Combining Signals by Weighted Least Square Error Method for Wireless Communication Networks

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***Abstract*—In this paper we presented weighted least square error method for signal combining for wireless communication in presence of unequal channels noise variance. The method has been commonly used for target tracking and in econometrics. System presented for signal combining by weighted least square error is a receiver with filters at each received branch for the error estimation (noise). The inverse of estimate of channels noise variance used as weights for signal combining to achieve wiener's solution. The presented scheme of signal combining is particularly useful when wireless communication subject to unequal noise variance which is very common in present wireless communication systems . The performance of our scheme is shown by computer simulation in Gaussian, Rayleigh flat fading and frequency selective wireless communication channel. The computer simulation performance of the system is about 10^{-3} bits at 8 dB SNR and about 10^{-4} at 16 dB SNR in wireless flat fading Raleigh channel with two receive antennas.**

***Keywords*- Signal combining, weighted least square error method, un weighted least square error, unequal noise variance**

I. INTRODUCTION

In wireless communication networks combining of receive signals at the destination node/base station or mobile phone with multiple antenna is of vital importance. The need of signal combining further increased with the development of next generation cooperative communication wireless networks, where each user in networks cooperate to transfer data. Many authors presented the equalization of channels and combining of the signals. Various techniques has been used in past literature for combining of received signals with multiple antennas, they are named as: selection diversity, equal gain combining, optimum combining[1][2][3], channel equalization and combining with decision feedback equalizer [4], maximum ratio combining (MRC)[5], adaptive combining is presented zero mean and unit variance[6][7][8][9] and adaptive combining for unequal noise variance is presented in our previous work [10]. The Wiener solution of signal combining for unequal and equal noise variances is described in [11][12].

In this paper we proposed to use the well known method of weighted least square for signal combining. The method has been previously used in estimation theory, for target tracking and in economics [13][14][15][16]. We presents the mathematical analysis and examined the performance by computer simulations. For mathematical analysis estimation theory been used and for computer simulation; ensemble average mean square error[17] and bit error rate (BER) [18] experiments been performed. From mathematical analysis and computer simulation it is observed that un weighted least square error method is equivalent to equal gain combining. And weighted least square combining scheme provide performance very close to optimum signal combining (Wiener Solution). With additional benefit of low computational complexity. Computer simulations shows that with the classical asumption of zero mean and unit variance, equal gain combining, adaptive combining with least square error algorithm and recursive least square algorithm and optimum combining provide equal performance and achieve wiener's solution of signal combining.

Communication mutiple antenna receivers are usually subject to unequal/different channels noise variance[10]. In such envirnoment the proposed scheme/method provide performance very close to optimum combining (Wiener solution). Where as, other combining schemes fail. To use weighted least square error combining scheme, we coupled an adaptive filter with each antenna element to estimate the noise variance of channel and then inverse of this estimate used as weights of combiner. However, one can also use other filters instead of adaptive filter for noise estimation.The performance of our system depend upon the accuracy of estimation of channels noise variance.

The contribution claimed in this paper are as follow:

- A signal combining method of weighted least square proposed for wireless communication networks.
- It is shown by mathematical analysis and simulation results shows that un weighted least square error is actually equal gain combining.
- The performance of weighted least square method achieve Wiener solution with unequal/different channels noise variance.

- Computer simulated Bit Error Rate (BER) performance presented for the system. It is about 10^{-3} bits at 8 dB SNR and about 10^{-4} at 16 dB SNR in wireless flat fading Rayleigh channels with two receive antennas with unequal/different channels noise variance.

