2014 Fifth International Conference on Intelligent Systems, Modelling and Simulation

Color Image Representation Using Multivector

Somasis Roy
Dept. of Computer Science
University of Calcutta

Anirban Mitra
Dept. of Computer Science
Academy of Technology, WBUT

Sanjit Kumar Setua
Dept. of Computer Science
University of Calcutta

***Abstract* – This paper discusses about Clifford Algebra and its significance in a color image representation. A color image following RGB color model is described by a multivector in 3D space. The color blades in the multivector define the colors. Different shades of color can be obtained by applying rotor operator on those blades. Clifford color space is introduced to define each color against a pixel of an image in form of a color blade. A multivector function is derived from this space for explaining the color image. In simpler way, it is proposed that a color image is stored in the Clifford color space. This paper is also useful in the reduction of computation time for color image processing as every Grade – k vectors are colors.**

Keywords* – *Clifford algebra, Wedge, Multivector, Color blade, Grade, Color vector, Clifford color space

I. INTRODUCTION

An image is defined as the finite set of spatial coordinates with attributes. It is described by the geometric shapes and their relations using the coordinates. Each coordinate is represented $f(x,y)$ by a two-dimensional function at (x,y), where the function returns a value known as the intensity of the coordinate. Each coordinate holds data of an image in various forms such as binary, gray-scale or color form. A digital image which is described by color is said to be a color image. It holds data in color form, and contains the highest level of information compare to other forms [1]. A color model or a color representation system is needed to analyze the data present in color image.

There are several existing models widely used in Computer Graphics like RGB model, HSI model, CMY model etc. Every distinct color model has the distinguished nature to define a pixel of an image. The most convenient color model used in digital image processing is the RGB color model. It is the additive color model whereas CMY (Cyan Magenta Yellow) is the subtractive color model. HSI (Hue-Saturation-Intensity) model provides best color description for human interpretation. In RGB model each pixel is represented by three values, the amount of Red, Green and Blue. It is spanned over three dimensional Euclidean space and every coordinate of this model defines a color. A color in this color model is expressed as a vector instead of coordinate system. In case of a gray-scale image a pixel holds a scalar (gray) value derived from the RGB vector. Mathematically, a pixel of an image holds data in form of a vector or scalar to make coordinate free. A vector holds higher amount of actual data compare to scalar that's why a RGB color image will use three times as much memory as a gray-scale image of the same pixel dimensions. The main objective is representation of an image as independent of coordinate system.

There are several mathematical tools independent of coordinate system to represent an image. By the traditional approach of linear algebra, the concept of homogeneous coordinate is successful for developing computer graphics packages [2]. The dissimilarity between matrix computation and geometric interpretation leads a problem in this approach. As mentioned earlier the safest and efficient way to get rid of coordinate system is the adoption of vector. A color image is defined as the summation of vectors against each pixel exists in that image [3]. It is observed that higher grade of vector is also responsible to define a color of a pixel in an image. In that case, Quaternion algebra [4] and Clifford algebra shows their power as a mathematical tool in the development of computer graphics. Both of these algebra is different from traditional linear algebra and both carefully handle the subspaces of any dimensions. A subspace may act as a point (scalar), line (vector), plane (bivector), volume (trivector) etc. It is developed in different vector space for representing a certain geometric object. Clifford algebra provides a strong framework to describe a color in RGB model by using subspaces [5].

William K. Clifford (1845-1879) introduced Clifford Algebra, also called Geometric Algebra where geometric product is the combination of the dot product and the outer product. Clifford's motivation was to combine Grassman's "associativity" and Hamilton's "anticommutativity" features into a single matter. Clifford also united the several advantages of quaternion algebra by introducing the subspaces. Clifford Algebra is used as a powerful mathematical tool in areas as computer vision, computer graphics, robotics, etc. As it is a coordinate-free tool it is easier to model geometric objects and their several transformations.

