Performance Analysis of Multi-Hop Decode and Forward Cooperative Communications under Rayleigh Fading Channel

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Abstract **—In this paper, we present the performance analysis of multi-hop decode-and-forward (DF) cooperative communications under Rayleigh fading channel. The source, multiple relays and destination nodes are equipped with single antenna transceiver while the transmission channels are assumed to be mutually independent. A time-division channel allocation scheme is occupied in order to realize orthogonal channelization, therefore no inter-relay interference is considered in the system model. The compact derivation form of the end-to-end signal to noise ratio (SNR) probability density function (pdf) is analyzed. We also derived the symbol error rate (SER) of M-ary phase shift keying (MPSK) and outage probability under slow flat Rayleigh fading. Moreover, the expression of spectral efficiency over multihop DF is derived. In this work, we study the probability of SNR gain of multi-hop DF system over the single-hop system. The numerical results show that decreasing the threshold SNR and increasing the number of hops (N) yields lower the outage probability of the system. It is obvious that the performance of the multi-hop system with the improvement of the average BER is proportional to the number of hop. Finally, probability of SNR gain can be achieved by increasing number of hops.**

Keywords – component, Decode-and-forward, cooperative communications, multi-hop, symbol error rate, outage probability, SNR gain

I. INTRODUCTION

Cooperative communications for wireless networks have gained much interest due to its ability to mitigate fading in wireless network through achieving spatial diversity. It is closely related with multiple-input multiple-output (MIMO) systems, which has been widely employed to achieve a diversity gain, provide higher quality of service, power savings, extended coverage area, and improve reliability in bit error rate (BER). However, in some scenarios the wireless terminals may not be able to support multiple transmit antenna due to the hardware limitations. For example, the user terminals can be too small for implementing an antenna array. So in this case, it is possible to enable single antenna mobile in a multi-user environment to share their antennas and generate a virtual multiple-antenna transmitter that allows them to achieve transmit diversity. Cooperative communication protocols make use of the broadcast nature of wireless channel, where a number of relay nodes are assigned to help a source in forwarding its information to its destination, hence forming a virtual antenna array.

II. LITERARURE REVIEW

In [1]-[3], Laneman et al., proposed several different cooperative protocols for realizing cooperative diversity, including amplify-and-forward (AF), decode-and-forward (DF), adaptive relaying protocol (ARP), etc. Many researchers focused on the channel capacity, diversity gain, and outage behavior in different scenarios. In [4] and [5], the symbol error rate (SER) performance for DF cooperation was analyzed under Rayleigh fading channel. In [6] and [7], the authors analyze the SER performance over Nakagami fading channels for both AF and DF protocols, respectively. However, they all considered the classical three node model.

The design and performance analysis of multi-hop DF relaying with single-input single-output (SISO) in each hop has well studied [8-10]. In [10], an upper bound for error probability of the multi-hop system was investigated and it also introduced a concept of multi-hop diversity. Also in [11], the authors analyzed error probability performance for multi-hop DF relaying over Rayleigh fading channel under M-ary Quadrature Amplitude Modulation (M-QAM).

In this paper, we present a performance analysis of multi-hop DF relaying system over Rayleigh fading channels. The main contribution of this work is the derivation of the compact form of pdf of the tightly approximated end-to-end SNR, which in then used to derive the closed form expressions for symbol error probability, bit error probability for MPSK. In addition, probability of SNR gain of the system is also considered. The approach employed in this paper offers a convenient and compact way to evaluate the system's performance such as probability of SNR gain and outage probability. The benefit achieved by using multi-hop relaying communications instead of the single hop communications is further investigated via the probability of SNR gain, which is defined as an average ratio of the end-to-end SNR of the multi-hop system to the SNR of the direct transmission.

The rest of this paper is organized as follows. In section III, the system model of cooperative communication with multi-hop DF is described. Then the system analysis including pdf of end-to-end SNR, symbol error rate, outage probability, and probability of SNR gain under Rayleigh fading is analyzed in section IV. Section V presents the results and discussion. Finally, we summarize our conclusion in section VI.