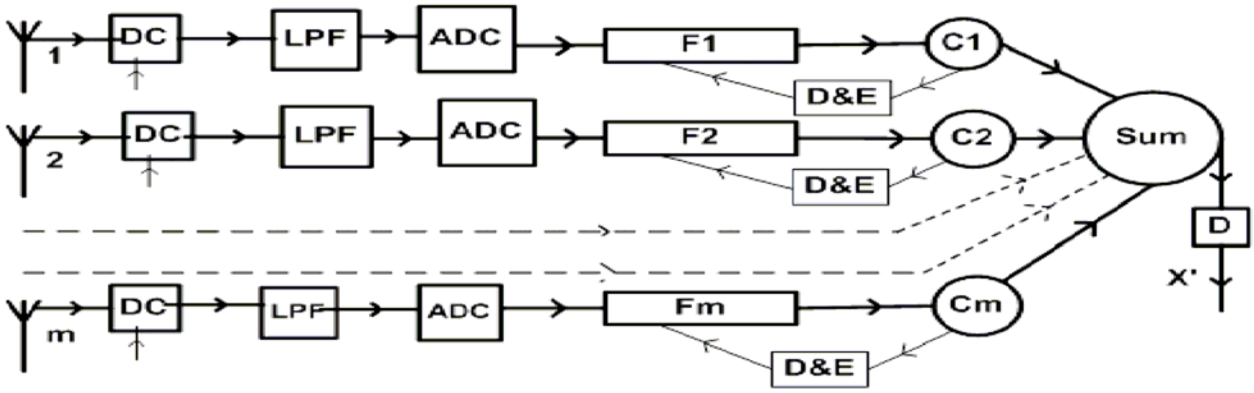

Figure: 1 System Model of Proposed System

The rest of the paper is divided into following sections; Section II describe the system model and optimum signal combining (Wiener). Section III presents classical MRC, section IV presents the proposed weighted least square method, section V describe performance comparison and numerical simulations and finally section VI is for conclusion.

II. SYSTEM MODEL AND OPTIMUM SIGNAL COMBINING (WIENER)

Fig. 1 represents baseband representation of the presented **m-**branch combiner. At given time 't', a signal **s(t)** is sent from the transmitter with multiple antenas.The channels $\mathbf{h_m}$ including the effects of the transmit chain, the air-link, and the receive chain may be modeled by a complex multiplicative distortion composed of a magnitude response and a phase response. The channels between the transmit antenna and the receive antennas are assumed Gaussian, flat fading Raleigh and frequency selective Raleigh with Gaussian noise $\mathbf{n_m}$ of variance $\boldsymbol{\sigma_m}$. For the Gaussian channels $\mathbf{h_m=1}$(Identity column matrix), since noise channel distortion is only due to Gaussian noise. Receive signals arrived from $\mathbf{m}^{th}$ antennas are frequency down converted by down converter(DC) fed to low pass filter (LPF) and digitilized by analogue to digital convertor (ADC) and further send to filters (F_m) for noise estimation. In our computer experiments we used these filters as adaptive for noise estimation, however any noise estimation filter can be used. The decision and error(DE) device provide interface between noise estimation filter and weight of combiner. The noise is the channels distortion noise, channels interference and additive Gaussian noise termed as error computed by usual training sequences. The inverse of channels noise variance are used as weights of the combiner, assumed to be remain constant for whole block of received data.The weight of combiner multiplied with each respective signal to combine singals accoring to signal to noise ratio. Let $\mathbf{r_m}$ **(t)** be the general form of $\mathbf{m}^{th}$ received analogue signal whose each symbol duration is T .Signals are digitalized by ADC and $\mathbf{r_m}$ **(n)** received digital output of respective symbol fed into the adaptive filters for equalization and noise estimation in time T_r . The estimate of symbols '**x′(n)**' after combining is given by

$$\mathbf{x'(n) = c^H(n)r_m(n)} \quad \mathbf{(1)}$$

where $\mathbf{c^H(n)}$ is m-dimentional complex value weight vector is given by

$$\mathbf{c_m (n) = [c_1 (n), c_2 (n).....c_m(n))]} \quad \mathbf{(a)}$$

The $\mathbf{r_m(n)}$ is received complex value vector is given by

$$\mathbf{r_m(n) = [r_1 (n), r_2 (n).....r_m(n))]} \quad \mathbf{(b)}$$

And it is given by following matrix equation

$$\mathbf{r_m(n)= h_m\, x(n)+ v_m (n)} \quad \mathbf{(2)}$$

Here channels are assumed to be time invariable, independent and identially distributed, therefore,

$$\mathbf{h_m = [h_1, h_2.....h_m)]} \quad \mathbf{(c)}$$
$$\mathbf{v_m (n) =[v_1 (n), v_2 (n).....v_m(n))]} \quad \mathbf{(d)}$$

