Modeling of Epilepsy EEG Signal using Prony's Method

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***Abstract* - Prony 's method is employed in spectral domain estimation to model epilepsy seizure from human's electroencephalograph (EEG).This method assumes that the original signal is a sum of damped complex exponential sinusoids. The method has the best frequency resolution compared to Auto Regression (AR) parametric methods. Modeling of the EEG Epilepsy signal is based on the poles of the signal. Computing the poles of the Prony's method, the signal Modeling is based on the location of the poles and number of poles. As we increase number of poles we can minimize the error due to signal reconstruction. The optimum number of poles is calculated to get the minimum error.**

Keywords - Electroencephalograph EEG, epilepsy, feature extraction, Prony's method, poles

I. INTRODUCTION

Epilepsy is a common brain disorder that affects about 1% of the population in the United States and is characterized by abnormal firing of neurons in the brain which may lead to recurrent and spontaneous seizures (with no apparent external cause or trigger). Approximately 30% of the epileptic population is not helped by medications [1]. Seizures can be categorized as general or partial depending on established conventions as explained below.

Generalized seizures occur due to simultaneous abnormal activity in multiple parts of both brain hemispheres from the beginning leading loss of consciousness. Partial (or focal) seizures are more common and initiate in one part of the brain, often leading to strange sensations, motor behavior, and even loss of memory. These seizures are further subdivided based on the part of the brain that contains the epileptogenic focus which determines the exact symptoms. Partial seizures can sometimes spread from the focus to other parts of the brain, leading to secondary generalized seizures.

Different parts of the brain are implicated in the generation of different types of seizures associated with various types of epilepsy. Partial seizures are attributed to localized disturbances in various areas of the brain. Hence, there is specific area of the brain that could generate all types of epileptic seizure[2].

However, in almost 33% of all epilepsy patients with partial seizures, the epileptogenic focus is located in the temporal lobe. This condition is termed temporal lobe epilepsy. TLE seizures are of primary clinical importance due to the frequency of occurrence and difficulty of diagnosis and treatment. TLE can be further categorized as medial or neocortical based on the location of the focus inside the temporal lobe. Epilepsy is often described as a group of disorders with many types, subtypes, and cross-classifications.

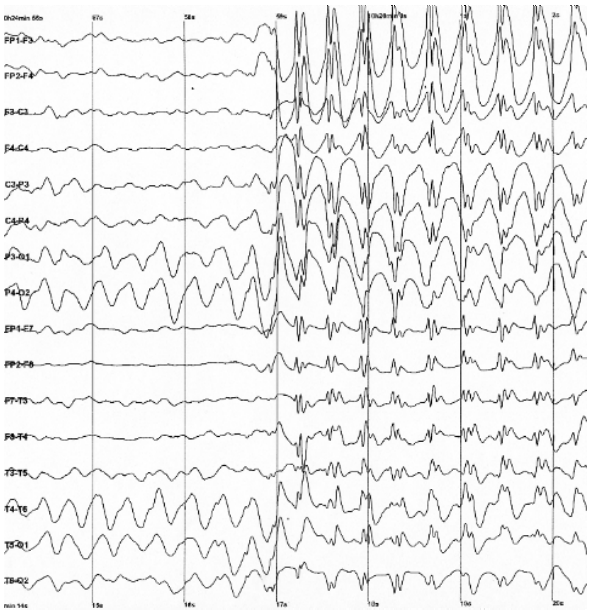

Fig. 1 EEG signals normal and abnormal Epilepsy.

Abnormal states, primarily observed in neurological disorders like seizures in epilepsy Figure (1). Most current research focuses on publicly available databases ,which are briefly described from MIT-BIT[3], This database, collected at the Children's Hospital Boston, consists of EEG recordings from pediatric subjects with intractable seizures. Subjects were monitored for up to several days following withdrawal of anti-seizure medication in order to characterize their seizures and assess their candidacy for surgical intervention. Recordings, grouped into 22 cases, were collected from 22 subjects (5 males, ages 3-22; and 17 females, ages 1.5-19) [3]. A survey on Epilepsy Seizure EEG signal modelling is available [5-9]. AR model is the most common technique used for EEG Modelling due to the feature extracted can easily be used to identify the epilepsy signal based on its poles and zeros.

II. PRONY METHOD

Prony's method is a technique for modelling sampled data $f(t_i)$ at D data points and is equated to a linear combination of exponential functions [4]. Thus, the signal can be represented as:

Prony introduced a technique for modelling sampled data as a linear combination of damped exponentials .This method has been applied to various areas, notably electromagnetic scattering, antenna problems signal processing, and radar target identification.