III. SYSTEM MODEL

We consider a wireless relay network consisting of one source, $N-1$ relays and one destination operating over Rayleigh fading channels. The source terminal (S) communicates with the destination (R) via N relay nodes denoted as $R_1,...,R_N$. Intermediate terminals always perform hard decisions on the received symbols before forwarding them to their respective successor node. We assume that every relay processes only the signals received from its preceding nodes, what allows for reducing the computational costs and hardware complexity at the receiver of each node. It is assumed that every channel between the nodes experiences slow, flat, Rayleigh fading. Due to Rayleigh fading, the channel power of each hop, denoted by $|h_j|^2$ is independent and exponential random variable whose mean is λ_j where h_j is the fading coefficient from the $(j-1)$ th node to the j th node where $j=1,...,N$. The average transmits powers for the source and the relays are denoted by ρ_j with $j=1,...,N$, respectively. Let us define $\gamma_j=\rho_j|h_j|^2$ as the instantaneous SNR for each hop. We further define the received SNR for the direct transmission as $\gamma_0=\rho_0|h_0|^2$ where ρ_0 and h_0 are the transmit power of the source in case of direct link and the fading coefficient of the channel between the source and the destination, respectively. It is assumed that the receivers at both the destination and relays have perfect channel state information (CSI). Finally, the system model is shown in Figure 1.

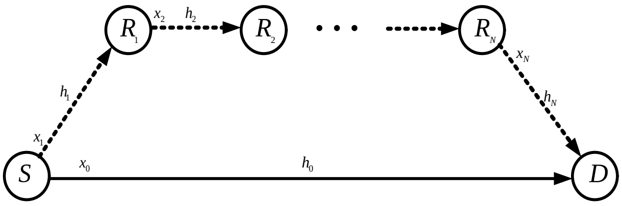

Figure 1. System model of multi-hop DF communications under Rayleigh fading channel.

IV. PERFORMANCE ANALYSIS

The performance analysis is organized into 2 part, non-cooperative and cooperative transmission.

A. Non-Cooperative Transmission

For non-cooperative transmission, source node transmit the information to the destination node, so the received signal is given by

$$y_{SD} = h_{SD}x_0+n_{SD}, \tag{1}$$

where $x_0 \in \left\{-\sqrt{2\mathcal{E}_s}, +\sqrt{2\mathcal{E}_s}\right\}$ for bit 0 and bit 1, respectively. $\mathcal{E}_s$ denotes the symbol energy, h_{SD} is the channel coefficient from S to D and assumes to be Gaussian random variable with zero mean and uniform phases distributed over $\left[0,2\pi\right]$. So h_{SD} is known as Rayleigh distribution. n_{SD} is an additive white Gaussian noise (AWGN) with zero mean and variance N_0, which can be noted as $n_{SD} \sim G\left(0,N_0\right)$. The average SNR of the S-D link is defined as

$$\overline{\gamma}_0 = \mathsf{E}[\gamma_0] = \frac{|h_0|^2\mathcal{E}_s}{N_0}, \tag{2}$$

where the pdf of γ_0 is given by

$$f_{\gamma_0}(\gamma) = \frac{1}{\overline{\gamma}}\exp\left(-\frac{\gamma}{\overline{\gamma}}\right), \tag{3}$$

with the expectation value λ_0. Using moment generating function, the average bit error rate for the non-cooperative transmission is given by

$$\overline{P}_b = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_\gamma\left[-\frac{1}{\sin^2\theta}\right]d\theta, \tag{4}$$

with $M_\gamma(s) = \dfrac{\lambda}{\lambda-s}$. After some manipulation, the BER of non-cooperative transmission is derived as follows:

$$\overline{P}_b = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{1}{1+\dfrac{\overline{\gamma}_0}{\sin^2\theta}}d\theta = \frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}_0}{1+\overline{\gamma}_0}}\right). \tag{5}$$

B. Cooperative Transmission

The following section shows the performance analysis of cooperative transmission.