There are many color spaces by which an image is represented but RGB color space is considered here. Geometrically, this RGB space is represented by a cube where each of the axis representing unique color components [4]. According to pure quaternion every pixel in a color image contains three components therefore the function is defined as:

$$f(x,y) = r(x,y)i + g(x,y)j + b(x,y)k$$

2166-0662/14 $31.00 © 2014 IEEE
DOI 10.1109/ISMS.2014.66

where i, j and k are the imaginary part of the quaternion [6]. The imaginary parts are actually mathematical representation of bivector. In Clifford algebra for n - dimension Euclidean space 2^n number of subspace are formed. The function for the color of a pixel $f(x,y) = r(x,y)e_1 + g(x,y)e_2 + b(x,y)e_3$ in three dimensional Euclidean space $\mathbb{R}^3$ where e_1,e_2 and e_3 are the unit vectors. As the function shows each of the pixel is represented by a vector, therefore to represent an image $M \times N$ number of vectors are needed, where M and N are the dimensions of the image. The wedge operator of Clifford Algebra has a significant role in handling subspaces. Each of these subspaces is defined as a wedge product or outer product. According to Clifford algebra every color is a subspace in RGB vector space. The motivation of this paper is to define each of the color in RGB space by appropriate subspaces and representation of color image in form of a multivector, combination of subspaces.

This paper gives an idea to represent an image in form of a multivector by using Clifford algebra. The rest of the paper is organized as follows. Section II discusses about the Clifford Algebra applications as the related works. Clifford Algebra is introduced in section III. In section IV, the geometrical transformations like reflection and rotation is discussed. Section V explores about image representation by homogenous vector. In section VI we discuss about the proposal, how to represent a color image by using multivector. Finally section VII contains conclusions and future scope from this work.

II. RELATED WORK

Clifford Algebra is applied in Computer Science to show new horizons in research and applications. It is a coordinate free algebra providing efficient framework to those applications. Image processing is one of the applied areas of Clifford Algebra. Color image representation and edge detection are the most inevitable part of color image processing. Several measures and approaches were proposed for the improvement of both the cases. This algebra authorizes number of operators to represent a color image effectively. Generally, image representation using RGB color space increases the dimensionality, non linearity, redundancy and reduces the efficiency of color image processing [1]. Clifford algebra resolves these constraints while considering RGB color space by introducing vector and multivector. It has invoked the vector concept to describe a color [7]. The colors in three dimensional RGB color space are manipulated and the color components are the vector parts in the multivector [3]. It solves real life problems such as removal of facial make up disturbances where a color is a multivector [8]. Clifford Algebra is used to define different color spaces and act as a natural unified language to solve multicolor image processing and pattern recognition problems [5]. It is empowered by two powerful operations like reflection and rotation. A rotor is responsible for representing a color at a particular pixel. Rotor edge detection based on RGB color space replaces traditional edge detection techniques effectively [9]. An edge detection algorithm was introduced that accepts color value triples as vectors [10]. Clifford algebra framework is used because it extends the traditional convolution and Fourier transform to vector fields.

III. CLIFFORD ALGEBRA

A. Definition

A vector space is $\mathbb{V}^n$ is considered for dimension n. A set of orthonormal basis vectors of $\mathbb{V}^n$ is $\{e_1, e_2 \ldots . e_n\}$. A new element can be obtained from the geometric product of the basis elements. Assuming any two basis vectors e_i and e_j, the geometric product $e_i e_j$ is formed and it is anticommutative. Mathematically, it is formulated as, $e_{ij} = e_i e_j = -e_j e_i = -e_{ji}$, $\forall\, i \neq j$

Squaring on the basis vectors results +1, -1 or 0. It suggests that there are nonnegative integers p, q and r such that $n = p + q + r$ and

$$e_i e_i = e_i^{\,2} = \begin{cases} +1\ for\ i = 1,2,\ldots\ldots, p-1, p \\ -1\ for\ i = p+1,\ldots\ldots, p+q \\ 0\ for\ i = p+q+r,\ldots\ldots, n \end{cases}$$

These operations give proof of the associativity of linear algebra with identity to define Clifford Algebra (Cl_n) of dimension $n = p + q + r$ generated by the vector space $\mathbb{V}^n$.