Starting with the basic derivation of Prony's method it is desired to determine an approximation of the form,

$$f(t_i) \cong \sum_{\alpha=1}^{P} R_\alpha \exp(s_\alpha t_i), \qquad \alpha = 1,2,3,...,P, \qquad i = 0,1,2,3,...,D-1, \tag{1}$$

Where $f(t_i)$ is the EEG signal defined at D sampling points $t_0, t_1, t_2, ..., t_{D-1}$, s_α is the αth pole, R_α is the αth residue's amplitude.

It is useful to express the equation (1) in discrete sampled data form as normally found in practice, thus,

$$f(t_i) = \sum_{\alpha=1}^{P} R_\alpha \exp(s_\alpha i \delta t) = \sum_{\alpha=1}^{P} R_\alpha (X_\alpha)^i \tag{2}$$

$$\alpha = 1,2,3,...,P. \qquad i = 0,1,2,3,...,D-1.$$

Where,

$\alpha = 1,2,3....P,$

$i = 0,1,2,...D-1$

s_α : *Complex pole*

R : Modal Amplitude

$\delta t = sampling\ interval$

Rs : Residual of the poles

$X_\alpha = \exp(s_\alpha \delta t)$, and the size of the sampling interval is defined as δt.

The above set of nonlinear equations (2) have both two sets of unknowns X_α's and R_α's.

If the constants X_α's were known, this set would comprise D linear equations in the P unknowns R_α's and could be solved exactly if $D = P$ or approximately, by using least square method if $D > P$. However, if the X_α's are to be determined, at least $2P$ equations are needed.

Using Prony's method procedure, one can define a polynomial $A(M)$ of order P in the variable M, having the same α roots appearing in equations (1) to (2), thus,

$$A(M) = a_0 + a_1 M + a_2 M^2 + ... + a_P M^P \tag{3}$$

Equation (3) can be written in terms of its roots as,

$$A(M) = (M - X_1)(M - X_2)...(M - X_P) = 0 \tag{4}$$

Where $X's$ are the roots of the above equation .

In order to determine the coefficients $a_0, a_1, a_2, ..., a_P$ in equation (3), the first equation in (2) will be multiplied by a_0, the second equation by a_1 , .. , the Pth equation by a_P, the result will give the following set of equations,

$$\left.\begin{aligned} a_0 f_0 &= a_0 R_1 + a_0 R_2 + ... + a_0 R_P \\ a_1 f_1 &= a_1 R_1 X_1 + a_1 R_2 X_2 + ... + a_1 R_P X_P \\ a_2 f_2 &= a_2 R_1 (X_1)^2 + a_2 R_2 (X_2)^2 + ... + a_2 R_P (X_P)^2 \\ &\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ a_P f_P &= a_P R_1 (X_1)^P + a_P R_2 (X_2)^P + ... + a_P R_P (X_P)^P \end{aligned}\right\} \tag{5}$$

Adding the above set of equations (5), yields,

$$A(X_1) + A(X_2) ... + A(X_P) = f_0 a_0 + f_1 a_1 ... + f_P a_P \tag{6}$$

Where $A(X_\alpha)$ is defined in equation (4), X_α is the roots of A, thus, equation (6) yields,

$$f_0 a_0 + f_1 a_1 + ... + f_P a_P = 0 \tag{7}$$

A set of $D - P - 1$ additional equation can be obtained similar in a way by repeating the steps explained above. Starting from f_1 to $f_{(D-P-1)}$, giving the following set of equations,

$$\left.\begin{aligned} & f_0 a_0 + f_1 a_1 + ... + f_P a_P = 0 \\ & f_1 a_0 + f_2 a_1 + ... + f_{P+1} a_P = 0 \\ & \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ & f_{D-P-1} a_0 + f_{D-P} a_1 + ... + f_{D-1} a_P = 0 \end{aligned}\right\} \tag{8}$$

Since the ordinates f_i are known, and by taking $a_p = 1$ (linear predictor constraint), equation (8) generally can be solved directly for the a if $D = 2P$, or solved approximately by using the least square method if $D > 2P$.

After computing $a's$ coefficients, the $x's$ can be calculated as the root of equation (3). Equation (2) then becomes a set of linear equations in R. Thus, R can be founded from the first P equations (2) or by applying the least square techniques to the entire set.

The poles (s_α) of the EEG signal can be directly calculated as,

$$s_\alpha = \frac{1}{\delta}\log\left(x_\alpha\right) \qquad (9)$$

III. EXPERIMENTAL RESULTS AND DISCUSSIONS

The EEG signal was obtained from the MIT-BIH \ Database [3]. The data set used for this work was composed of 23 epilepsy cases. The epilepsy signals were sampled at 250 sample/sec.

Applying Prony's method to epilepsy yields to reliable results only if the following considerations are employed,

Optimal order of polynomial is chosen in equation (1) result mean square error $4 * .10^{-8}$ .It is possible to increase the order of the reconstructed signal polynomial up to 30 to get minimum error very close to zero . Power spectral Figure (2) and Figure (3) show the difference between the epilepsy and normal signals.