The error (noise) **e(n)** between the reference signal and the output of filter is for the n^{th} symbol is

$$\mathbf{e(n) = (x(n).-.x'(n))} \quad \textbf{(3)}$$

Here **x(n)** is digital reference training sequence known at receiver filter. From equation **(1)** and equation **(2)**

$$\mathbf{e(n) = (x(n) - c^H(n) r_m (n))} \quad \textbf{(4)}$$

The mean square error (MSE) is given as

$$J = E[\mathbf{e(n)e^*(n)}] \quad \textbf{(5)}$$

Where '*E*'($\mathcal{E}$) represent expectation

From equation (4) and equation (5) we have

$$\mathbf{J} = E[\mathbf{(x(n) - c^H (n) r_m (n))(x(n) - c^H (n) r_m (n))^*}] \quad \textbf{(6)}$$

If **z** is the expectation M by 1 cross-correlation matrix vector between the received components and the reference sequence, and it is given by the expectation,

"$E[\mathbf{x(n) r_m^H (n)}] = \mathbf{z^H}$" and $[\mathbf{c_m (n)}]_{opt}$ optimal weight vector.

Then $$\mathbf{R[c_m (n)]_{opt} = z} \quad \textbf{(7)}$$

It is wiener equation or normal equation (optimum combining) for zero mean and unit noise variance described by J Winter in [1] and unequal noise variance presented in [12] by Ali H Saeed. The one possible solution of this equation is matrix inversion of **R**, mathematically

$$\mathbf{[c_m (n)]_{opt} = R^{-1} z} \quad \textbf{(8)}$$

The optimal weight matrix of $\mathbf{x'(n)}$, for received singnals $\mathbf{r_1}$ **(n)**, $\mathbf{r_2}$ **(n), ...**$\mathbf{r_m}$ **(n)** with unequal/different noise variances is given by $[\mathbf{c_m (n)}]_{opt}$. It depends upon the variances of noise of receive signals and **z**.

For two receive antenna (m = 2), **R** is given by

$$\mathbf{R} = \mathcal{E}[\mathbf{r}_m(n)\mathbf{r}_m^H(n)] = \begin{bmatrix} \mathcal{E}[\mathbf{r}_1(n)]^2 & \mathcal{E}[\mathbf{r}_1(n)\mathbf{r}_2^*(n)] \\ \mathcal{E}[\mathbf{r}_2(n)\mathbf{r}_1^*(n)] & \mathcal{E}[\mathbf{r}_2(n)]^2 \end{bmatrix} \quad (9)$$

III. MAXIMUM RATIO COMBINING (MRC)

The classical maximum ratio combining (MRC) [5] is been used for mathematical analysis and computer simulation performance comparison. The MRC is the most popular and commonly used scheme in present communication systems.

Consider again the two received signals $\mathbf{r_1}$ **(n)** and $\mathbf{r_2}$ **(n)**. The classical MRC is given by well known equation of MRC equalization.

$$\hat{x}(n) = \frac{\mathbf{h}_1^*(n)\mathbf{r}_1(n) + \mathbf{h}_2^*(n)\mathbf{r}_2(n)}{\mathbf{h}_1^*(n)\mathbf{h}_1(n) + \mathbf{h}_2^*(n)\mathbf{h}_2(n)} \quad \textbf{(10)}$$

In MRC equation the simplest situation could be Gaussian channel, when $\mathbf{h_1}$ and $\mathbf{h_2}$ is equal to unity and channel distortion is only due to Gaussian noise.

Then symbol estimate **x′** or '$\hat{x}(n)$' is

$$\mathbf{x'=0.5\ [\ r_1\ (n)]+0.5[\ r_2(n)]} \quad \textbf{(11)}$$

Above equation shows that the MRC equaly weight the received signal without considering the factor of different noise variance in $\mathbf{r_1}$ **(n)** and $\mathbf{r_2}$ **(n).** Actually MRC only combine the signal according to signal power. Where as, for optimum performance noise factor should be considered in the MRC formula. The symbol combining in Raleigh channel further degrade in perfornance of MRC as noise factor not consider accurately. In section V we presented the weighted least square method of singal combining. The presented/proposed system keep the noise factor in mathematical formula, therefore it provide better performance. Also in our previous presented adaptive combining algorithm [10] this noise factor has been considered.