The elements belong to the Clifford Algebra or Geometric Algebra is called multivectors. The space which contains all the n dimensional multivectors is called the n dimensional Clifford Algebra (Cl_n). A multivector is generated as an entity containing several grades of elements of the basis set of Cl_n. Such as,

$$A = \langle A_0 \rangle + \langle A_1 \rangle + \langle A_2 \rangle + \cdots .. + \langle A_n \rangle \qquad (1)$$

where the multivectors $A \in Cl_n$ is expressed as the summation of $\langle A_0 \rangle$ as 0-vector (scalar), $\langle A_1 \rangle$ as 1-vector (vector), $\langle A_2 \rangle$ as 2-vector (bivector), $\langle A_3 \rangle$ as 3-vector (trivector) and so on up to $\langle A_n \rangle$ n-vector (pseudoscalar).

B. Two dimensional Approach

Vectors are the elements of the n dimensional vector space $\mathbb{V}^n$. In Clifford Algebra, the higher dimensional oriented subspaces are the basic elements of computation [12]. Cl_n is represented as an extension of the n dimensional Euclidean vector space $\mathbb{R}$. For n = 2, the space is bounded in a plane $\mathbb{R} \times \mathbb{R}$ into linear space $\mathbb{R}^2$.

Considering an orthonormal basis (e_1,e_2) of $\mathbb{R}^2$ having the characteristics of these basis are ${e_1}^2 = {e_2}^2 = 1$ and $e_{12}= -e_{21}$.

Assuming two generic vectors a and b of $\mathbb{R}^2$ are expressed as linear combinations of the basis elements: $a = a_1e_1 + a_2e_2$ and $b= b_1e_1 + b_2e_2$. After multiplication of a and b, the Clifford product will be

$$\begin{aligned} ab &= (a_1e_1 + a_2e_2)(b_1e_1 + b_2e_2) \\ &= (a_1b_1 + a_2b_2) + (a_1b_2 - a_2b_1)\, e_{12} \end{aligned} \qquad (2)$$

The product is the summation of a scalar and a bivector. A bivector represents the oriented plane generated by two vectors a and b. The Clifford product of two vectors of $\mathbb{R}^2$ is formulated as the summation of scalar product (dot product) and wedge product.

$$ab = a.b + a \wedge b \qquad (3)$$

Comparing (2) and (3), it can be concluded

$$a.b = a_1b_1 + a_2b_2$$

$$a \wedge b = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} e_{12}$$

The wedge product is considered as the outer product and it is anticommutative, $a \wedge b = -b \wedge a$. Whereas the scalar product is commutative, $a.b = b.a$. In other form (2) can be written as, $ab = \frac{1}{2}(ab + ba) + \frac{1}{2}(ab - ba)$. So,

$$a.b = \frac{1}{2}(ab + ba) \text{ and } a \wedge b = \frac{1}{2}(ab - ba).$$

If $a \parallel b$ then $ab = ba$, that suggest $a \wedge b = 0$ and when $a \perp b$ then $ab = -ba$, suggesting $a.b = 0$.

C. Three dimensional Approach

For $n = 3$, the Euclidean vector space is bounded into a linear space $\mathbb{R}^3$. It has an orthonormal basis consisting of three orthogonal unit vectors (e_1,e_2,e_3). Considering two generic vectors a and b of $\mathbb{R}^3$ can be expressed as the linear combinations of the basis elements: $a = a_1e_1 + a_2e_2 + a_3e_3$ and b= $b_1e_1 + b_2e_2 + b_3e_3$. For Cl_3, the Clifford product between these two vectors a and b is

$$\begin{aligned} ab &= (a_1e_1 + a_2e_2 + a_3e_3)(b_1e_1 + b_2e_2 + b_3e_3) \\ &= \textstyle\sum_{i=1}^{3} a_ib_i + \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} e_{12} + \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} e_{13} + \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} e_{23} \end{aligned} \qquad (4)$$

It follows the same characteristics of the orthonormal basis, ${e_1}^2 = {e_2}^2 = {e_3}^2 = 1$ and $e_{12}= -e_{21}, e_{13}= -e_{31}$, $e_{23}= -e_{32}$. If we consider three generic vectors a, b, c that results an entry of new subspace as an element trivector in the Clifford product. A Clifford Algebra space (Cl_3) consists of all the subspaces scalar, vector, bivector and trivector. Generally, e_{123} represents the oriented volume element in $\mathbb{R}^3$ and it is called trivector.