If we compare this new method to the method employed by Min Han [5] ,we find that Prony's method is faster compared to the AR model .The modelling is based on poles location and binary classifier could be used instead of RVM at Min Han[5].Further investigation on classification methods is expected in future work .

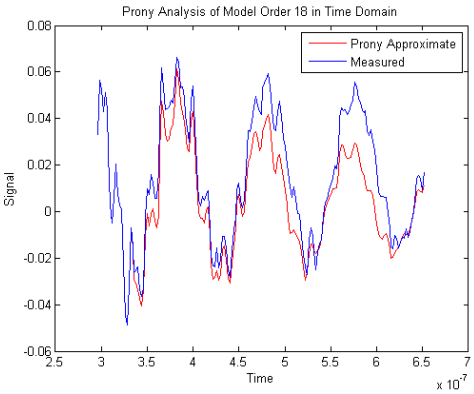

Fig. 2 Error due to signal reconstruction.

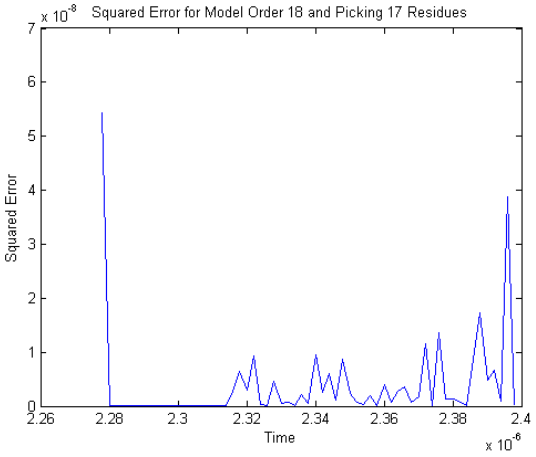

Fig.3 Error due to signal reconstruction.

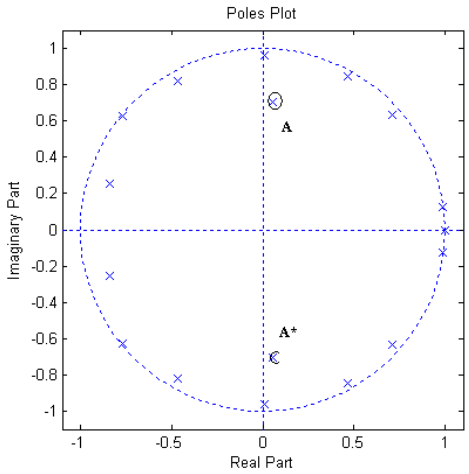

Fig. 4 Poles location of the Normal Patient

It can be seen from figure (4) that two conjugate poles (A, A*) disappear in figure (5) . Also it can be seen from figure (5) that a new pole D appears on the real axis and another two conjugate pairs (B , C) and (B* , C*) appears on the second and the third quadrant. Further investigation on different cases is expected in future work for more enhancement classification problem.

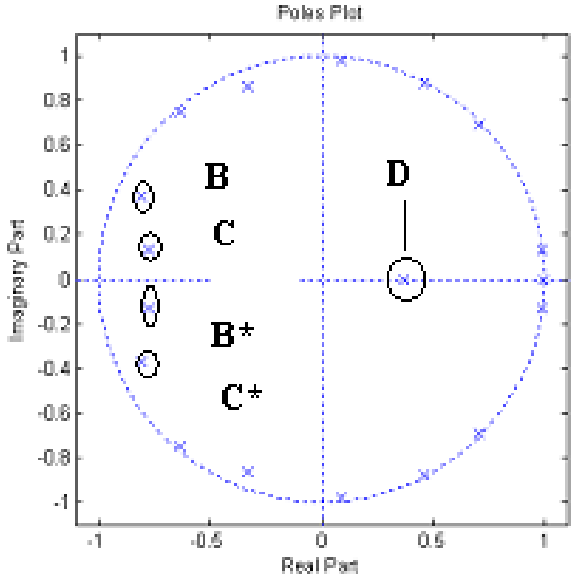

Fig.5 Poles location of the Abnormal Patient

V. CONCLUSION

Prony's method is used to calculate the poles of the EEG signal .Poles of the epilepsy seizure signal is used to model this disorder. New poles appear in the case of epilepsy while other poles disappear compared to normal EEG signal.

The signal model is based on the location of the poles and number of poles. As we increase number of poles we can minimize the error due to signal reconstruction and get faster identification method compared to the AR method.

The result mean square error $4 *.10^{-8}$ .It is possible to increase the order of the reconstructed signal polynomial up to 30 to get minimum error very close to zero.

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