IV. UN-WEIGHTED LEAST SQUARE ERROR METHOD OF

Least square error method is a concept of fitting curve to get best line, to get best solution. The error matrix $\mathbf{e_m(n)}$ between the reference signal and the received signal from the output of filter is for the m[th] symbol is in n time interval.

$$\mathbf{e_m(n) = (x_m (n) - r_m (n))} \quad \textbf{(12)}$$

Here $\mathbf{x_m(n)}$ is reference sequence (training) matrix with all entries equal to **x(n)**. To remove the effect of channel equalization performed at each branch of receiver. Consider we have m number of filters for noise estimation. We used adaptive filter for that purpose. The sum of errors of received signals at the receive antenna for un weighted least square error **E** is given by the following formula.

$$\mathbf{E\ |= e_m^T(n)\ e_m(n) = e_1^2(\ n)+\ e_2^2(n) \ldots .\ e_m^2(n)} \quad \textbf{(13)}$$

For the sufficeintly long training sequence the sum of errors of the signals not provide minimum error, because above equation equaly weighing all the error with unit noise variance. This formula is nothing but equal gain combining, that is equalvalent to unwieghted least square error combining. If we assume that noise is Gaussian with zeo mean and unit variance then above equation provide optimum perfornance of signal combining but in reality we cannot ignore the unequal non unity channels noise variance while implementing the system. More over the interference and channel distrotion cannot be ignored which contribute in channels noise. In reality to minimize error we have to weigh differently the signals according to the quality, specially when unequal channel noise variances are on receive branches. Therefore, weighted least square error method is needed.

V. WEIGHTED LEAST SQUARE ERROR METHOD OF SIGNAL COMBINING

As disscussed earlier that in practice received communication signals have different level of curruption due to noise, therefore, weighing equally the errors not provide least square error. To obtain least square error **E** by theory of weighted least square estimation we have to multiply each signal's square of error '$\mathbf{e_m^2}$' to its inverse of noise variance. The weighted least square estimation method equation **(13)** becomes

$$\mathbf{E = e_m^T(n)\, e_m(n) = C_1e_1^2(n)+ C_2e_2^2(n) .... C_me_m^2(n)} \quad \textbf{(14)}$$

Where $\mathbf{C_m}$ represent the respective weight to minimize the error. The value of $\mathbf{C_m}$ according to weighted least square estimation theory is inverse of noise variance of the respective channel. Hence, the proposed weighted least square equation for m branches of combiner receiver is given by

$$\mathbf{E \ |= 1/\square_1[e_1^2(n)]+ 1/\square_2[e_2(n)] .... 1/\square_m[e_m(n)]} \quad \textbf{(15)}$$

Here $\square_m$ represents the channel noise variance.
The above equation weigh the signals according to the level of error (noise) of signal.

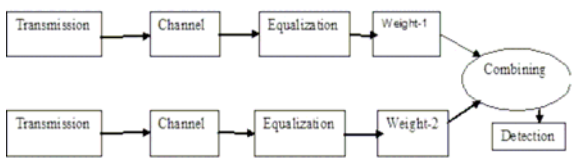

Figure 2 Computer Simulation Model

VI. PERFORMANCE COMPARISON AND NUMERICAL SIMULATIONS

A. Simulation Model

Simulation model is shown in Fig. 2. Our simulation are aimed at determining the error of two combined signals in Gaussian, flat fading Rayleigh, frequency selective fading Rayleigh channels environment with unequal noise variances. The bit error rate (BER) performance is computed from the difference of received combined signal and refernce training sequence. The following conditions exist in all simulations: a) Un-coded coherent BPSK modulation is used and we transmit power taken as variable b) Independent fading characteristics assumed on each channel when Raleigh flat fading channel is used and path losses are assumed to be negligible c) The training

