfr is change in Feed Rate, Δdc is change in Depth of Cut, ε is the error due to truncation of higher order terms and ΔMRR is change in Material Removal Rate.

An algorithm is stable if the results produced under the variation of fr and dc satisfies equation (6).

The robustness of the AQiEA is also studied on the same three test cases as given in [16]:

Case 1: $5 \pm rn \leq dc \leq 30 \pm rn$
Case 2: $8.6 \pm rn \leq fr \leq 13.4 \pm rn$
Case 3: $5 \pm rn \leq dc \leq 30 \pm rn$ & $8.6 \pm rn \leq fr \leq 13.4 \pm rn$

where rn is a random number between 0 and 1.

Table V shows the result of post-optimal robustness analysis of AQiEA. There are no constraint violation in the result and no substantial change in the MRR due to small changes in fr and / or dc as shown in Table V. All the Cases satisfy the inequality given in Equation (6). Therefore, AQiEA is a stable optimization algorithm as per the post optimal robustness analysis mentioned in [16] for Ceramic Grinding Optimization Problem.

VII. CONCLUSIONS AND FUTURE WORK

The Ceramic Grinding Process Optimization Problem is important for successful commercialization of advance structural ceramics. The model for this process has been improved by formulating it as Mixed Integer nonlinear constrained optimization problem in which material removal rate is maximized subject to constraints on the surface roughness and the number of defects. Adaptive quantum inspired evolutionary algorithm has been designed and used for solving the problem. The algorithm uses two qubits representation instead of one and utilizes the quantum mechanical features of entanglement and superposition. It uses Feasibility Rules for constraint handling but still does not require any other technique like mutation for maintaining diversity. AQiEA successfully solved the problem. Further, it is shown to be more robust and efficient than the known state of art techniques like Genetic Algorithm, Differential Evolution, Particle Swarm Optimizers and Nelder Mead Particle Swarm Optimization Algorithm.

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TABLE I. RESULT OF CERAMIC GRINDING PROCESS MODELLED AS NONLINEAR CONTINUOUS CONSTRAINED OPTIMIZATION PROBLEM WITH AQIEA (MEDIAN)

Exp.	Constraints		Variables			Actual Value		Median MRR
	SRmax	*NDmax*	*fr*	*Dc*	*M*	*SRmax*	*NDmax*	
1	0.30	7	13.40	5.607	500.0000	0.2959	7	75.1342
2		9	12.20	8.488	500.0000	0.3000	9	103.5048
3		11	11.20	11.58	500.0000	0.3000	11	129.7162
4	0.35	7	13.40	5.607	500.0000	0.2959	7	75.1342
5		9	13.40	10.25	500.0000	0.3326	9	137.3293
6		11	12.89	15.35	500.0000	0.3500	11	197.8883
7	0.40	7	13.40	5.607	500.0000	0.2959	7	75.1342
8		9	13.40	10.25	500.0000	0.3326	9	137.3293
9		11	13.40	16.59	500.0000	0.3652	11	222.2834

TABLE II. RESULTS WITH MODEL AS NONLINEAR MIXED INTEGER CONSTRAINED OPTIMIZATION PROBLEM WITH AQIEA

Exp.	Constraints		Variables			Actual Value		Best MRR	Std. Dev. MRR
	SRmax	*NDmax*	*fr*	*Dc*	*M*	*SRmax*	*NDmax*		
1	0.30	7	13.40	5.607	500	0.2959	7	75.1342	1.14E-13
2		9	12.20	8.488	500	0.3000	9	103.5048	6.42E-06
3		11	11.20	11.58	500	0.3000	11	129.7162	1.39E-06
4	0.35	7	13.40	5.607	500	0.2959	7	75.1342	1.14E-13
5		9	13.40	10.25	500	0.3326	9	137.3293	2.00E-13
6		11	12.89	15.35	500	0.3500	11	197.8883	3.00E-05
7	0.40	7	13.40	5.607	500	0.2959	7	75.1342	1.14E-13
8		9	13.40	10.25	500	0.3326	9	137.3293	2.00E-13
9		11	13.40	16.59	500	0.3652	11	222.2834	5.14E-13

TABLE III. STATISTICAL ANALYSIS OF EFFICIENCY OF AQIEA ON MIXED INTEGER MODEL

Exp.	Constraints		Best NFE	Median NFE	Worst NFE	Mean NFE	Std. Dev.	Feasibility Rate (%)	Success Rate (%)
	SR (max)	*ND (max)*							
1	0.30	7	653	1839	5095	2230	1221	100	100
2		9	1348	3922	5795	3748	1098	100	100
3		11	1294	4467	6143	4268	1256	100	100
4	0.35	7	485	1098	1720	1140	296	100	100
5		9	506	1159	1788	1178	295	100	100
6		11	2106	4584	6161	4534	1121	100	100
7	0.40	7	549	1073	1689	1086	265	100	100
8		9	553	1009	1367	1000	232	100	100
9		11	411	1092	1603	1075	291	100	100