D. Multivector Approach in 2D and 3D

A multivector over Clifford Algebra space Cl_n is defined as the collection of heterogeneous elements. The elements of a multivector in the Cl_3 are the oriented subspaces like scalar, vector, bivector and trivector. The general concept for n-dimensional space 2^n number of elements or subspaces are possible. The generic element or multivector is a linear combination with real coefficients of the 2^n basis elements. In case of $n = 3$, Cl_3 space consists of 2^3 elements.

A multivector contains the different subspaces in it. Each of these subspaces is called blade. For a Euclidean vector space $\mathbb{R}^2$ the orthonormal basis is (e_1,e_2). The elements are 1 (scalar), e_1,e_2 (vector), e_{12} (bivector) form a basis $(1, e_1, e_2, e_{12})$ for the Clifford Algebra space Cl_2 of $\mathbb{R}^2$. Therefore the generic multivector for Cl_2 is

$$a = a_0 + a_1e_1 + a_2e_2 + a_{12}e_{12} \qquad (5)$$

According to (1) the above said (5) can be written as

$$a = \langle a_0 \rangle + \langle a_1 \rangle + \langle a_2 \rangle \qquad (6)$$

Equation (5) represents an element obtained by containing basis vectors of $\mathbb{R}^2$ and it is a multivector in Cl_2.

Similarly for Clifford Algebra space Cl_3, the generic multivector is written as

$$\begin{aligned} a = a_0 + \sum_{i=1}^{3} a_ie_i + a_{12}e_{12} + a_{13}e_{13} \\ + a_{23}e_{23} + a_{123}e_{123} \end{aligned} \qquad (7)$$

For Cl_3, (1) is now expressed as

$$a = \langle a_0 \rangle + \langle a_1 \rangle + \langle a_2 \rangle + \langle a_3 \rangle \qquad (8)$$

IV. GEOMETRIC TRANSFORMATIONS – REFLECTION & ROTATION

The reason behind Clifford Algebra's acceptance is due to its operators' representational power. The geometric transformation like reflection and rotation in

Clifford Algebra space can be expressed as algebraic operators.

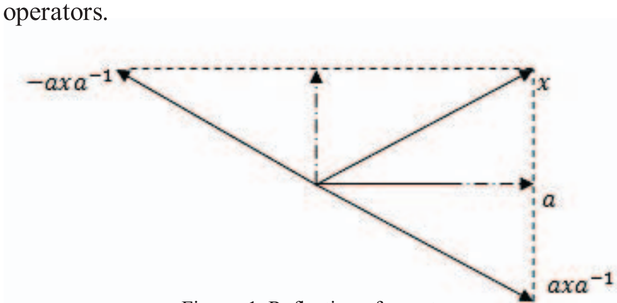

Figure. 1. Reflection of a vector.

The reflected component of vector x with respect to a fixed vector a is axa^{-1} shown in Figure 1. When a plane is perpendicular to a then the reflected component will be $-axa^{-1}$. In simpler form a blade X is reflected with respect to the vector a. It forms a vector X' by using a formula $X' \mapsto aXa^{-1}$. X' is the reflection of X in vector a.

Rotation is represented as the pair of reflections. Two successive reflection of a vector makes that vector rotated at a certain angle. In Figure 2, a and b are the unit vectors separated by an angle φ. We wish to rotate a vector x to $x"$. The vector x makes an angle θ_1 with a where $\theta_1 < \varphi$. After reflection through angle θ_1, x becomes x'.