1) Pdf of end-to-end SNR

Consider the j th hop and the received signal at node j can be written as

$$y_j = h_jx+n_j \quad ; j=1,...,N, \tag{6}$$

where h_j is a zero-mean complex Gaussian random variable with Rayleigh-distributed amplitude and a uniformly distributed phase angle, x is a complex baseband transmitted signal and n_j is a zero-mean complex Gaussian random variable representing the AWGN with variance N_0 which is the one-sided power spectral. The probability density function and probability distribution function (CDF) of γ_j can be defined as

$$f_{\gamma_j}(\gamma) = \frac{1}{\bar{\gamma}_j}\exp\left(-\frac{\gamma}{\bar{\gamma}_j}\right), \qquad (7)$$

where, $\bar{\gamma}_j = \mathrm{E}\left[\gamma_j\right] = \mathrm{E}\left[\rho_j \left|h_j\right|^2\right] = \rho_j\lambda_j$. Note that, owing to the imperfect detection at the relay, it may forward incorrectly decoded signals to the destination. Hence, similarly as in [12], the multi-hop DF channel can be modeled as an equivalent single hop whose output SNR γ_{eq} can be tightly approximated as

$$\gamma_{eq} = \min_{j=1,\dots,N}\gamma_j . \qquad (8)$$

Under the assumption that the hops are subject to independent but not necessarily identically distributed Rayleigh fading, order statistics give the CDF of γ_{eq} as

$$F_{\gamma_{eq}}(\gamma) = 1-\Pr\left[\gamma_1>\gamma,\dots,\gamma_N>\gamma\right]$$

$$= 1-\prod_{j=1}^{N}\left[1-F_{\gamma_j}(\gamma)\right]. \qquad (9)$$

By differentiating (9) yields the joint pdf of γ_{eq} for N hops, which is denoted as

$$f_{\gamma_{eq}}(\gamma) = \sum_{j=1}^{N} f_{\gamma_j}(\gamma)\prod_{\substack{i=1\\ i\neq j}}^{N}\left[1-F_{\gamma_i}(\gamma)\right]. \qquad (10)$$

Substituting (7) into (10), the pdf of γ_{eq} can be determined as follows:

$$f_{\gamma_{eq}}(\gamma) = \sum_{j=1}^{N}\frac{1}{\bar{\gamma}_j}\exp\left(-\frac{\gamma}{\bar{\gamma}_j}\right)\prod_{\substack{i=1\\ i\neq j}}^{N}\left[1-1-\exp\left(-\frac{\gamma}{\bar{\gamma}_i}\right)\right]$$

$$= \sum_{j=1}^{N}\frac{1}{\bar{\gamma}_j}\exp\left(-\gamma\sum_{j=1}^{N}\frac{1}{\bar{\gamma}_j}\right). \qquad (11)$$

Let $\chi = \dfrac{1}{\sum_{j=1}^{N}\bar{\gamma}_j}$, so equation (11) can be written as

$$f_{\gamma_{eq}}(\gamma) = \chi\exp(-\gamma\chi). \qquad (12)$$

2) Symbol Error Rate

In this paper, we consider the SER of MPSK modulation, where $M = 2^k$. In general, the SER of MPSK in the presence of AWGN channel is given by [13]

$$P_s\left(e\middle|\gamma_s\right) = 2Q\left(\sqrt{2\gamma_s}\sin\frac{\pi}{M}\right), \qquad (13)$$

where γ_s is known as the signal to noise ratio per symbol. So the exact expression can be shown as [13]

$$P_s\left(e\middle|\gamma_s\right) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\exp\left(-\frac{g\gamma_s}{\sin^2\theta}\right)d\theta , \qquad (14)$$

with $g = \sin^2\left(\dfrac{\pi}{M}\right)$. By averaging the SER for AWGN channel as describe by (14) over the pdf of the SNR, as shown in (12). Finally, the uncondional SER under slow flat Rayleigh fading channel can be derived as

$$\bar{P}_s = \int_0^{\infty}P_s\left(e\middle|\gamma\right)f_{\gamma_{eq}}(\gamma)d\gamma$$

$$= \int_0^{\infty}\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\exp\left(-\frac{g\gamma_s}{\sin^2\theta}\right)d\theta f_{\gamma_{eq}}(\gamma)d\gamma$$

$$= \frac{1}{\pi}\int_0^{\infty}\int_0^{\frac{(M-1)\pi}{M}}\exp\left(-\frac{g\gamma_s}{\sin^2\theta}\right)f_{\gamma_{eq}}(\gamma)d\gamma d\theta . \qquad (15)$$