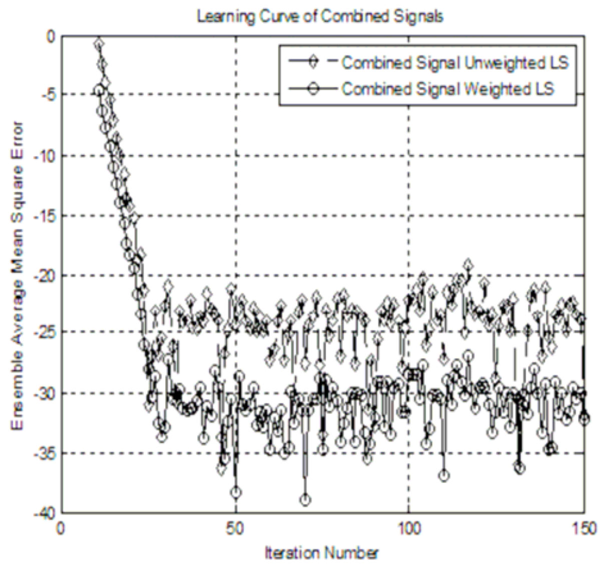

Figure 3 Average error preformance in Gaussian Channels
1. Un weighted least square error combining.
2. Weighted least square error combining.

sequences are generated independently using uniformly distributed pseudo-random number generators. e) Different level of noise variance '$\square_m$' taken on each channel. f) We used adaptive transversal finite impulse response filters on each branch for channel equalization and noise estimation. g) Least mean square algorithm used to estimate error by equalization and noise estimation. h) Signal to noise ratio normalized at each branch of combiner receiver.

In most of the computer simulation work for wireless communication system figure/graph presented between signal to noise ratio (SNR) and bit error rate. We used alternative approach which is not very common but used in [19]. In this approach, received power taken from 2dB to 20dB range on x-axis .

B. Results

In Fig.3 computer simulation clearly shows the difference of ensemble average mean square error curves for two methods, unweighted and weighted least square combining in Gaussian communication channels. We kept the unequal channel noise variance for the computer simulation. As expected from mathemtical theory, there is significant difference in error curves of unweighed (equal gain combining) and weighted least square error for siganl combining. This simulation experiment carried out to design the system for the combiner to measure further the bit error rate (BER) performance. In this simulation case the weighted least square error provide about 4dB gain in minimizing the error, because unweighted equally weight the signals with unit variance. Where as weighted least square having a multiplicative factor inverse of channel noise variance at each branch of combiner

Fig. 4 represent BER performance for two combining two signals in Gaussian channels with their different and unequal noise variances. In this specific example when 4dB power received at each branch of combiner

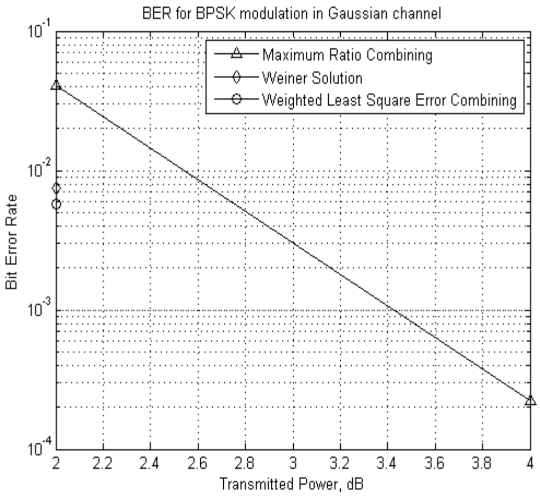

Figure 4 BER preformance in Gaussian Channels
1. Maximum ratio combining.
2. Wiener solution (optimum combining).
3. Proposed Weighted Least Square Error Combining.

receiver. The presented simulated BER performance is in Gaussian channel with different/unequal channel noise variance of 1.43 and 9.04 respectively and the signal received on one branch is 10log10(1.43) = 4.46dB and on the second branch the signal is 10log10 (9.04) = −3.5dB. We determined the performance of MRC, Optimum(Wiener) and weighted least square error combining for comparision in same channel conditions. The proposed scheme and optimum scheme (wiener) almost produce no error. Where as MRC scheme is unable to achieve optimum performance, due to channel noise. From the figure it is clear that proposed scheme provide no error for all transmit power from 2.2 dB to 4dB and it is expected that beyond 4dB we get consistent results. It is improtant to note that thhe MRC has linear performance improvement from 10^{-2} to 10^{-4} bits but still far behind from optimum.