Now, for second reflection, the angle between vector x' to vector b is $(\varphi + \theta_1)$. Therefore after reflection through $(\varphi + \theta_1)$, the vector x' becomes the rotated vector $x"$. Mathematically, the two reflections are described by, $x' = axa^{-1}$ and $x" = bx'b^{-1}$.

In simpler form, the rotated vector

$$x" = b(axa^{-1})\, b^{-1}$$
$$= b(a^{-1}xa)b^{-1} = (ba^{-1})x(ab^{-1})$$
$$= RxR^{-1} \text{ where } RR^{-1} = 1.$$

Here R suggests a multivector and it is also known as Rotational operator or Rotor. If we want to rotate by an angle φ around the origin, then $R = \cos\varphi + I\sin\varphi$. I is the pseudoscalar for the corresponding Clifford Algebra space.

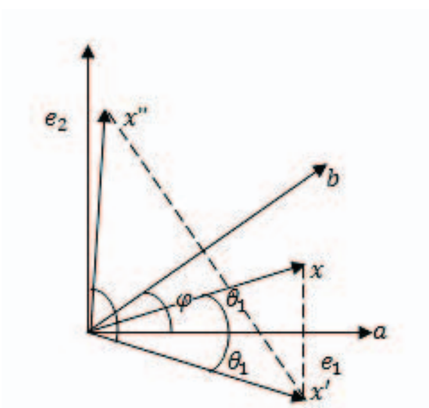

Figure. 2. Rotation of a vector.

It can be written as $R = e^{I\varphi}$ and $R^{-1} = e^{-I\varphi}$. The general form of rotation in the higher dimensional subspaces, a rotor is used to an arbitrary blade through this formula $X \mapsto RXR^{-1}$.

V. IMAGE REPRESENTATION USING VECTOR

The RGB color model reflects a 3D Euclidean space. Each pixel in an image is represented by a triplet that is Red, Green and Blue component. Each of these colors reacts as a vector where x axis suggests Red component, y axis Green and z axis Blue respectively.

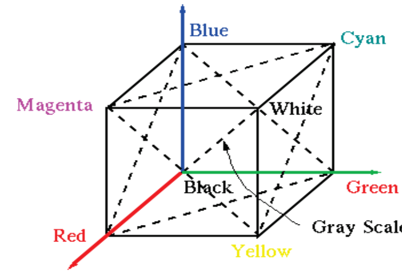

Figure. 3. RGB Color Model

It is observed that eight different colors is possible to draw against RGB component shown in Figure 3. According to Venn diagram in Figure 4 of the RGB color model, the colors are Black, Red, Green, Blue, Yellow, Magenta, Cyan and White. Each of the intersection of R, G and B is forming Cyan, Magenta and Yellow. At the middle of all intersection is showing White. The space beyond Red, Green and Blue set is considered as Black.

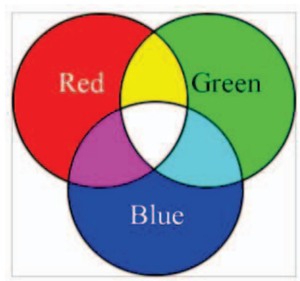

Figure. 4. Venn Diagram representation of RGB Color Model

Every pixel of an image is represented by a 3D Euclidean color space. In $\mathbb{R}^3$ the orthonormal basis are e_1, e_2 and e_3 representing Red, Green and Blue components respectively. So, the real value of RGB will be associated with this orthonormal basis to form a homogeneous vector. Say, at pixel (x, y) the vector

$$col(x,y) = r(x,y)e_1 + g(x,y)e_2 + b(x,y)e_3 \quad (9)$$

where $r(x,y)$, $g(x,y)$ and $b(x,y)$ are RGB real value at pixel (x, y). Therefore, for an image having $M \times N$ pixels the function will be

$$col^{MN}() = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} r(x,y)e_1 + g(x,y)e_2 + b(x,y)e_3 \quad (10)$$

According to (9) there will be $M \times N$ number of homogenous vectors for each of the pixels in an image.