Using moment generating function (MGF) [13], the MGF of γ_{eq} can be shown as

$$M_{\gamma_{eq}}(s) = \int_0^{\infty}\chi\exp(-\gamma\chi)\exp(s\gamma)d\gamma = \frac{\chi}{\chi-s}. \qquad (16)$$

Finally, the SER of MPSK modulation under Rayleigh slow flat fading is represented in term of MGF as

$$\overline{P}_s = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_{\gamma_{eq}}\left(-\frac{g}{\sin^2\theta}\right)d\theta$$

$$= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{\sin^2\theta}{\sin^2\theta+\frac{g}{\chi}}d\theta. \qquad (17)$$

Since [14],

$$\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{\sin^2\theta}{\sin^2\theta+c}d\theta = \left(\frac{M-1}{M}\right)\left[1-\sqrt{\frac{c}{1+c}}\,\frac{M}{(M-1)\pi}\tan^{-1}\left(\sqrt{\frac{1+c}{c}}\tan\frac{(M-1)\pi}{M}\right)\right]$$

$$= \left(\frac{M-1}{M}\right)\left[1-\sqrt{\frac{c}{1+c}}\,\frac{M}{(M-1)\pi}\left[\frac{\pi}{2}+\tan^{-1}\left(\sqrt{\frac{c}{1+c}}\cot\frac{\pi}{M}\right)\right]\right]. \qquad (18)$$

So, the SER in (17) is now written as

$$\overline{P}_s = \left(\frac{M-1}{M}\right)\left[1-\sqrt{\frac{\frac{g}{\chi}}{1+\frac{g}{\chi}}}\,\frac{M}{(M-1)\pi}\left[\frac{\pi}{2}+\tan^{-1}\left(\sqrt{\frac{\frac{g}{\chi}}{1+\frac{g}{\chi}}}\cot\frac{\pi}{M}\right)\right]\right]. \qquad (19)$$

With BPSK Modulation, by letting $M=2$ and $g=1$ the BER can be expressed as

$$\overline{P}_b = \frac{1}{2}\left[1-\sqrt{\frac{1}{\chi+1}}\right]. \qquad (20)$$

3) Outage probability

The outage probability P_{out} is defined as the probability that the output SNR γ_{eq} of the equivalent single hop falls below a certain predetermined threshold SNR γ_{th}. Hence, the outage probability of the system can be obtained by integrating the pdf of γ_{th} as

$$P_{out} = \Pr\left\{\gamma_{eq} < \gamma_{th}\right\}$$

$$= \int_0^{\gamma_{th}} f_{\gamma_{eq}}(\gamma)\,d\gamma$$

$$= 1-\exp\left(-\gamma_{th}\chi\right). \qquad (21)$$

4) Probability of SNR Gain

The objective of this topic is to study the probability of SNR gain of multi-hop DF systems over the single-hop system. The probability of SNR gain achieved by multi-hop DF systems over direct transmission is defined as [15]

$$\Omega = \Pr\left\{\frac{\gamma_{eq}}{\gamma_0} > \mu\right\}$$

$$= 1-\int_0^{\infty}\Pr\left\{\gamma_{eq} < \mu\gamma_0 \middle| \gamma_0=\gamma\right\} f_{\gamma_0}(\gamma)\,d\gamma$$

$$= 1-\int_0^{\infty} F_{\gamma_{eq}}(\mu\gamma)\, f_{\gamma_0}(\gamma)\,d\gamma. \qquad (22)$$

where μ is a pre-determined SNR gain. Since

$$F_{\gamma_{eq}}(\gamma) = \int_0^{\gamma} f_{\gamma_{eq}}(\gamma)\,d\gamma$$

$$= 1-\exp(-\gamma\chi). \qquad (23)$$

Substitute (23) in (22), the probability of SNR gain is shown as

$$\Omega = 1-\int_0^{\infty}\left[1-\exp\left(-\gamma\mu\chi\right)\right] f_{\gamma_0}(\gamma)\,d\gamma$$