TABLE IV. COMPARISON OF AQIEA WITH GA, DE, PSO AND NM-PSO

Exp.	Constraints		Algorithm	Best MRR	Mean MRR	Worst MRR	Std. Dev.	NFE
	SR (max)	*ND (max)*						
1	0.30	7	GA [4]	75.1269	75.0220	74.6560	9.95E-02	10000
			DE [17]	**75.1342**	**75.1342**	**75.1342**	0	10000
			PSO [16]	**75.1342**	**75.1342**	**75.1342**	**7.18E-14**	10000
			NM- PSO [5]	**75.1342**	**75.1342**	**75.1342**	2.94E-10	6784
			AQiEA	**75.1342**	**75.1342**	**75.1342**	1.14E-13	**3153**
2		9	GA [4]	102.7890	101.6700	99.0790	8.84E-01	10000
			DE [17]	102.9204	102.9204	102.9204	7.19E-06	10000
			PSO [16]	102.9200	102.9200	102.9200	1.44E-14	10000
			NM- PSO [5]	**103.5048**	**103.5048**	**103.5048**	9.12E-06	6812
			AQiEA	**103.5048**	**103.5048**	**103.5048**	**6.42E-06**	**4151**
3		11	GA [4]	128.8610	127.3100	124.9300	1.00E-00	10000
			DE [17]	128.9838	128.9838	128.9838	1.08E-05	10000
			PSO [16]	128.9840	128.9840	128.9840	0.00	10000
			NM- PSO [5]	**129.7162**	**129.7162**	**129.7162**	1.00E-04	6804
			AQiEA	**129.7162**	**129.7162**	**129.7162**	**3.73E-06**	**5432**
4	0.35	7	GA [4]	75.1266	75.0510	74.8750	5.57E-01	10000
			DE [17]	**75.1342**	**75.1342**	**75.1342**	0	10000
			PSO [16]	**75.1342**	**75.1342**	**75.1342**	**7.18E-14**	10000
			NM- PSO [5]	**75.1342**	**75.1342**	**75.1342**	4.11E-10	6812
			AQiEA	**75.1342**	**75.1342**	**75.1342**	1.14E-13	**2305**
5		9	GA [4]	137.2880	136.9900	136.1200	2.96E-01	10000
			DE [17]	**137.3293**	**137.3293**	**137.3293**	4.60E-06	10000
			PSO [16]	137.3290	137.3290	137.3290	8.61E-14	10000
			NM- PSO [5]	**137.3293**	**137.3293**	**137.3293**	4.07E-08	6802
			AQiEA	**137.3293**	**137.3293**	**137.3293**	**2.00E-13**	**3042**
6		11	GA [4]	196.7130	195.4200	192.8300	9.25E-01	10000
			DE [17]	196.7708	196.7708	196.7708	6.85E-06	10000
			PSO [16]	196.7710	196.7710	196.7710	1.44E-13	10000
			NM- PSO [5]	**197.8883**	197.8879	197.8867	7.00E-04	6792
			AQiEA	**197.8883**	**197.8883**	**197.8881**	**3.68E-05**	**4002**
7	0.40	7	GA [4]	75.1336	75.0660	74.9490	5.05E-02	10000
			DE [17]	**75.1342**	**75.1342**	**75.1342**	0	10000
			PSO [16]	**75.1342**	**75.1342**	**75.1342**	**7.18E-14**	10000
			NM- PSO [5]	**75.1342**	**75.1342**	**75.1342**	2.73E-09	6804
			AQiEA	**75.1342**	**75.1342**	**75.1342**	1.14E-13	**2589**
8		9	GA [4]	137.3200	137.1000	136.2800	2.01E-01	10000
			DE [17]	**137.3293**	**137.3293**	**137.3293**	4.60E-06	10000
			PSO [16]	137.3190	137.3190	137.3190	8.61E-14	10000
			NM- PSO [5]	**137.3293**	**137.3293**	**137.3293**	2.33E-09	6798
			AQiEA	**137.3293**	**137.3293**	**137.3293**	**2.00E-13**	**3093**
9		11	GA [4]	222.2720	221.9000	220.60	3.14E-01	10000
			DE [17]	**222.2833**	**222.2833**	**222.2833**	1.01E-03	10000
			PSO [16]	222.2830	222.2830	222.2830	1.15E-13	10000
			NM- PSO [5]	**222.2834**	**222.2834**	**222.2834**	2.30E-08	6850
			AQiEA	**222.2834**	**222.2834**	**222.2834**	**5.14E-13**	**3901**

TABLE V. RESULT OF POST-OPTIMAL ROBUSTNESS ANALYSIS

Case	Constraints		Variables			Actual Value		Best MRR	‖ΔMRR / MRR‖	‖Δfr/f‖ + ‖Δdc/dc‖
	SRmax	***NDmax***	***fr***	***dc***	***M***	***SRmax***	***NDmax***			
1.1	0.3	7	13.4013	5.6081	500	0.29594	7	75.1552	2.80E-04	2.93E-04
1.2		9	12.1946	8.4878	500	0.30000	9	103.5048	3.05E-07	4.71E-04
1.3		11	11.1977	11.5842	500	0.30000	11	129.7162	2.53E-07	5.72E-04
2.1		7	13.4000	5.6070	500	0.29591	7	75.1342	3.53E-07	4.97E-06
2.2		9	12.1946	8.4878	500	0.30000	9	103.5048	3.05E-07	4.71E-04
2.3		11	11.1977	11.5842	500	0.30000	11	129.7162	2.53E-07	5.72E-04
3.1		7	13.4013	5.6081	500	0.29594	7	75.1552	2.80E-04	2.85E-04
3.2		9	12.1946	8.4878	500	0.30000	9	103.5048	3.05E-07	4.71E-04
3.3		11	11.1977	11.5842	500	0.30000	11	129.7162	2.53E-07	5.72E-04