The bivector basis is used to represent a color for $\mathbb{R}^3$ against a pixel [7]. The color vector is written as

$$col_{m,n} = r_{m,n}e_{23} + g_{m,n}e_{31} + b_{m,n}e_{12}.$$

It is observed that the color of a pixel is being represented by pure quaternion [6]. The above function follows quaternionic concept because each of the $e_{ij}{}^2 = -1$.

VI. IMAGE REPRESENTATION USING MULTIVECTOR

In the previous section an image is expressed as homogenous vector where each of the basis vectors is representing R, G and B color axis. For a pixel (x, y) in an image (9) is rewritten as;

$$col(x,y) = r(x,y)e_r + g(x,y)e_g + b(x,y)e_b \quad (11)$$

where e_r, e_g & e_b are the basis vector of Red, Green and Blue component respectively. If we look at RGB color model Figure 3, eight different colors are observed at the corner. And each of these unique colors is forming by the unique combination of Red, Green & Blue component. All of these colors are coordinate dependent according to the color model. We know that the value of R, G and B lies within 0 to 255. A coordinate at any point in RGB color model defines a color. For example, color at (0, 0, 0) is Black, at (255, 255, 0) is Yellow or at (255, 0, 0) is Red. A coordinate in RGB color model is a color. At any pixel (x, y) of an image is defined by (11) as positional value of $r(x,y)$, $g(x,y)$ and $b(x,y)$ in color model. A homogeneous vector is responsible to define a color in this context.

It is declared that the color model is providing 3D Euclidean space. Color spaces in RGB model are the oriented subspaces in 3D Clifford Algebra (Cl_3). For Cl_3 space there would be 2^3(eight) numbers of subspaces possible. Therefore it assigns Black as scalar, (Red, Green, Blue) as vector, (Cyan, Magenta, Yellow) as bivector and White as trivector. The reason behind the bivector is, Cyan is being formed by Blue and Green component with their highest magnitude. That suggests Cyan lies in the plane formed by Blue and Green vectors. In Cl_3, the plane formed by any of the vectors is bivector. Similarly, Magenta and Yellow are the bivector against Red-Blue and Red-Green respectively. Black is not carrying any of the components of RGB treated as scalar. White is the volume element in Cl_3 as each of the vectors is associated with their highest magnitude, so white is treated as the trivector. As Clifford Algebra has coordinate free characteristics, we tried to represent each color by using a multivector. One of the subspace among all the subspaces in the multivector is responsible to define a color. In our proposal, RGB color model is replaced by Cl_3 space because it consists of scalar, vector, bivector and trivector instead of homogeneous vector. To represent a single pixel, a multivector is introduced in which all possible grades of dimension exist. Say, at pixel (x, y) the multivector:

$$col(x,y) = P + r(x,y)e_r + g(x,y)e_g + b(x,y)e_b + \alpha e_{rg} + \beta e_{gb} + \gamma e_{rb} + Q e_{rgb} \quad (12)$$

where P is Black in which no RGB component exists that is no vector component is required to define this color. Q is White consists of highest magnitude of Red, Green & Blue defined as the volume element e_{rgb}. And α, β, γ are the real values associated with the bivectors. e_{rg}, e_{gb} & e_{rb} bivectors represent the plane Red-Green, Green-Blue and Red-Blue respectively. So, it can be said that a color is represented as a wedge product. A color is a Grade - 1 multivector when e_r, e_g & e_b is not null and associated real values are lesser than maximum but not zero. But it is also true that when single component is missing then bivector represents a color. Say, at any pixel (x, y) where there is no blue component, $\alpha e_{rg} = r(x,y)e_r \wedge g(x,y)e_g$. If a pixel (x, y) gives (255,255,0) then α reflects Yellow color and e_{rg} suggests the subspace in which Yellow color belongs. It is also possible for Cyan & Magenta. To represent White color, a trivector is needed because it consists of RGB components in highest magnitude at pixel (x, y), so

$$Qe_{rgb} = r(x,y)e_r \wedge g(x,y)e_g \wedge b(x,y)e_b.$$

This function proposed in (12) is applicable for all the 2^{24} colors possible for this color model.