$$= 1-\frac{1}{\overline{\gamma}_0}\int_0^{\infty}\left[\exp\left(-\frac{\gamma}{\overline{\gamma}_0}\right)-\exp\left(-\gamma\left(\mu\chi+\frac{1}{\overline{\gamma}_0}\right)\right)\right]d\gamma$$

$$= \frac{1}{\mu\chi\overline{\gamma}_0+1}. \qquad (24)$$

5) Spectral efficiency

The spectral efficiency is an important parameter. According to communication theory, spectral efficiency is defined as the channel capacity and evaluated form the average value of instantaneous SNR [14]

$$C = \mathrm{E}_{\gamma_{eq}}\left[\frac{1}{N}\log_2(1+\gamma_{eq})\right]$$

$$= \frac{1}{N}\,\frac{1}{\ln 2}\,\mathrm{E}_{\gamma_{eq}}\left[\ln(1+\gamma_{eq})\right]$$

$$= \frac{1}{N}\,\frac{\chi}{\ln 2}\int_0^{\infty}\ln(1+\gamma)\exp(-\gamma\chi)\,d\gamma$$

$$= \frac{1}{N}\,\frac{\chi}{\ln 2}\left[\frac{\ln(1+\gamma)}{\chi}\exp(-\gamma\chi)\Big|_0^{+\infty} + \frac{1}{\chi}\int_0^{\infty}\frac{\exp(-\gamma\chi)}{1+\gamma}\,d\gamma\right]$$

Using the changing of variable method, by letting $u = 1+\gamma$ then $du = d\gamma$. Finally

$$C = \frac{1}{N}\,\frac{1}{\ln 2}\int_0^{\infty}\frac{\exp(-\gamma\chi)}{1+\gamma}\,d\gamma$$

$$= \frac{1}{N}\,\frac{1}{\ln 2}\int_1^{\infty}\frac{\exp\left[-\chi(u-1)\right]}{u}\,du$$

$$= \frac{\exp(\chi)}{N\ln 2}\int_1^{\infty}\frac{\exp(-\chi u)}{u}\,du$$

Define the following variable $t = \chi u$ then $dt = \chi du$. The spectral efficiency is defined as

$$C = \frac{1}{N}\,\frac{1}{\ln 2}\,\frac{\exp(\chi)}{\chi}\int_1^{\infty}\frac{\exp(-\chi u)}{u}\,du$$

$$= \frac{\exp(\chi)}{N\ln 2}\,\mathrm{Ei}(\chi) \qquad (25)$$

where $\mathrm{Ei}(x) = \int_x^{\infty}\frac{\exp(-t)}{t}\,dt \;;\; x > 0$ is defined as exponential integral function (EIF).

V. NUMERICAL RESULTS

We consider a linear network consisting of multiple nodes and assume that the transmitted power of the sender nodes (source and relay) is equal to one watt. For a fair comparison to direct transmission, the overall distance of all hops is normalized to be one and uniform power allocation is employed in order to keep the total power constraints, i.e. $\{\rho_j\}_{j=0}^{N} = \frac{\rho_0}{K}$.

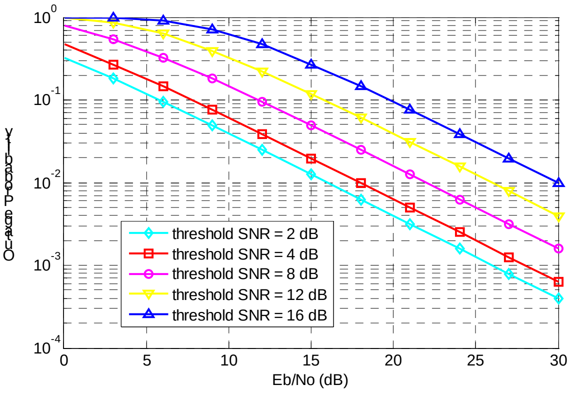

Figure 2. Outage Probability for Multi-hop Systems.

Figure 2 illustrates the outage probability of the multi-hop systems by letting constant number of hop. Herein, under the assumption that there is no latency processing in each node, i.e., decoding and then forwarding the received the signal to the next node. Let's consider in case of 3 relay nodes which is equal to 4 multi-hops at the different value of threshold SNRs, we observe that the lower threshold SNR the lower outage probability of the system.