In Fig.5 we simulated the BER performance in wireless Raleigh channesl[20] with different channel noise variance of 1.43 and 9.04. We found the performance of MRC, Optimum(Wiener) and weighted least square error combining. The proposed scheme and optimum scheme almost produce same performance. The presented scheme performance is only 0.4 dB less than that the performance of the optimum combining for all SNRs. Where as MRC method of signal combinig is 4dB to 6dB laging behind the optimum combining performance.

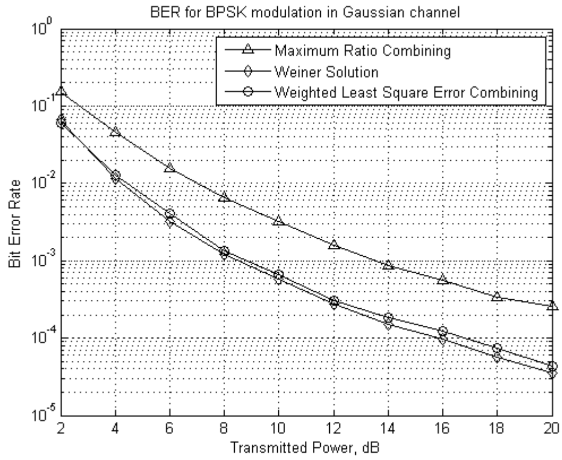

Figure 5 BER preformance in flat fading Rayleigh Channels.
1. Maximum ratio combining.
2. Wiener solution (optimum combining) .
3. Proposed Weighted Least Square Error.

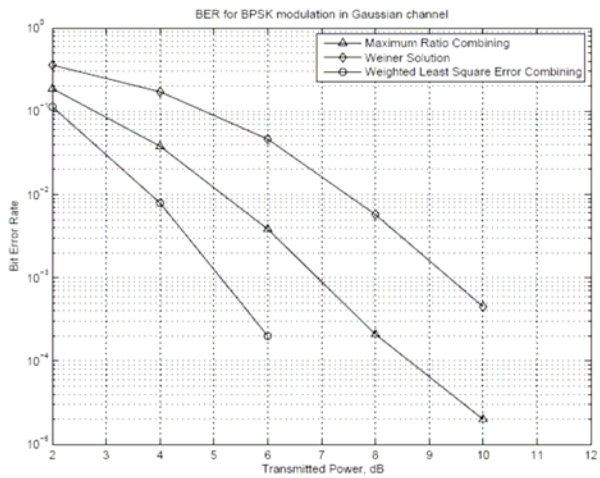

Figure 6 BER preformance in frequency selective Rayleigh Channels.
1. Maximum ratio combining.
2. Wiener solution (optimum combining).
3. Proposed Weighted Least Square Error Combining.

In Fig. 6 we computer simulation carried out for two received signals. In this experiment BER performance is meaured in frequency selective Raleigh channel with channel impairment (tap) on each branch **0.5** and **0.3** respectively. Results shows that even the optimum combining (wiener solution) failed to combine in such situation. It is again important to note that MRC provide inferior performance than proposed method. And at 6dB the proposed system provide almost zero BER.

VII. CONCLUSION

A method of signal combining by weighted least square is presented in this paper. The proposed method is linear and simplest among all other combining schemes, where only error (noise) estimation is required to obtain near optimum (wiener) performance of signal combining. The performance of presented method depend upon the accuracy of noise estimation. Our presented technique is computational simple and useful in practical communication combiner design, where receive antennas system of combiner subject to unequal channel noise variances. Where as un weighted least square error method of signal combining provide equivalent perfornance to equal gain combining when we assume channels noise with zero mean and unit variance.

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