Therefore, for an image having $M \times N$ pixels the function will be

$$col^{MN}(\ ) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} P + r(x,y)e_r + g(x,y)e_g + b(x,y)e_b + \alpha e_{rg} + \beta e_{gb} + \gamma e_{rb} + Q e_{rgb} \quad (13)$$

It suggests the general form of a function in form of multivector in Cl_3 space. The above (13) reflects a multivector which is responsible to represent an image pixel by pixel. It is obvious different shades of Red, Green & Blue value comes in an image. In those shades it is possible that at any pixel (x, y) may not contain any RGB component. So, in that case the function $col(x, y) = P$, that is the color is black. Another case may occur where no Red component is present then $col(x, y) = \beta e_{gb}$. It is observed that the function (12) can be represented minimum by one term and maximum by three terms. From Clifford Algebra perspective the multivector behaves like any of the Grade - k vector. In Cl_3 space say for Black Grade - 0 (scalar), for Yellow Grade - 2 (bivector) is the behavior of multivector. So, according to the proposal under Cl_3 space the color blades are forming. These color blades are responsible for representing a color.

A color blade is actually the subspace in Cl_3. The function in (13) shows how an image is completely defined by a multivector. In contrast with the quaternion approach [7], this paper does not limit the colors among the bivector basis. Every subspace will be responsible to define a color not only the bivectors. For every different color against each pixel there must have different bivector definitions. In many cases, a color is simply defined by a vector implies avoidance to represent that color by bivector. Higher dimensional subspaces rotations are always complicated. To avoid that complication and reduce the computational time it is necessary to minimize the number of bivectors representing an image. That's why this paper defines a color of a pixel as Grade - k vector.

Section IV highlighted about the theory of reflection and rotation. It is observed how the successive reflection makes a vector rotated. The rotation operator has the powerful application in representing an image by multivector. A color blade in a multivector is representing a color. In our approach we mentioned that every color exists in Cl_3 space in form of color blade. If a color blade is rotated for certain angle then it is obvious a new color will form. And definitely that color will belong in Cl_3 space in form of color blade with same or different grade.

The rotation operation performs on four types of grades in Cl_3 space. Considering scalar (Grade – 0 vector) element representing black color. The rotation cannot be performed as it doesn't contain any basis element. A vector is used to rotate on a plane results a rotated vector on that plane. The nature of the rotated vector may vary from its originality. Similarly, when a vector is rotating in a space it is obvious that its rotated vector is closed into that space. Therefore, a color with Grade – 1 vector after rotation remains in its own Clifford Algebra space. But the nature of the color may change. Angle of rotation plays the key role for changing the color's nature. The rotor operator discussed helps to evaluate the rotated vector. Say, a color vector x rotates an angle φ, mathematically the resulting rotated color vector $x^{rot} = e^{-I\varphi/2} x e^{I\varphi/2}$ where $I = e_{rgb}$ in Cl_3 space. Considering a plane (Grade – 2 vector) is rotated into a space where the plane after rotation is also belonging into that space. Therefore a color defined by a bivector in Cl_3 space after rotation defines a new color with same or different grade. Say, a bivector e_{rg} is rotated at an angle $\pi/2$ in Cl_3 space. After rotation the bivector transformed into e_{gb} by using the formula $X \mapsto RXR^{-1}$. It suggests a plane with Red & Green component after rotation is forming a plane (or bivector) containing Green & Blue component. It proves that the nature of the color changes with rotation and it is closed into the space. A plane is rotated by the help of rotor R to evaluate the nature of rotated plane [11]. When a volume element is rotated as the highest dimension subspace in Cl_3 space then the whole space will rotate and it doesn't change into a new volume element. So, according to the proposal the volume element (Grade – 3 vector) is white color. After rotation this color blade remains its color property intact.

This paper introduces Clifford color space as it holds the multivector in it. A Clifford color space is a Cl_3 space representing a color as a subspace against each of the pixel of an image. Therefore, it can be said that an image is a Clifford cube in precise form. All the information of an image following RGB color model is stored in this cube in form of a multivector. In generalized form, for $M \times N$ pixels of an image, it is defined as the summation of $M \times N$ number of Grade – k vectors.