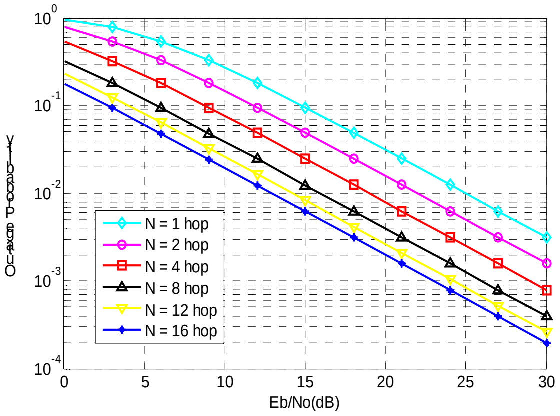

Figure 3. Outage Probability for Multi-hop Systems.

Figure 3 shows the outage probability of the multi-hop systems by letting constant threshold SNR. In this figure, threshold SNR is equal 5 dB and observes the outage probability at the different of hops. We notice that increasing the number of hops (N) reduces the outage probability of the system. As in this figure, at the outage probability 10^{-2} , with 4 hop systems use lower SNR as compare with 1 hop (non-cooperative) about 7 dB.

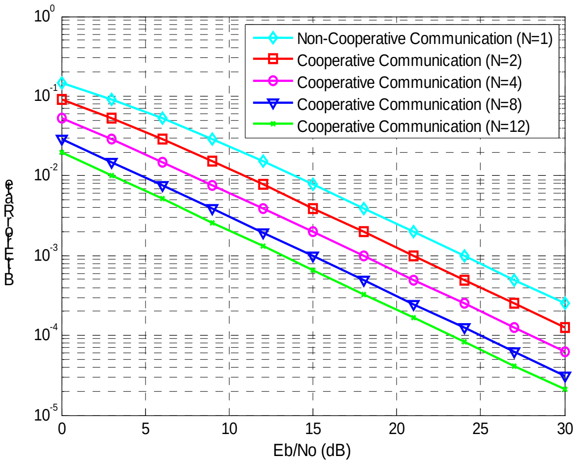

Figure 4. Bit Error Rate for BPSK modulation scheme of Multi-hop Systems.

Figure 4 shows the average bit error rate (BER) of the systems for BPSK modulation scheme as functions of the average signal to noise ratio. In addition, we compare the performance of the dual hop as for the cooperative transmission with the non-cooperative transmission (direct transmission). We notice that at BER of 10^{-3} , dual hop requires lower SNR about 2.5 dB as compare with non-cooperative communication. For all values of the SNRs, the performance of cooperative communication yields better bit error probability. It is obvious that the performance of the multi-hop system with the improvement of the average BER is proportional to the number of hops (N).

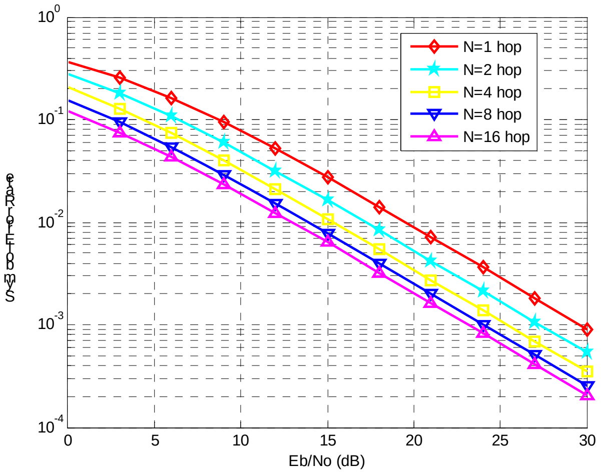

Figure 5. Symbol Error Rate for QPSK modulation scheme of Multi-hop Systems

In figure 5 and 6, we consider the performance of the system in term of symbol error rate as function of average signal to noise ratio per bit. In addition, we compare the performance of multi-hop decode and forward system with direct transmission. For all values of SNRs, the performance of the proposed system is better than the other (direct transmission and dual hop systems). It is obvious that the performance of the multi-hop systems with the improvement of the average SER or average BER is proportional to the number of hop (N). For example, at the average SER 10^{-3} , multi-hop decode and forward (with 4 hops) outperforms direct transmission and dual hop with transmit power gain about 4.8 and 2 dB, respectively.

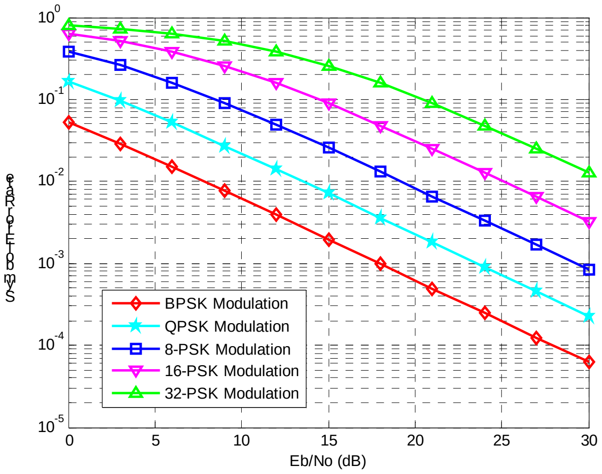

Figure 6. Symbol Error Rate for MPSK modulation scheme of Multi-hop Systems.

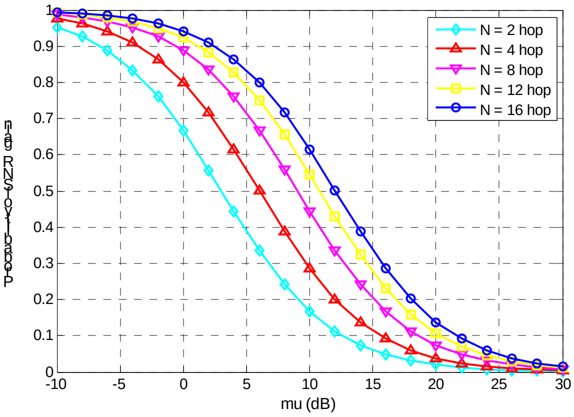

Figure 7. Probability for SNR gain for Multi-hop systems.

In figure 7, we present the probability of the SNR gain achievable by cooperative communication environments. We can see that the probability of SNR gain benefits by increasing number of hops. The spectral efficiency of multihop decode and forward cooperative communications is presented in Figure 8. We notice that increasing number of hops yielding the lower channel capacity or less spectral efficiency. This is the disadvantage of cooperative communications. For example, at 15 dB, the spectral efficiency with dual hop (N=2) is better than N=4 about 1.075 bit/s/Hz.

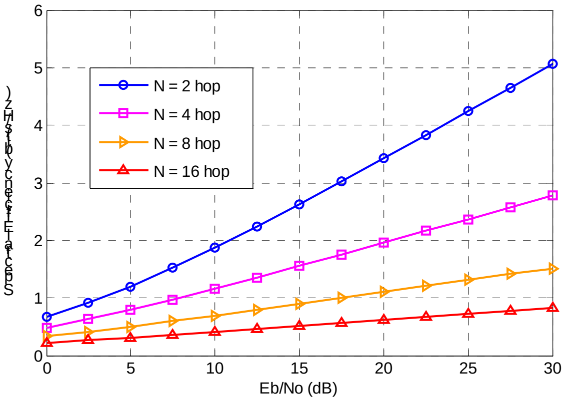

Figure 8. Spectral efficiency for cooperative communication.

VI. CONCLUSION

In this paper, we analyzed the close form expressions of the symbol error rate for M-ary phase shift keying and bit error rate of the multi-hop system with decode and forward over Rayleigh fading channel. Moreover, the outage probability and probability of SNR gain of the system are also investigated. We observe that decreasing the threshold SNR and increasing the number of hops (N) reduces the outage probability of the system. It is obvious that the performance of the multi-hop system with the improvement of the average BER is proportional to the number of hop (N). Finally, probability of SNR gain can be achieved by increasing number of hops. We also notice that the disadvantage of cooperative communication is in defined in term of spectral efficiency. In real situation, we should be selected the mode of transmission according to the following parameters, outage probability, bit error rate, symbol error rate, probability of SNR gain, and spectral efficiency.

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