VII. CONCLUSION

In our study we observed that a color image is represented by the multivector. We introduced the concept of color blade where each and every blade in the multivector is representing a color. Clifford Algebra is the only coordinate free mathematical tool that helps us to think differently. With contrast to previous research works in this context, we approached uniquely to use every subspaces of 3D Clifford Algebra. Our proposal suggests that image representation will be much easier and more effective. An edge detection algorithm uses rotor operator and each color is represented in form of bivector basis. It will take more time to compute for $M \times N$ bivectors. According to our proposal the computation time can be reduced as all the colors are categorized into different color blades. We introduced Clifford color space where every color of RGB color model exists. An image is mapped in a Clifford cube for easy implementation. A simple multivector function generated from Clifford cube is sufficient to represent an image. Our focus is only on the RGB color model that is additive color model. There are several color models discussed earlier is not our point of interest.

ACKNOWLEDGMENT

The authors extend sincere thanks to the Department of Computer Science and Engineering, University of Calcutta West Bengal, India and Academy Of Technology, Hooghly, West Bengal, India for using the infrastructure facilities for developing the technique.

REFERENCES

[1] A.K.Ray, Kishor M. Bhurchandi, P.M.Nawghare "*An Analytical Approach for Sampling the RGB Color Space Considering Physiological Limitations of Human Vision and its Application for Color Image Analysis*", Indian Conference on Computer Vision, Graphics and Image Processing, December 20-22, 2000 Bangalore pp 44-49

[2] Mann Stephen, Nathan Litke, and Tony DeRose. "*A Coordinate Free Geometry ADT*", Research Report CS-97-15, University of Waterloo, 1997

[3] Patrice Denis and Philippe Carré. "Colour *gradient using geometric algebra*" EUSIPCO2007, 15th European Signal Processing Conference. 2007.

[4] J. Angulo. "*Geometric algebra colour image representations and derived total orderings for morphological operators - Part I: Colour Quaternions*". Journal of Visual Communication and Image Representation, Vol. 21, 33--48, 2010

[5] Valeriy Labunets, "*Clifford Algebras as Unified Language for Image Processing and Pattern Recognition*". Computational Noncommutative Algebra and Applications, NATO Science Series II: Mathematics, Physics and Chemistry Volume 136, 2005, pp 197-225

[6] S.J.Sangwine and T.A.EII, "*Colour image filters based on hypercomplex convolution*", Vision, Image and Signal Processing, IEE Proceedings - Volume:147 , Issue: 2 , pp. 89 – 93, Apr 2000

[7] Biswajit Mishra, Peter Wilson and Bashir Al-Hashimi, "*Advancement in Color Image Processing using Geometric Algebra*". In, *EUSIPCO* (2008)

[8] Wenming Cao, Tiancheng He, "*An Algorithm for Removing Facial Makeup Disturbances Based on Clifford Algebras*", International Conference on Computational Intelligence and Security Workshops, 2007. CISW 2007., pp. 275 – 278,2007

[9] B.Mishra and P.Wilson, "*Color Edge Detection Hardware based on Geometric Algebra*," in 3rd European Conference on Visual Media Production CVMP 2006. IET, London, 2006, pp. 115–121.

[10] Silvia Franchini, Antonio Gentile, Filippo Sorbello, Giorgio Vassallo, Salvatore Vitabile, "*Clifford Algebra based Edge Detector for Color Images*" Sixth International Conference on Complex, Intelligent and Software Intensive Systems (CISIS), 2012, pp. 84 – 91

[11] Leo Dorst and Stephen Mann, "*Geometric algebra: a computational framework for geometrical applications (part II: applications)*" Computer Graphics and Applications, IEEE (Volume:22 , Issue: 4), pp. 58 -67

[12] Leo Dorst and Stephen Mann. "*Geometric algebra: A computational framework for geometrical applications (part I)*".Computer Graphics and Applications, IEEE (Volume: 22, Issue: 3), pp. 24-31,2002.